

# Porter-Thomas distribution in unstable many-body systems

Alexander Volya

*Department of Physics, Florida State University, Tallahassee, FL 32306-4350, USA*

(Dated: November 25, 2010)

We use the continuum shell model approach to explore the resonance width distribution in unstable many-body systems. The single-particle nature of a decay, the few-body character of the interaction Hamiltonian, and collectivity that emerges in non-stationary systems due to the coupling to the continuum of reaction states are discussed. Correlations between structures of the parent and daughter nuclear systems in the common Fock space are found to result in deviations of decay width statistics from the Porter-Thomas distribution.

PACS numbers: 24.60.Lz, 24.10.Cn 24.60.Dr

The Porter-Thomas distribution (PTD) [1] of transition strengths is a central aspect of complex systems. This statistical law was noted by many authors [2] to be valid more generally than other predictions of the Random Matrix Theory from which it originates. The PTD emerges under the assumption that the relative orientation of the two states involved in the overlap describing a transition covers the  $\Omega$ -dimensional sphere in the Hilbert space uniformly. While this is true by definition for the Gaussian Orthogonal Ensemble (GOE), the validity of the PTD extends much farther as it constitutes the central limit theorem (CLT). Being a sum of a large number of uncorrelated components, the transitional amplitude is indeed expected to have a Gaussian distribution. There is a large volume of work on this subject; see reviews [2–6] and references therein. Generally, there is a consensus among authors that while the specifics of an ensemble and the physics of transitions do matter for certain observables, any deviations from the PTD are quickly defeated by even small stochastic components due to the robust nature of the CLT; see for example Ref. [7]. Any claims to the contrary, either experimental [8] or theoretical [9, 10], have always ignited debates and discussions [11].

Here we will not consider the data handling procedures, which on many occasions, have been deemed to be the most likely reasons for the deviations observed experimentally [6]. Instead, we focus on the possible reasons for high correlation between the transitioning states from the theoretical perspective. We analyze the feasible scenarios with the help of the continuum shell model approach [12, 13] which is one of the most equipped methods to address the structure-reaction physics of interest microscopically. In a unified picture we review the super-radiance (SR) effects [9, 14], the wave-function localization effects in the two-, three-, and four-body ensembles [15], the role of rotational symmetry, and other parent-daughter structural correlations that emerge in a decay [7].

The dynamics of an unstable many-body system projected onto the intrinsic space spanned by the bound (shell-model) states is generated by the effective, energy-

dependent Hamiltonian [12, 16, 17]

$$\mathcal{H} = H - i \sum_{c(\text{open})} \wp^c |c\rangle\langle c|. \quad (1)$$

Here  $H$  is the Hermitian part that is identified with the traditional shell model Hamiltonian. The second, imaginary term reflects the irreversible decays into the continuum of states excluded by the Feshbach projection. This factorized operator contains the kinematic penetrability factor  $\wp^c$  and the set of channel vectors  $|c\rangle$ . For simplicity we omit the angular momentum, isospin, and other labelings; detailed notations are found in Ref. [12]. The problem is non-stationary; the Hamiltonian (1) is understood as a component of the propagator and is dependent on the scattering energy. The Hermitian component includes the coupling to the continuum of reaction states via virtual excitations; the penetrability also depends on energy through the kinematics of the decay process. Away from thresholds the energy dependence is smooth, and its exact form mainly pertains questions of the experimental data analysis. We ignore this dependence here, and further assume that the penetrability is the same for all channels. The eigenvalues of the effective Hamiltonian (1) are complex,  $\mathcal{E} = E - i\Gamma/2$ , and represent the poles of the scattering matrix in the complex energy plane. These complex energies are associated with resonances and their widths.

Let us first consider weak decays, for which the imaginary component in (1) can be treated perturbatively. In this case the shell model eigenstate  $|I\rangle$  defined by  $H|I\rangle = E_I|I\rangle$  is not modified by the decay instability, and the corresponding decay width is

$$\Gamma_I = 2\wp \gamma_I, \quad \text{where} \quad \gamma_I = \sum_{c(\text{open})} |\langle I|c\rangle|^2$$

is the reduced width. The PTD of reduced widths

$$P_\nu(\gamma) = \frac{1}{\gamma} \left( \frac{\nu\gamma}{2\bar{\gamma}} \right)^{\nu/2} \frac{1}{\Gamma(\nu/2)} \exp\left(-\frac{\nu\gamma}{2\bar{\gamma}}\right) \quad (2)$$

emerges under the uniform Hilbert space coverage assumption for  $|I\rangle$ . Here  $\nu$  is the dimension spanned by the channel vectors, and  $\bar{\gamma}$  is the average reduced width. For the orthogonal and normalized channels  $\bar{\gamma} = \nu/\Omega$ .

In this work we examine situations with only one open channel. The strength of the continuum coupling is defined via the average decay width relative to the level spacing. Here we express this coupling using the parameter  $\kappa = \varphi/\lambda$ , where  $\lambda^2 = \Omega^{-1}\text{Tr}(H^2)$  is the variance of the density of states distribution of  $H$ .

In Fig. 1 we consider an example of GOE+SR, where  $H$  in (1) is represented by the GOE. Here the PTD is reproduced numerically in the limit  $\varphi \rightarrow 0$ . The density of states in this limit has a semicircular distribution bound by a radius  $2\lambda$ . The imaginary component in (1) is factorized, which is pertinent to the unitarity of the scattering matrix. The non-Hermitian component, when large, gives rise to the collectivity often referred to as superradiance (SR). A similar collectivity due to the factorized Hermitian interaction describes giant resonances. The resulting deformed random ensembles are discussed in Refs. [2, 18, 19]. As coupling to the continuum increases, and the average decay width relative to the level spacing  $\kappa$  becomes large, the resonances start to overlap, thus reorienting the intrinsic structure. This could be hypothesized to result in the PTD being violated [9]. An in-depth examination, however, shows that the SR mechanism alone is unlikely to cause a significant change to the PTD. Indeed, for  $\kappa \ll 1$ , the PTD simply follows from the definition of GOE. For  $\kappa \gg 1$ , in full mathematical equivalence to the deformed ensembles, the Hilbert space is separated into the SR channel space, which is one-dimensional here with a single eigenstate  $\mathcal{E} = -i\varphi$ , and the orthogonal statistical (compound resonance) space of dimension  $\Omega - 1$ . Because  $H$  is orthogonally invariant the reduced-space dynamics is represented by the GOE. With the perturbation theory built in this limit one finds that the reduced widths for the compound resonances follow the PTD with  $\bar{\gamma} = 1/(\Omega\kappa^2)$ . For large  $\Omega$  the single SR state with a reduced width  $\gamma_{SR} = 1 - \kappa^{-2}$  has no effect on the PTD. As seen in Fig. 1, numerical study confirms the PTD for both small and large values of  $\kappa$ . Moreover, the slight deviation from the PTD for couplings  $\kappa$  between around 0.4 and 2 is due to a small fraction of very broad states with  $\gamma > 10\bar{\gamma}$ . This localized effect is shown in the inset of Fig. 1. This subset of an exceptionally broad states is difficult to identify experimentally. Investigations [2, 18, 19] of deformed ensembles further confirmed a good agreement of the decay width distribution with the PTD and of the density of states distribution with the semicircular shape.

Experience with the realistic nuclear structure and some theoretical arguments [2–6] suggest that the effective Hamiltonian involves only few-nucleon interactions, thus, the two-body random ensembles (TBRE) appear to be more appropriate. Many features of these ensembles are different from those of GOE, nevertheless the eigenvectors form a uniform coverage of the Hilbert space. Numerical studies confirm that, in agreement with the CLT, this leads to the PTD of transition strengths toward an uncorrelated channel vector [7]. This logic, however, does not take into account the correlations that ex-

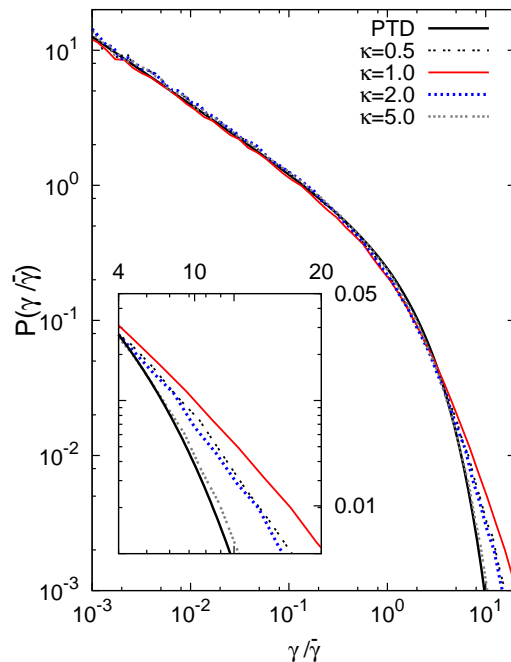


Figure 1: (Color online) The normalized distribution of probabilities of the reduced widths for the GOE+SR ensemble with  $\Omega = 10^4$ . The curves for different continuum couplings  $\kappa = 0.5, 1, 2$ , and  $5$  are compared with the PTD. The corresponding average widths  $\Omega\bar{\gamma} = 0.76, 0.40, 0.20$ , and  $0.05$ , are computed after the very broad state(s) are removed from consideration. For  $\kappa > 2$  this amounts to exclusion of a single superradiant state in which case  $\Omega\bar{\gamma} = \kappa^{-2}$ . With the exception of very broad states, there is perfect agreement with PTD, the differences being magnified in the inset where the region of  $\gamma > 4\bar{\gamma}$  is shown. The largest deviation is observed for the transitional coupling strength  $\kappa = 1$ . For  $\kappa = 5$  the distribution is already indistinguishable from the PTD.

ist in the variable particle-number Fock space. Strong parent-daughter correlations emerge due to the microscopic physics of decay. Indeed, in the single-particle reaction processes all nucleons, except for one, are spectators, and the decay channels for the  $N$ -particle system  $|c; N\rangle$  are built from the  $(N - 1)$ -particle eigenstates of the daughter nucleus  $|F; N - 1\rangle$  that follow from the same two-body Hamiltonian. Thus,  $|c; N\rangle = \{a^\dagger|F; N - 1\rangle\}$ , where  $a^\dagger$  is a single-particle creation operator corresponding to the decaying nucleon, and brackets  $\{\dots\}$  indicate normalization to unity and appropriate symmetry coupling. The correlation between eigenstates and channels leads to the violation of PTD. Single-particle removal amplitudes are related to the independent-particle basis, where departure from the PTD has been demonstrated in the past [10].

In Fig. 2 the EGOE+SR ensemble is considered, where  $H$  in Eq. (1) is given by an embedded two-body GOE (EGOE), and by definition does not include any symmetries. The previous example shows that it is important to draw attention to the statistics of the nar-

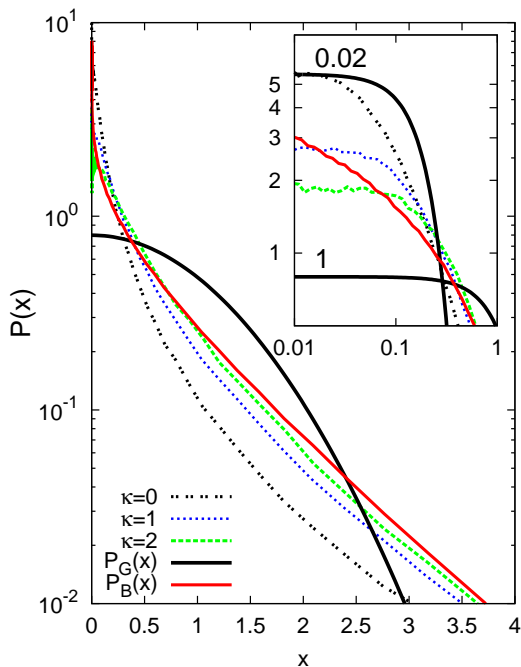


Figure 2: (Color online) The normalized distribution of probabilities of decay amplitudes in the EGOE+SR ensemble. The decay of a 7-particle system to the ground state of a 6-particle system is considered. There are 16 single-particle states, thus  $\Omega = 11440$ . The curves for different continuum couplings  $\kappa = 0, 1, \text{ and } 2$  are compared with the Gaussian and Bessel distributions. The curves are normalized so that the average widths do not include the single SR state.  $\Omega\bar{\gamma} = 1, 0.19, \text{ and } 0.06$  for  $\kappa = 0, 1 \text{ and } 2$  respectively. The region of very narrow widths is shown in the inset using a log-log scale. While all observed distributions for very narrow states seem to approach a constant, they are still not described by Gaussian distribution of different variances (or  $\bar{\gamma}$ 's). The inset includes two Gaussian curves with variances 1 and 0.02 as labeled.

row states. In this work we do not present our fits of the observed distributions to the PTD (2) treating  $\nu$  as a parameter. We found that attempts to do so could produce misleading results. The effective  $\nu$  is directly related to the normalization  $\bar{\gamma}$  which is disproportionately influenced by statistically unimportant collective state(s). In an attempt to avoid any potential misjudgments, here we show the distribution of absolute values of amplitudes  $x = \sqrt{\gamma/\bar{\gamma}}$ . Then the Gaussian distribution  $P_G(x) = \sqrt{2/\pi} \exp(-x^2/2)$  that corresponds to the PTD is easy to separate from a distribution provided by the Bessel function  $P_B(x) = (2/\pi)K_0(x)$ . In contrast to the CLT, the latter distribution emerges when the transitional overlap is possible only due to a single component in the wave-function along some direction in the Hilbert space given by a vector  $|1\rangle$ ; so that  $\langle I|c\rangle = \langle I|1\rangle\langle 1|c\rangle$ , where both  $\langle I|1\rangle$  and  $\langle 1|c\rangle$  are distributed normally (agree with PTD).

From Fig. 2 we find that none of the EGOE+SR results follow the PTD. In contrast to the Gaussian curve

$P_G(x)$  the distributions have sharp peaks at low amplitudes and an extended exponential tail. In the SR limit of large  $\kappa$  the distributions appear to approach the one given by the Bessel function  $P_B(x)$ .

The rank of the force beyond the two-body interaction, and symmetries, such as rotational, may have additional influences. To examine this we consider a model where  $N$  identical fermions occupy a single- $j$  level. This has been a popular model for exploring the properties of TBRE [3, 4, 20]. The resemblance of the low-lying spectra to those observed in realistic nuclei is the most intriguing feature. For our demonstration we select  $j = 19/2$  and discuss widths of the decay of many-body states in 9-particle systems. The final state is the ground state of the system with 8 nucleons. All states, in both parent and daughter nuclei, are eigenstates of the same Hamiltonian given by the  $n$ -body Random Ensemble ( $n$ -BRE) [21]. In this work we restrict our consideration to the two-, three-, and four-body forces,  $n = 2, 3, \text{ and } 4$ , respectively. We select only those realizations where the daughter system has ground state spin  $F = 0$ ; and thus the channel spin is  $I = 19/2$ . The fractions of such realizations are 42%, 64%, and 83% for  $n = 2, 3, \text{ and } 4$  respectively.

The resemblance between random ensembles with symmetries and realistic nuclei extends to parentage relations. The low-lying states in the odd-particle parent nucleus are predominantly of the single-particle nature. If  $F = 0$  for the even-particle core then the ground state of a system with an extra nucleon is likely to carry the single-particle quantum numbers  $j = I = 19/2$ . This is indeed observed, and the corresponding probabilities are 21%, 47%, and 37% for  $n = 2, 3, \text{ and } 4$  respectively. The correlation between the parent and daughter ground states is demonstrated in Fig. 3 which shows the distribution of reduced widths for the decay from ground state to ground state when  $F = 0$  and  $I = 19/2$ . Both parent and daughter systems have correlated structures because they are eigenstates of the same Hamiltonian for different number of particles. In the distribution of spectroscopic factors this correlation is seen as a peak near  $\gamma = 1$ . As the rank of interaction  $n$  becomes higher, more remote configurations can be admixed, which reduces the ground state to ground state transitional collectivity. For totally uncorrelated systems the transitional strength due to the CLT is expected to follow the PTD.

Since most of the transitional strength is concentrated in a few states at the low end of the spectrum, here the PTD is not generally expected; however, there is also no agreement with this law for the widths of highly excited compound resonances. In Fig. 4 the distribution of reduced decay amplitudes is shown for the two-, three-, and four-body random ensembles with rotational symmetry. The curves are quite close to each other and are similar to the EGOE result ( $\kappa = 0$  in Fig. 2). All findings indicate violation of the PTD.

This study is motivated by the long-standing debate in relation to the Porter-Thomas distribution and possibility of its violation [7–11]. The PTD is a robust prediction

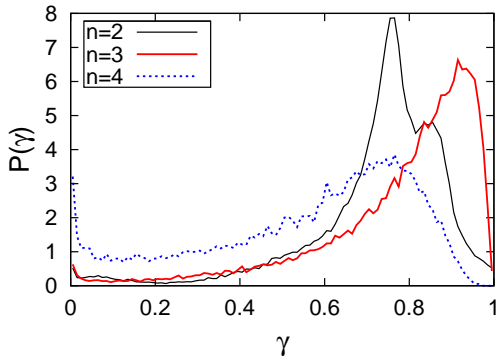


Figure 3: (Color online) Distribution of the reduced decay widths (spectroscopic factors) for the ground state  $N = 9$  spin  $I = 19/2$  parent decaying to the ground state  $N = 8$  spin  $F = 0$  daughter system. The  $n$ -BRE of identical nucleons in  $j = 19/2$  level is considered. Three curves correspond to two-, three-, and four-body ( $n = 2, 3,$  and  $4$ ) random ensembles with rotational symmetry.

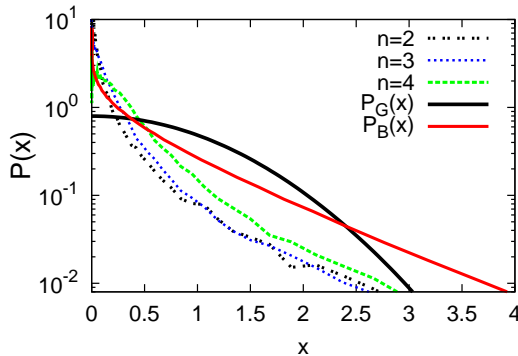


Figure 4: (Color online) Same model as in Fig. 3 The distribution of spectroscopic amplitudes for all 204 states with spin  $I = 19/2$  in the parent 9-nucleon system is shown in the decay to the spin  $F = 0$  ground state of the daughter nucleus. The results are compared with the Gaussian and Bessel distributions. The curves do not change if states only in a certain energy region are considered.

justified by the central limit theorem; it is easily confirmed for different random matrix ensembles. This, however, is a purely structural approach that does not take into account the microscopic physics of reactions. To address this we use a continuum shell model approach where violations of the PTD may result from one or a combination of the following: coherence in structure due to factorized nature of the effective Hamiltonian that reflects unitarity of the scattering matrix, the so called super-radiance mechanism; parent-daughter relation between decaying systems in the common Fock space; few-body low rank interaction forces; and significant variations in the energy dependence of the effective Hamiltonian. We examine all of these possibilities, with the exception of the last, which is to be discussed elsewhere.

In agreement with the studies of deformed ensembles we find that the SR mechanism by itself does not lead to violations of the PTD. However, the distribution, unambiguously different from PTD, is observed in random ensembles with a particle decay and with few-body intrinsic Hamiltonians. This remains true even when subsets of states are considered: restricted by a certain energy region, or by some reasonable limits on the value of the decay width itself. The latter allows for any collective or unmeasurably broad states to be excluded. The parent-daughter relation in the decay process appears to be central for this phenomenon, the results being only slightly influenced by SR, additional rotational symmetry or the rank of forces.

The author is thankful to Kirby Kemper for motivating discussions. Support from the U. S. Department of Energy, grant DE-FG02-92ER40750 is acknowledged. The computing resources were provided by the Florida State University shared High-Performance Computing facility.

- 
- [1] C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).
  - [2] T. A. Brody, J. Flores, J. B. French, P. A. Mello, A. Pandey, and S. S. M. Wong, Rev. Mod. Phys. **53**, 385 (1981).
  - [3] V. Zelevinsky and A. Volya, Phys. Rep. **391**, 311 (2004).
  - [4] T. Papenbrock and H. A. Weidenmüller, Rev. Mod. Phys. **79**, 997 (2007).
  - [5] G. E. Mitchell, A. Richter, and H. A. Weidenmüller, Rev. Mod. Phys. **82** (2010).
  - [6] H. A. Weidenmüller and G. E. Mitchell, Rev. Mod. Phys. **81**, 539 (2009).
  - [7] S. M. Grimes, Phys. Rev. C **28**, 471 (1983).
  - [8] P. E. Koehler, F. Bečvář, M. Krtička, J. A. Harvey, and K. H. Guber, Phys. Rev. Lett. **105**, 072502 (2010).
  - [9] G. L. Celardo, N. Auerbach, F. M. Izrailev, and V. G. Zelevinsky, *Distribution of Resonance Widths and Dynamics of Continuum Coupling*, arXiv:1010.4342v1 [cond-mat.mes-hall].
  - [10] R. R. Whitehead, A. Watt, D. Kelvin, and A. Conkie, Phys. Lett. B **76**, 149 (1978).
  - [11] E. S. Reich, Nature **466**, 1034 (2010).
  - [12] A. Volya, Phys. Rev. C **79**, 044308 (2009).
  - [13] A. Volya and V. Zelevinsky, Phys. Rev. C **67**, 054322 (2003).
  - [14] A. Müller and H. L. Harney, Phys. Rev. C **37**, 2435 (1988).
  - [15] L. Kaplan and T. Papenbrock, Phys. Rev. Lett. **84**, 4553

- (2000).
- [16] H. Feshbach, *Theoretical nuclear physics : nuclear reactions* (Wiley, New York, 1991).
- [17] C. Mahaux and H. A. Weidenmüller, *Shell-model approach to nuclear reactions* (North-Holland Pub. Co., Amsterdam, London, 1969).
- [18] A. Pandey and J. B. French, *J. Phys. A* **12**, L83 (1979).
- [19] V. V. Sokolov and V. G. Zelevinsky, *Phys. Lett. B* **202**, 10 (1988).
- [20] C. W. Johnson, G. F. Bertsch, and D. J. Dean, *Phys. Rev. Lett.* **80**, 2749 (1998).
- [21] A. Volya, *Phys. Rev. Lett.* **100**, 162501 (2008).