

# Pion stability in a hot dense media

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Pions may remain stable under certain conditions in a dense media at zero temperature in the normal phase (non pion superfluid state). The stability condition is achieved when the in-media pion width vanishes. However, thermal fluctuations will change this stable regime. For low temperature pions will remain in a metastable state. Here we discuss the different possible scenarios for leptonic pion decays at finite temperature, taking into account all the different chemical potentials involved. The neutrino emission due to pions in a hot-dense media is calculated, as well as the cooling rate of a pion-lepton gas.

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The study of pions in a superfluid state has called the attention of physicist during many years in different frameworks [1–10]. In particular, compact stars may provide a natural scenario for such a state of matter. However, in the absence of a stable superfluid state, pions in the normal phase could behave as metastable particles. In fact, for high densities, an appropriate combination of chemical potentials can avoid the normal pion decay process. Indeed, if in cold matter the leptons states are completely filled up to the Fermi level, there will be no possible allowed final states for the decay of charged pions, unless thermal fluctuations are present. Pion decay properties in a hot and/or dense media have been vastly studied [11–16]. The present work is focused on the existence of possible pion metastable states in the normal phase at high densities. As a consequence, we will study how these metastable states affect the neutrino emission rate. The cooling rate for a pion-lepton gas is also determined. The temperature considered is much less than the pion mass. We will consider baryon chemical potential values lower than  $\sim 1$  GeV, in order to avoid other possible media effects like color superconductivity. The possible scenario for such a process are protonneutron stars [18, 19].

## I. LOW ENERGY QCD AT FINITE DENSITY

As the 99.9877% of the charged pions decay into muons and muonic neutrinos, we will refer specifically to this process. Our discussion will be based on the following low-energy Lagrangian:  $\mathcal{L} = \mathcal{L}_\mu + \mathcal{L}_\chi$ , where

$$\mathcal{L}_\mu = \sum_{f=\mu,\nu_\mu} \bar{\psi}_f (i\cancel{\partial} + \mu_f \gamma_0 + m_f) \psi_f \quad (1)$$

corresponds to the lepton free Lagrangian,  $\mu_f$  being the associated lepton chemical potential. The second term in the low-energy Lagrangian corresponds to the  $\mathcal{O}(p)^2$  chiral Lagrangian [20, 21]

$$\mathcal{L}_\chi = \frac{F^2}{4} \text{tr}(D_\mu U)^\dagger D U^\mu + \frac{G}{2} \text{tr}(U^\dagger \mathcal{M} + \mathcal{M}^\dagger U). \quad (2)$$

The  $F$  and  $G$  terms are the tree level pion decay constant and the tree level chiral condensate, respectively. The  $U$  fields contain the pion fields as  $U = \exp(i\pi^a \tau^a / f)$  and the covariant derivative  $D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu$  includes external right and left currents. When  $\mathcal{M} = \text{diag}(m_u, m_d)$  we break explicitly the chiral symmetry. In this article we are not interested in computing pion radiative corrections. Therefore, higher order chiral Lagrangian terms will not be considered. However, those corrections have been already calculated at finite temperature and isospin chemical potential [16], and therefore, if we want to incorporate these contributions, it is enough to replace the masses, decay constant and chemical potential terms by temperature and isospin chemical potential dependent dressed terms [22].

We will use the effective Fermi model for the leptonic weak coupling. The leptonic weak currents and the isospin chemical potential are introduced by setting the external currents in the chiral Lagrangian as

$$r_\alpha = \frac{1}{2} \mu_I \tau^3 \delta_{\alpha 0} \quad (3)$$

$$l_\alpha = G_F [\bar{\psi}_{\nu_\mu} \gamma_\alpha (1 - \gamma_5) \psi_\mu \tau^- + \bar{\psi}_\mu \gamma_\alpha (1 - \gamma_5) \psi_{\nu_\mu} \tau^+] + \frac{1}{2} \mu_I \tau^3 \delta_{\alpha 0} \quad (4)$$

with  $\mu_I = \mu_u - \mu_d$  is the isospin chemical potential and where we use the following combination of Pauli matrices:  $\tau^\pm = \frac{1}{\sqrt{2}}(\tau^1 \pm i\tau^2)$ . We will concentrate only on the normal phase where  $|\mu_I| < m_\pi$ . In the superfluid phase,  $|\mu_I| > m_\pi$ , one of the charged pions condenses, and therefore, another treatment is needed [2, 3].

## Baryon chemical potential

Two different regions in the literature have been considered when the Baryon chemical potential  $\mu_B = \frac{3}{2}(\mu_u + \mu_d)$  is introduced in the frame of chiral perturbation theory:

*Small  $\mu_B$ .* The baryon chemical potential can be taken as  $\mathcal{O}(p)$  in the power counting in chiral perturbation theory. In this case  $\mu_B$  appears in the Wess-Zumino-Witten

anomalous term, which turns out to be relevant only in higher order radiative corrections [23].

*Very high  $\mu_B$ .* In this case an expansion in powers of  $\mu_B^{-1}$  is performed (asymptotically infinite chemical potential). Here, effects of color superconductivity and color flavor locked phase are present, due to the appearance of diquark pairing [15, 24–26]. We can construct an effective Lagrangian including  $\mu_B$  in the absence of diquark effects, by considering effective  $F(\mu_B)$  and  $G(\mu_B)$  constants in the chiral Lagrangian in Eq. (2) and introducing a Lorentz symmetry breaking term:  $(D_\mu U)^\dagger D U^\mu \rightarrow (D_0 U)^\dagger D_0 U - v^2 (\mathbf{D}U)^\dagger \cdot \mathbf{D}U$ . Results obtained in the frame of the Nambu–Jona-Lasinio model show that for  $\mu_B \lesssim 1$  GeV the  $F$ ,  $G$  and  $v$  parameters do not suffer significant changes [7, 14, 17]. We can neglect then baryon chemical potential effects in this work.

Once the chiral Lagrangian has been expanded in terms of the pion fields, the canonical quantization procedure is the standard one, keeping in mind that the energy of the charged pions and lepton fields are shifted due to the chemical potentials. Then, we proceed to calculate the decay width of the charged pions and the corresponding neutrino emissivity.

## II. CHARGED PIONS DECAY WIDTHS

The decay width for charged pions, including finite temperature and density effects, is given by

$$\Gamma_{\pi^\pm} = \frac{1}{2m_\pi} \int dq_{\mu^\pm} dk_{\nu_\mp} (2\pi)^4 \delta^{(4)}(p - q - k) \times |\mathcal{M}_\pm|^2 [1 - n_F(q_0) - n_F(k_0)], \quad (5)$$

where the on-shell pions are in the rest frame,  $p = (m_\pi \mp \mu_I, \mathbf{0})$ , and where  $n_F(z) = (e^{z/T} + 1)^{-1}$  is Fermi-Dirac distribution. The phase space measure is defined as

$$dk_P = \frac{d^4 k}{(2\pi)^3} \theta(k_0 + \mu_P) \delta((k_0 + \mu_P)^2 - \mathbf{k}^2 - m_P^2), \quad (6)$$

where  $P$  stands for the different particles involved: pions, fermions and antifermions. The transition probability matrix was abbreviated as

$$|\mathcal{M}_\pm|^2 = \sum_{\text{spin}} |\langle \mu^\pm \nu_\mp^\mp | H_{\text{int}} | \pi^\pm \rangle|^2. \quad (7)$$

The corresponding chemical potentials are  $\mu_{\pi^\pm} = \pm \mu_I$ , and  $\mu_{f^\mp} = \pm \mu_f$  where  $f^-$  ( $f^+$ ) denotes a fermion (antifermion).

This definition of the decay width corresponds to the imaginary part of the thermal one-loop weak interaction corrections to the pion propagator. The decay width includes the decay of pions into leptons as well as recombination of leptons and neutrinos. By considering massless neutrinos, the decay width for charged pions is then

$$\Gamma_{\pi^\pm} = \Gamma_\pi \left[ 1 - n_F(e_{\mu^\pm}) - n_F(e_{\nu_\mp^\mp}) \right] \theta(m_\pi \mp \delta\mu - m_\mu)$$

$$\times \left[ \frac{1 - m_\mu^2 / (m_\pi \mp \delta\mu)^2}{1 - m_\mu^2 / m_\pi^2} \right]^2, \quad (8)$$

with

$$\Gamma_\pi = \frac{f_\pi^2 G_F^2}{4\pi} m_\pi m_\mu^2 [1 - m_\mu^2 / m_\pi^2]^2, \quad (9)$$

$$\delta\mu \equiv \mu_I + \mu_\mu - \mu_{\nu_\mu}, \quad (10)$$

$$e_{\mu^\pm} \equiv \frac{(m_\pi \mp \delta\mu)^2 + m_\mu^2}{2(m_\pi \mp \delta\mu)} \pm \mu_\mu, \quad (11)$$

$$e_{\nu_\mu^\pm} \equiv \frac{(m_\pi \mp \delta\mu)^2 - m_\mu^2}{2(m_\pi \mp \delta\mu)} \pm \mu_{\nu_\mu}, \quad (12)$$

$\Gamma_\pi$  being the vacuum pion decay width.

For the non chemical equilibrium case, when  $|\delta\mu| > m_\pi - m_\mu$ , one of the charged pions remains stable since its decay width vanishes, as can be seen from the Heaviside function in Eq. (8). We will consider then, in the non-equilibrium case, that  $|\delta\mu| < m_\pi - m_\mu \approx 34$  MeV. Here we used  $m_\pi = 139.6$  MeV and  $m_\mu = 105.6$  MeV.

Stable states at zero temperature, i.e. those where their decay width vanish, will now develop a small decay width due to thermal fluctuations. From Eq. (8), considering that  $n_F(x) \rightarrow \theta(-x)$  when  $T \rightarrow 0$ , we find two metastable cases:

$$\text{metastable } \pi^- \quad \mu_\mu > \frac{(m_\pi + \delta\mu)^2 + m_\mu^2}{2(m_\pi + \delta\mu)}, \quad (13)$$

$$\text{metastable } \pi^+ \quad \mu_{\nu_\mu} > \frac{(m_\pi - \delta\mu)^2 - m_\mu^2}{2(m_\pi - \delta\mu)}. \quad (14)$$

### Leptonic and beta equilibrium

In terms of quark degrees of freedom, the beta equilibrium condition is  $\mu_d = \mu_u + \mu_e - \mu_{\nu_e}$ , or, in terms of the isospin chemical potential,  $\mu_I + \mu_e - \mu_{\nu_e} = 0$ . If we consider a degenerate gas, which is the case for leptons in compact stars, the condition  $\mu_\mu - \mu_{\nu_\mu} = \mu_e - \mu_{\nu_e}$  arises in order to equilibrate the Fermi levels [19]. As a consequence of this leptonic chemical equilibrium, the beta-equilibrium condition will produce  $\delta\mu = 0$ .

From Eqs. (13) and (14), we can see that metastable states in beta equilibrium will occur for  $\mu_\mu > \mu_\mu^*$  for  $\pi^-$  mesons and  $\mu_{\nu_\mu} > \mu_{\nu_\mu}^*$  for  $\pi^+$  mesons, where

$$\mu_\mu^* \equiv \frac{m_\pi^2 + m_\mu^2}{2m_\pi} \approx 109.74 \text{ MeV}, \quad (15)$$

$$\mu_{\nu_\mu}^* \equiv \frac{m_\pi^2 - m_\mu^2}{2m_\pi} \approx 29.9 \text{ MeV}. \quad (16)$$

Note that in beta equilibrium, the  $\pi^-$  meson condenses if the lepton chemical potential is high enough such that  $\mu_\mu - \mu_{\nu_\mu} \geq m_\pi$ . On the other side, if the neutrino chemical potential is high enough, such that  $\mu_{\nu_\mu} - \mu_\mu \geq m_\pi$ , then a  $\pi^+$  meson condenses [42].

In order to explore the phenomenological consequences of these pion metastable states, we will discuss next the

neutrino emission and cooling rate of a pion-lepton gas in leptonic- and beta- equilibrium. Our discussion will show in a clear way the influence of metastable states on the cooling rate.

### III. NEUTRINO EMISSIVITY

Neutrino emission is perhaps the most relevant phenomena associated to the temperature evolution of compact stars. The neutrino emissivity  $\epsilon$  is defined as the energy loss through neutrino emission per unit time and unit volume. For the decaying pions, the neutrino emissivity must include the probability of finding a pion in the media as well as the Pauli blocking for the emerging muons

$$\epsilon_{\nu_{\mu}^{\pm}} = \int dp_{\pi^{\mp}} dq_{\mu^{\mp}} dk_{\nu_{\mu}^{\pm}} (2\pi)^4 \delta^{(4)}(p - q - k) \times |\mathcal{M}_{\pm}|^2 k_0 n_B(p_0) [1 - n_F(q_0)], \quad (17)$$

where  $n_B(z) = (e^{z/T} - 1)^{-1}$  is the Bose-Einstein distribution and the other terms were defined in the previous section.

Hereafter, we will consider the degenerate case where  $\mu_{\mu} \geq m_{\mu}$  and also beta-equilibrium. As we discussed in the last section, these two assumptions imply  $\delta\mu = 0$ .

The neutrino emissivity, then, is given by

$$\epsilon_{\nu_{\mu}^{\pm}} = \frac{\Gamma_{\pi} m_{\pi}^3}{\pi^2 (m_{\pi}^2 - m_{\mu}^2)} \int_{m_{\pi}}^{\infty} dE_{\pi} \int_{E_{\mu}^{\mp}}^{E_{\mu}^{\pm}} dE_{\mu} (E_{\pi} - E_{\mu}) \times n_B(E_{\pi} \mp (\mu_{\mu} - \mu_{\nu_{\mu}})) [1 - n_F(E_{\mu} \mp \mu_{\mu})], \quad (18)$$

where the limits for the muon energy integral are

$$E_{\mu}^{\pm} = \frac{1}{2m_{\pi}^2} \left[ (m_{\pi}^2 + m_{\mu}^2) E_{\pi} \pm (m_{\pi}^2 - m_{\mu}^2) \sqrt{E_{\pi}^2 - m_{\pi}^2} \right]. \quad (19)$$

The temperature and the muon chemical potential tend to favor the anti-neutrino emission due to the  $\pi^{-}$  decay. The  $n_B$  factor in Eq. (18), which gives the probability of finding a  $\pi^{-}$  meson, grows as function of temperature and lepton chemical potential. On the other hand, the  $(1 - n_F)$  factor gives the probability of finding an accessible state for the emerging muon. Only thermal fluctuations will conspire against the Pauli blocking, allowing then the decay of  $\pi^{-}$ . The probability of finding a  $\pi^{+}$  meson becomes smaller for higher chemical potential, being suppressed by the Bose factor.

We are interested to extract the leading terms for the low temperature behavior of the emissivity. Since we are considering a degenerate gas ( $\mu_{\mu} \geq m_{\mu}$ ), the main contribution is given by the emission of antineutrinos, being the neutrino emission highly suppressed by an exponential factor as we mentioned previously. In order to extract the main contribution to the total emissivity, we expand Eq. (18) in the low temperature region at the leading order. The Fermi-Dirac distribution becomes

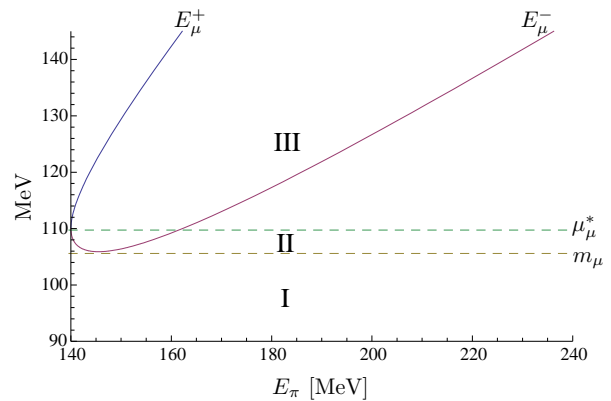


FIG. 1.  $E_{\mu}^{\pm}$  as a function of  $E_{\pi}$ , indicating three different regions for  $\mu_{\mu}$  (horizontal lines) which determine the integration regions of the  $E_{\pi}$  variables for the emissivity at low temperature, ruled by the condition  $E_{\mu}^{\pm} > \mu_{\mu}$  in Eq. (20).

then  $n_F(E_{\mu} - \mu_{\mu}) \approx \theta(\mu_{\mu} - E_{\mu})$ , obtaining

$$\epsilon_{\bar{\nu}_{\mu}} \approx \int_{m_{\pi}}^{\infty} dE_{\pi} [g_{+} \theta(E_{\mu}^{+} - \mu_{\mu}) + g_{-} \theta(E_{\mu}^{-} - \mu_{\mu})], \quad (20)$$

with

$$g_{\pm} = \pm \frac{\Gamma_{\pi} m_{\pi}^3 n_B(E_{\pi} - \mu_{\mu} + \mu_{\nu_{\mu}})}{\pi^2 (m_{\pi}^2 - m_{\mu}^2)} \left[ E_{\pi} E_{\mu}^{\pm} - \frac{1}{2} E_{\mu}^{\pm 2} \right]. \quad (21)$$

We can separate the antineutrino emissivity in three different regions, depending on the value of the lepton chemical potential. Fig. 1 shows the functions  $E_{\mu}^{\pm}$  plotted as a function of  $E_{\pi}$ . On the vertical axes, the three regions are indicated for specific values of the lepton chemical potential. The integration limits in equation (20) are determined by the condition  $E_{\mu}^{\pm} > \mu_{\mu}$ .

In region **I**, where  $\mu_{\mu} \leq m_{\mu}$ , the argument of the theta functions is always positive, then

$$\epsilon_{\bar{\nu}_{\mu}}(\mathbf{I}) = \int_{m_{\pi}}^{\infty} dE_{\pi} g_{+} + \int_{m_{\pi}}^{\infty} dE_{\pi} g_{-}. \quad (22)$$

In region **II**, where  $m_{\mu} < \mu_{\mu} < \mu_{\mu}^{*}$ , the function  $E_{\mu}^{+}$  is positive for all values of  $E_{\pi}$ . However, from Eq. (20), the condition  $E_{\mu}^{-} > \mu_{\mu}$  will exclude a region in the integral:

$$\epsilon_{\bar{\nu}_{\mu}}(\mathbf{II}) = \int_{m_{\pi}}^{\infty} dE_{\pi} g_{+} + \int_{m_{\pi}}^{m_{-}} dE_{\pi} g_{-} + \int_{m_{+}}^{\infty} dE_{\pi} g_{-}, \quad (23)$$

where

$$m_{\pm} = \frac{1}{2m_{\mu}^2} \left[ (m_{\pi}^2 + m_{\mu}^2) \mu_{\mu} \pm (m_{\pi}^2 - m_{\mu}^2) \sqrt{\mu_{\mu}^2 - m_{\mu}^2} \right] \quad (24)$$

are the solutions of the equation  $E_{\mu}^{\pm}(E_{\pi}) = \mu_{\mu}$ , giving as a result  $E_{\pi} = m_{\pm}$ . This is an intermediate region between low and high lepton chemical potential.

In region **III**, where  $\mu_{\mu} > \mu_{\mu}^{*}$ , the condition  $E_{\mu}^{\pm} > \mu_{\mu}$  will exclude some values in the integration limits:

$$\epsilon_{\bar{\nu}_{\mu}}(\mathbf{III}) = \int_{m_{-}}^{\infty} dE_{\pi} g_{+} + \int_{m_{+}}^{\infty} dE_{\pi} g_{-}, \quad (25)$$

where  $m_{\pm}$  was previously defined in Eq. (24) and corresponds to the solutions of the equation  $E_{\mu}^{\pm}(E_{\pi}) = \mu_{\mu}$ , giving  $E_{\mu}^{+}(m_{-}) = E_{\mu}^{-}(m_{+}) = \mu_{\mu}$ .

As we can see, the integrals above can be written in the form

$$I = \int_m^{\infty} dE_{\pi} f(E_{\pi}) n_B(E_{\pi} - \mu_{\mu} + \mu_{\nu}), \quad (26)$$

where the integrand  $f n_B = g_{\pm}$  in Eq. (21), and where  $m$  stands for  $m_{\pi}, m_{\pm}$ . In order to extract the leading terms in the low temperature region, if  $m > \mu_{\mu} - \mu_{\nu_{\mu}}$ , we can expand the Bose-Einstein distribution, and through an appropriate change of variables, we find

$$I = \sum_{n=1}^{\infty} e^{-\beta n(m - \mu_{\mu} + \mu_{\nu})} T^{\alpha} F_{\alpha, n}(T, m) \quad (27)$$

$$\approx e^{-\beta(m - \mu_{\mu} + \mu_{\nu})} T^{\alpha} F_{\alpha, 1}(0, m) \quad (28)$$

with

$$F_{\alpha, n}(T, m) \equiv \int_0^{\infty} dx \frac{f(Tx/n + m)}{nT^{\alpha-1}} e^{-x}, \quad (29)$$

and with  $\alpha$  such that the last integral remains finite in the limit  $T \rightarrow 0$ . Due to the exponential factor, the integrand in the above equation will be dominated by low  $x$ -values. If the condition  $m - \mu_{\mu} - \mu_{\nu_{\mu}} > T$  is satisfied, we can keep only the first term  $n = 1$  in the series. If  $m \approx \mu_{\mu} - \mu_{\nu_{\mu}}$ , we need to sum the whole series.

As a result, the low temperature behavior of the emissivity becomes

$$\epsilon_{\bar{\nu}_{\mu}} \approx \begin{cases} A T^{3/2} e^{-(m_{\pi} - \mu_{\mu} + \mu_{\nu})/T} & \text{for } \mu_{\mu} \approx m_{\mu} \\ B T e^{-(m_{-} - \mu_{\mu} + \mu_{\nu_{\mu}})/T} & \text{for } \mu_{\mu} > \mu_{\mu}^{*} \end{cases} \quad (30)$$

where

$$A = \Gamma_{\pi} m_{\pi}^4 (1 - m_{\mu}^2/m_{\pi}^2) (2\pi m_{\pi})^{-3/2} \quad (31)$$

$$B = \Gamma_{\pi} m_{\pi}^4 \frac{\mu_{\mu} (2m_{-} - \mu_{\mu})}{\pi(m_{\pi}^2 - m_{\mu}^2)} (2\pi m_{\pi})^{-1}. \quad (32)$$

The intermediate region  $m_{\mu} < \mu_{\mu} \leq \mu_{\mu}^{*}$  will be a combination of terms  $\sim T^{3/2}$  and  $\sim T$ . When the chemical potential grows, the linear term in  $T$  starts to dominate.

The neutrino emissivity, at low temperature, is strongly suppressed by a term  $\exp[-(m_{\pi} + \mu_{\mu} - \mu_{\nu_{\mu}})/T]$ . We are interested in the high chemical potential region since it increases the emissivity in the low temperature region. Our numerical analysis suggests that these approximations will be valid for temperatures less than 50 MeV.

#### IV. COOLING RATE

In order to explore some effects in the metastable region of the  $\pi^{-}$ , Eq. (13), we will now calculate the cooling rate due to muonic neutrino emission for a pion-lepton gas, at constant volume and charge. We will

consider that  $\mu_{\nu_{\mu}} = 0$ , which means that all the neutrinos will escape from the gas. We also consider  $\beta$ -equilibrium, lepton-equilibrium and neglect the process of neutrino emission through muon decay. In other words  $-\mu_I = \mu_{\mu} = \mu_e$ . This model is a simplification, eventually valid as isolated bubbles inside the nuclear media of compact stars, although finite volume effects should be taken into account.

The cooling time  $t$  is defined as

$$t = t_0 - \int_{T_0}^T \frac{c_V}{\epsilon} dT, \quad (33)$$

where  $t_0$  and  $T_0$  are the initial time and temperature, respectively.  $c_V$  and  $\epsilon$  correspond to the specific heat and the emissivity, respectively.

The specific heat per unit of volume is given by

$$c_V = \frac{T}{V} \left( \frac{\partial S}{\partial T} \right)_{V, N} \quad (34)$$

where  $S$  is the entropy, and  $N$ , in our case, is the charge number. In a non-interacting degenerated gas, at low temperature and high chemical potential, the charge number is dominated by fermions, and the leading term in the temperature expansion is constant:

$$N \approx \frac{V}{3\pi^2} \left[ (\mu_e^2 - m_{\mu}^2)^{3/2} + (\mu_e^2 - m_e^2)^{3/2} \right]. \quad (35)$$

So, for such low temperature approximation, the constant  $N$  condition is equivalent to consider a constant  $\mu$ . The specific heat per unit of volume can be written as

$$c_V = T \left( \frac{\partial^2 p}{\partial T^2} \right)_{V, \mu}, \quad (36)$$

where  $p$  is the pressure.

The contribution to the specific heat per unit of volume for noninteracting fermions is

$$c_{V_f} = \frac{g}{T^2} \int \frac{d^3 p}{(2\pi)^3} (E - \mu)^2 n_F(E - \mu) n_F(\mu - E), \quad (37)$$

with  $g = 2$ . The formula for the specific heat per unit of volume, for bosons, is the same as for fermions but with  $g = -1$  and changing  $n_F$  by  $n_B$ . The relevant contribution comes essentially from electrons and muons, where the specific heat per unit of volume for a degenerated fermion gas of mass  $m$  and chemical potential  $\mu$  is  $c_{V_f} = \frac{1}{3} \mu \sqrt{\mu^2 - m^2} T$  if  $\mu > m$ . In the case of muons, when  $\mu_e \approx m_{\mu}$ , their contribution will be  $\approx 0.4(m_{\mu} T)^{3/2}$ .

Fig. 2 shows the temperature dependence, as function of the logarithm of time in seconds, for three different values of the electron chemical potential:  $\mu_e = m_{\mu}$  (muon degeneracy),  $\mu_e = m_{\pi}$  (pion condensation) and another value in between:  $\mu = 122.6$  MeV. It can be seen from Fig. 2 that starting from an initial temperature,  $T = 5$  MeV, the needed time to reach 1 MeV is extremely short, fraction of seconds, in the metastable

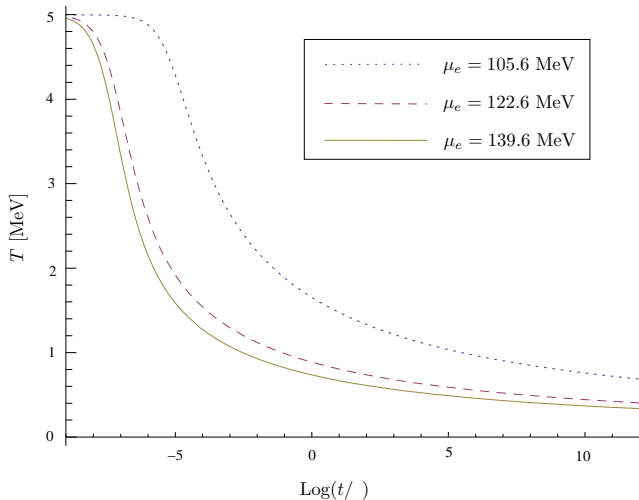


FIG. 2. Temperature as function of the logarithm of time in seconds, of a pion-lepton gas in  $\beta$ -equilibrium with zero neutrino chemical potential. The initial temperature is 5 MeV.

as well in the condensed case. On the other hand, it will take thousands of years to diminish the temperature from 1 MeV to fractions of MeV. The cooling process is not so fast if we consider values of the electron chemical potential lower than 109 MeV. Notice that, in spite of the fact that  $\mu_e = 122.6$  MeV is an average between  $\mu_e = m_\mu$  and  $\mu_e = m_\pi$ , the corresponding cooling time curve is notoriously closer to the beginning of the pion superfluid phase.

## V. CONCLUSIONS

In this paper we have calculated the charged pions decay widths in dense matter at finite temperature, analyzing the pion metastable condition. We have calculated also the neutrino emissivity through the leptonic pion decay in  $\beta$ -equilibrium for a degenerated system. We obtained the main contributions in the low temperature region for pions decaying into muons and muonic neutrinos. Finally, we estimated the cooling rate of a pion lepton gas in  $\beta$ -equilibrium for three different values of the electric chemical potential.

From our results, we argue that it is possible to find stable pions, even if they are not condensed, if the lepton chemical potential reaches a value higher than the muon mass, which is the case in compact stars. Under such conditions, the pions might only decay through thermal fluctuations. The muonic neutrino emissivity will grow with the lepton chemical potential, varying from  $\sim T^{3/2}$  to  $\sim T$ , both with an exponential suppressing term. The contribution to the cooling process in a neutron star, due to the emission of muonic neutrinos from pions, has not been much considered yet in the normal phase. In fact, from our estimation of the cooling time of the lepton pion gas, it can be seen that this process is relevant for temperatures higher than 1 MeV which corresponds to the cooling process of a protoneutron star. The pion superfluid case will be explored elsewhere.

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