N_f=2+1 flavour equation of state

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We conclude our investigation on the QCD equation of state (EoS) with $2+1$ staggered flavors and one-link stout improvement. We extend our previous study [JHEP 0601:089 (2006)] by choosing even finer lattices. These new results [for details see arXiv:1007.2580] support our earlier findings. Lattices with $N_t = 6, 8$ and 10 are used, and the continuum limit is approached by checking the results at $N_t = 12$. A Symanzik improved gauge and a stout-link improved staggered fermion action is taken; the light and strange quark masses are set to their physical values. Various observables are calculated in the temperature (T) interval of 100 to 1000 MeV. We compare our data to the equation of state obtained by the "hotQCD" collaboration.

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Introduction The study of QCD thermodynamics and that of the phase diagram are receiving increasing attention in recent years. A transition occurs in strongly interacting matter from a hadronic, confined system at small temperatures and densities to a phase dominated by colored degrees of freedom at large temperatures or densities. A systematic approach to determine properties of this transition is through lattice QCD. Lattice simulations indicate that the transition at vanishing chemical potential is merely an analytic crossover [1]. This field of physics is particularly appealing because the deconfined phase of QCD can be produced in the laboratory, in the ultrarelativistic heavy ion collision experiments at CERN SPS, RHIC at Brookhaven National Laboratory, ALICE at the LHC and the future FAIR at the GSI. The experimental results available so far show that the hot QCD matter produced experimentally exhibits robust collective flow phenomena, which are well and consistently described by near-ideal relativistic hydrodynamics. These hydrodynamical models need as an input an EoS which relates the local thermodynamic quantities.

Most of the results on the QCD EoS have been obtained using improved staggered fermions. This formulation does not preserve the flavor symmetry of continuum QCD; as a consequence, the spectrum of low lying hadron states is distorted. Recent analyses performed by various collaborations [2, 3] have pointed out that this distortion can have a dramatic impact on the thermodynamic quantities. To quantify this effect, one can compare the low temperature behavior of the observables obtained on the lattice, to the predictions of the Hadron Resonance Gas (HRG) model.

Lattice framework Here we present our results for thermodynamic observables: pressure (p), trace anomaly (I=ε-3p) and speed of sound (c_s), for n_f=2+1 dynamical quarks. We improve our previous findings [4] by choosing finer lattices ($N_t = 8$, 10 and a few checkpoints at $N_t = 12$). We work again with physical light and strange quark masses: we fix them by reproducing the physical ratios f_K/m_π and f_K/m_K and by this procedure [5, 6, 3] we get $m_s/m_{u,d}$=28.15.
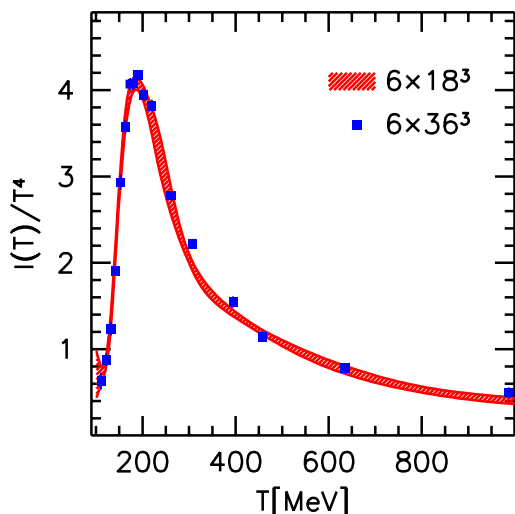
Figure 1: The trace anomaly on lattices with different spatial volumes: $N_s/N_t = 3$ (red band) and $N_s/N_t = 6$ (blue points).
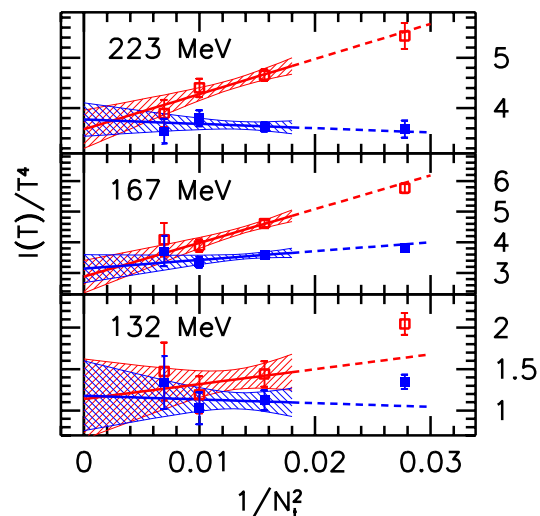
Figure 2: I=ε-3p at three different T-s as a function of $1/N_t^2$. Filled/open symbols represent results with/without tree-level improvement.

We checked that there were no significant finite size effects, by performing two sets of simulations in boxes with a size of 3.5 fm and 7 fm around T_c. Figure 1 shows the comparison between

the two volumes for the normalized trace anomaly I/T^4. Let us note here, that the volume independence in the transition region is an unambiguous evidence for the crossover type of the transition.

To decrease lattice artefacts, we apply tree-level improvement for our thermodynamic observables: we divide the lattice results with the appropriate improvement coefficients. These factors can be calculated analytically for our action and in case of the pressure we have the following values on different N_t's: N_t=6 gives 1.517, N_t=8 gives 1.283, N_t=10 gives 1.159 and N_t=12 gives 1.099. Using thermodynamical relations one can obtain these improvement coefficients for the energy density, trace anomaly and entropy, too. The speed of sound receives no improvement factor at tree level. Note, that these improvement coefficients are exact only at tree-level, thus in the infinitely high temperature, non-interacting case. As we decrease the temperature, corrections to these improvement coefficients appear, which have the form $1 + b_2(T)/N_t^2 + ...$ Empirically one finds that the $b_2(T)$ coefficient, which describes the size of lattice artefacts of the tree-level improved quantities, is tiny not only at very high temperatures, but throughout the deconfined phase. Figure 2 illustrates at three temperature values ($T = 132$, 167 and 223 MeV) the effectiveness of this improvement procedure. We show both the unimproved/improved values of the trace anomaly for N_t=6,8,10 and 12 as a function of $1/N_t^2$. The lines are linear continuum extrapolations using the three smallest a-s. The $a \rightarrow 0$ limit of both the unimproved and the improved observables converge to the same value. The figure confirms the expectations, that lattice tree-level improvement effectively reduces the cutoff effects. At all three T-s the unimproved observables have larger cutoff effects than the improved ones. Actually, all the three values of $b_2(T)$, which indicate the remaining cutoff effects after tree-level improvement, differ from zero by less than one standard deviation.

The most popular technique to determine the EoS is the integral method. For large homogenous sytems p is proportional to the logarithm of the partition function. Its direct determination is difficult. Instead, one determines the partial derivatives with respect to the bare lattice parameters. Finally, p is rewritten as a multidimensional integral along a path in the space of bare parameters. To obtain the EoS for various m_π, we simulate for a wide range of bare parameters on the plane of $m_{u,d}$ and β (m_s is fixed to its physical value). Having obtained this large set of data we generalize the integral method and include all possible integration paths into the analysis [7, 8].

An additive divergence is present in p, which is independent of T. One removes it by subtracting the same observables measured on a lattice, with the same bare parameters but at a different T value. Here we use lattices with a large enough temporal extent, so it can be regarded as $T = 0$.

Hadron Resonance Gas Model The Hadron Resonance Gas model has been widely used to study the hadronic phase of QCD in comparison with lattice data. The low temperature phase is dominated by pions. Goldstone's theorem implies weak interactions between pions at low energies. As the temperature T increases, heavier states become more relevant and need to be taken into account. The HRG model has its roots in the theorem of Ref. [9], which allows to calculate the microcanonical partition function of an interacting system, in the thermodynamic limit $V \rightarrow \infty$, to a good approximation, assuming that it is a gas of non-interacting free hadrons and resonances [10]. Staggered lattice discretization has a considerable impact on the hadron spectrum. In order to investigate these errors, we define a "lattice HRG" model, where in the hadron masses lattice discretization effects are taken into account.

In order to compare the HRG model results with our additional lattice simulations at larger-than-physical quark masses, we need the pion mass dependence (see e.g. [11] for a recent lattice

result) of all hadrons and resonances included in the calculation. As we already did in [3], we assume that all resonances behave as their fundamental state hadrons as functions of the pion mass. For the fundamental hadrons, we use the pion mass dependence from Reference [12]. For larger-than-physical quark masses, the taste symmetry violation at finite lattice spacing (a) has a milder impact on the pressure. This also motivates to take a pion mass of about 700 MeV as the starting point of the integration of the pressure at $T = 100$ MeV.

Results As we will see different sets of data corresponding to different N_t nicely agree with each other for all observables under study: thus, we expect that discretization effects are tiny.

On Figure 3 we show the T dependence of ε-3p for $n_f = 2+1$. We have results at four different "a"-s. Results show essentially no dependence on "a", they all lie on top of each other. Only the coarsest $N_t = 6$ lattice shows some deviation around ~ 300 MeV. On the same figure, we zoom in to the transition region. Here we also show the results from the HRG model: a good agreement with the lattice results is found up to $T \sim 140$ MeV. One characteristic temperature of the crossover transition can be defined as the inflection point of the trace anomaly. This and other characteristic features of the trace anomaly are the following: the inflection point of $I(T)/T^4$ is 152(4) MeV; the maximum value of $I(T)/T^4$ is 4.1(1), whereas T at the maximum of $I(T)/T^4$ is 191(5) MeV.

On Figure 4 we show p(T). We have results at three different "a"-s. The $N_t = 6$ and $N_t = 8$ are in the T range from 100 up to 1000 MeV. On Figure 5 we present the energy density. On Figure 6 c_s^2(T), the speed of sound is shown. One can also read off the characteristic points of this curve: the minimum value of $c_s^2(T)$ is 0.133(5); T at the minimum of $c_s^2(T)$ is 145(5) MeV; whereas ε at the minimum of $c_s^2(T)$ is 0.20(4) GeV/fm^3.

As it was already discussed in Reference [6], there is a disagreement between the results of current large scale thermodynamical calculations. The main difference can be described by a $\sim$ 20-30 MeV shift in the temperature. This means, that the transition temperatures are different: the temperature values that we obtain are smaller by this amount than the values of the "hotQCD" collaboration. References [12] and [3] presented a possible explanation for this problem: the more severe discretization artefacts of the "asqtad" and "p4fat" actions used by the "hotQCD" collaboration lead to larger transition temperatures.

It is interesting to look for this discrepancy in the equation of state as well. On Figure 7 we compare the trace anomaly obtained in this study with the trace anomaly of the "hotQCD" collaboration. We plot the $N_t = 8$ data using the "p4fat" and "asqtad" actions, which we took from References [13] and [14]. As it can be clearly seen, the upward going branch and the peak position are located at ~ 20 MeV higher temperatures in the simulations of the "hotQCD" group. This is the same phenomenon as the one, which was already reported for many other quantities in Reference [6]. We also see, that the peak height is about 50% larger in the "hotQCD" case.

Conclusions We determined the equation of state of QCD by means of lattice simulations. Results for the $n_f = 2+1$ flavor pressure, trace anomaly and for the speed of sound were presented on figures. The results were obtained by carrying out lattice simulations at four different "a"-s, at $N_t = 6, 8, 10$ and 12 in the T range of $T = 100 \ldots 1000$ MeV. In order to reduce the lattice artefacts we applied tree-level improvement for all of the thermodynamical observables. We found that there is no difference in the results at the three finest lattice spacings. This shows that the lattice discretization errors are not significant and the continuum limit can be reliably taken. The details of the present work can be found in Ref. [7]
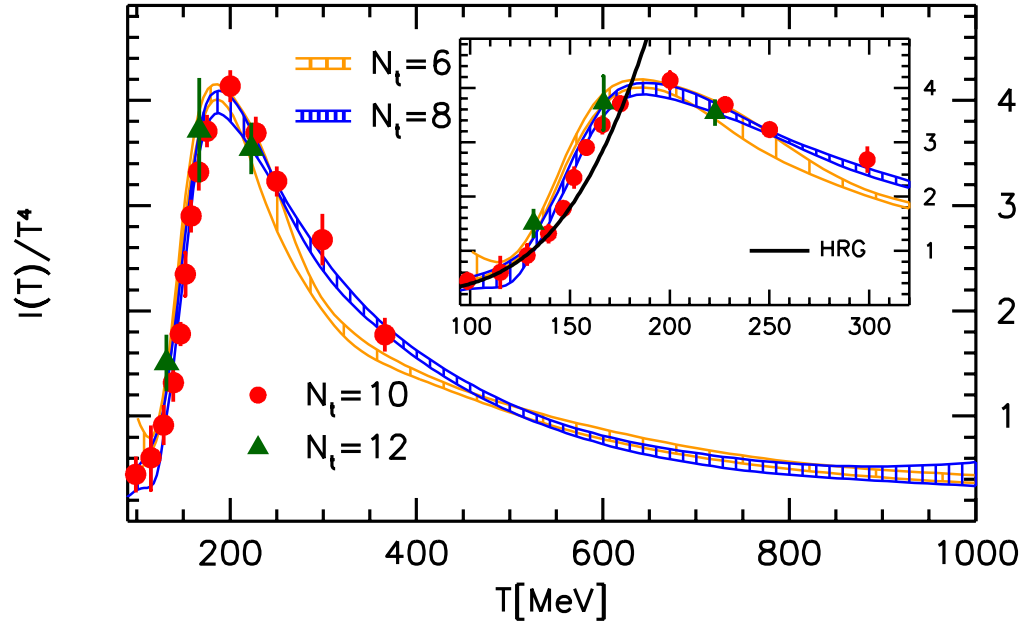
Figure 3: The trace anomaly normalized by T^4 as a function of T on $N_t = 6, 8, 10$ and 12 lattices.
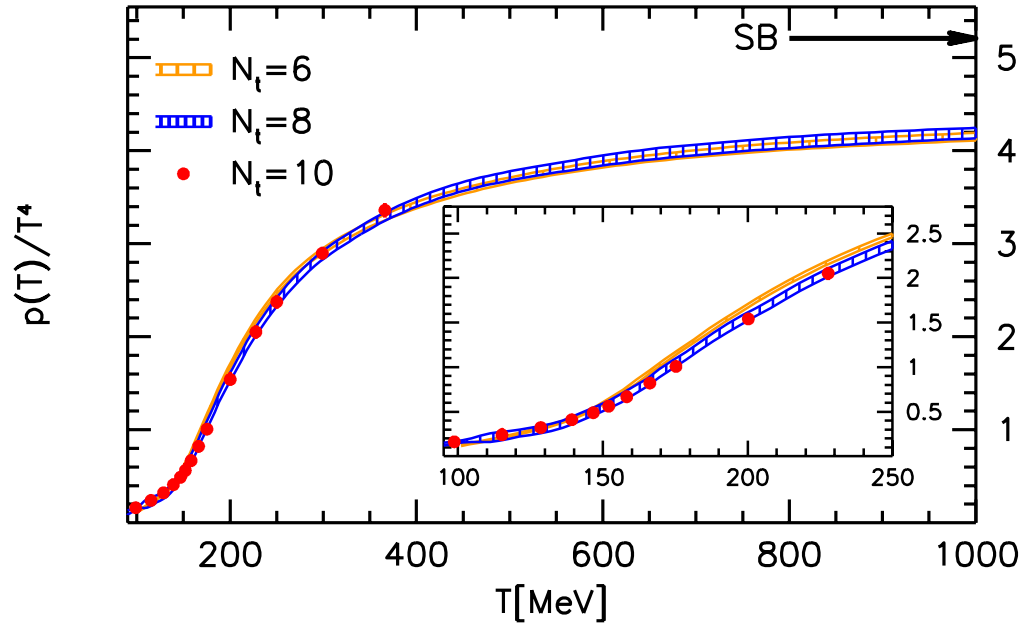

Figure 4: p(T) normalized by T^4 as a function of the temperature on $N_t = 6, 8$ and 10 lattices. The Stefan-Boltzmann limit $p_{SB}(T) \approx 5.209 \cdot T^4$ is indicated. At $T = 1000$ MeV p(T) is almost 20% below this limit.
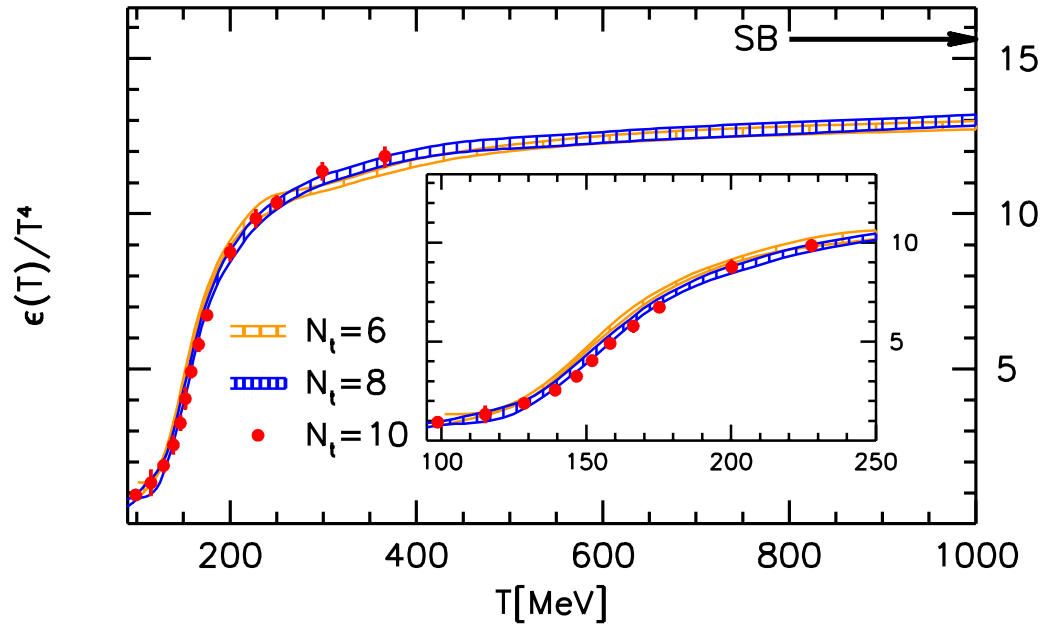
Figure 5: The energy density normalized by T^4 as a function of the temperature on $N_t = 6, 8$ and 10 lattices. The Stefan-Boltzmann limit $\varepsilon_{SB} = 3p_{SB}$ is indicated by an arrow.
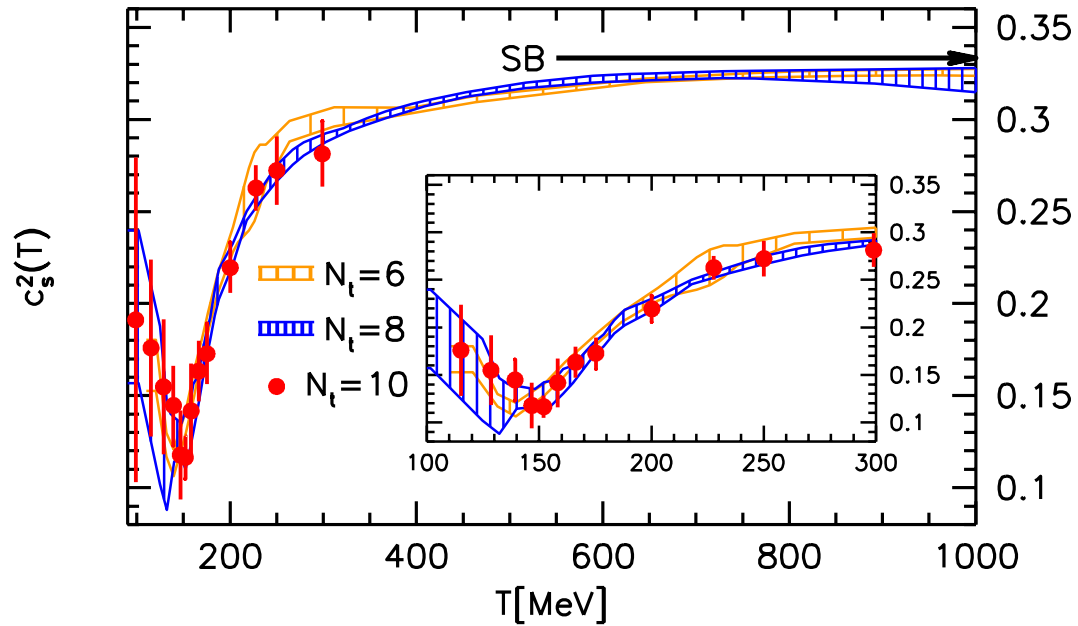
Figure 6: The speed of sound squared as a function of the temperature on $N_t = 6, 8$ and 10 lattices. The Stefan-Boltzmann limit is $c^2_{s,SB} = 1/3$ indicated by an arrow.
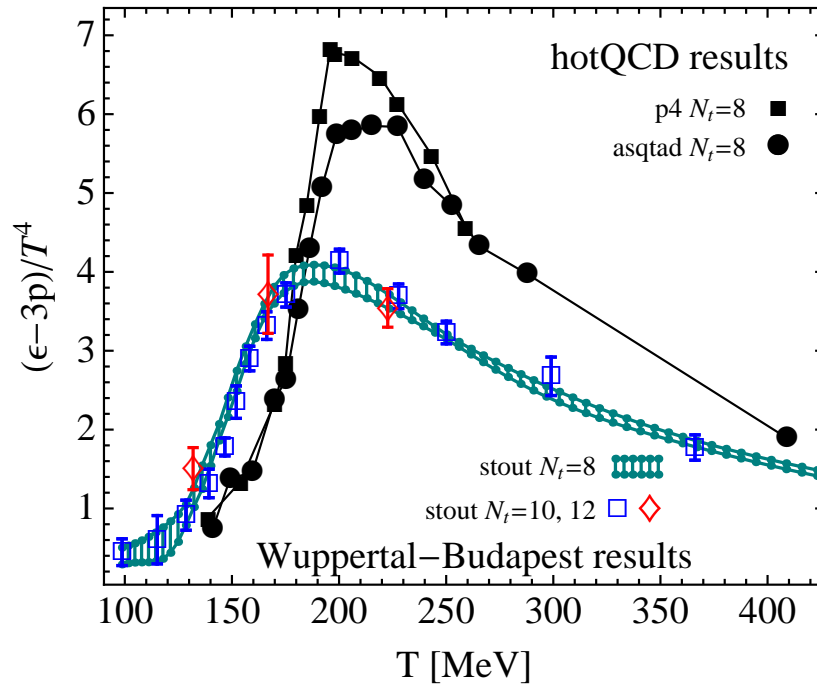
Figure 7: The normalized trace anomaly obtained in our study is compared to recent results from the "hotQCD " collaboration [13, 14].

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