Statistical Anisotropy and the Vector Curvaton Paradigm

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Abstract

The vector curvaton paradigm is reviewed. The mechanism allows a massive vector boson field to contribute to or even generate the curvature perturbation in the Universe. Contribution of vector bosons is likely to generate statistical anisotropy in the spectrum and bispectrum of the curvature perturbation, which will soon be probed observationally. Two specific models for the generation of superhorizon spectra for the components of an Abelian vector field are analysed. Emphasis is put on the observational signatures of the models when the vector fields play the role of vector curvatons.

1 Introduction

Cosmic inflation is arguably the most compelling way to overcome, or at least ameliorate, the so-called horizon and flatness problems of the hot big bang cosmology. However, these problems are successfully addressed by all models of inflation, provided that the inflationary expansion lasts long enough. Therefore, discrimination between inflation models is based on another important consequence of inflationary expansion, namely the generation of the curvature perturbation ζ in the Universe, which is responsible for the formation of structures such as galaxies and galactic clusters and which is reflected onto the Cosmic Microwave Background (CMB) radiation through the Sachs-Wolfe effect.

The latest CMB observations appear to confirm the "vanilla" predictions of inflation with respect to ζ: scale-invariance, Gaussianity and statistical homogeneity and isotropy. However, a period of accelerated expansion of space (this is the definition of cosmic inflation) is not enough to guarantee scale-invariance for the curvature perturbation. Indeed, inflation is required to be of the quasi-de Sitter type, where the density of the Universe remains roughly constant. Also, the CMB observations seem to suggest that exact scale-invariance is not favoured (although it is not ruled out) with the spectral index of ζ, satisfying $n_s - 1 = -0.037 \pm 0.014$ (at 1σ) [1] when ΛCDM cosmology is assumed.[2] Most likely, this reveals some dynamics for inflation (the density is not exactly constant), which is indeed expected by model-builders. Similarly, a high degree of Gaussianity in the curvature perturbation is expected since this reflects the randomness of quantum fluctuations, on which the particle production process is feeding, in order to generate the superhorizon spectrum of the fields which eventually give rise to ζ. However, the Gaussianity of ζ crucially depends on the linearity of the process which translates these field perturbations to ζ.

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[2]Exact scale invariance corresponds to $n_s = 1$.

This is quantified through the so-called non-linearity parameter $f_{\rm NL}$. The latest CMB observations provide a hint of non-zero non-Gaussianity since, in the squeezed configuration, $f_{\rm NL} = 32 \pm 21$ (at 1σ) [1], which again appears to deviate from the "vanilla" prediction.[3]

In the same spirit, observations suggest that there may be deviations from statistical homogeneity, since there seem to be a difference in the power of ζ as large as 10% between hemispheres in the CMB [2]. Finally, statistical isotropy in ζ is also questioned by observations. Indeed, there is tantalising evidence of a preferred direction on the microwave sky. This is the so-called "Axis of Evil" observation [3], which amounts to an alignment of the quadrupole and octupole moments in the CMB, which is statistically extremely unlikely [4] and has been shown to persist beyond foreground removal [5].

Thus, we see that the precision of the cosmological observations is such that begins to enable us to explore beyond the "vanilla" predictions of inflation and use the observed deviations from them to discriminate between classes of models and paradigms. There is already a huge literature on deviations from exact scale-invariance and Gaussianity for the curvature perturbation. In this paper we discuss possible deviations from statistical isotropy and we present a compelling paradigm for their generation; namely the Vector Curvaton Paradigm.

Throughout the paper we consider a metric with negative signature and use natural units where $c = \hbar = k_B = 1$ and Newton's gravitational constant is $8\pi G = m_P^{-2}$, with $m_P = 2.4 \times 10^{18}$ GeV being the reduced Planck mass.

2 Statistical anisotropy in the curvature perturbation

Statistical anisotropy amounts to direction dependent patterns in the CMB. It can be quantified as follows. The spectrum $\mathcal{P}_\zeta$ of the curvature perturbation is defined through the two-point correlator as

$$\langle \zeta(\boldsymbol{k})\zeta(\boldsymbol{k'})\rangle = (2\pi)^3\delta(\boldsymbol{k}+\boldsymbol{k'})\frac{2\pi^2}{k^3}\mathcal{P}_\zeta(\boldsymbol{k}), \tag{1}$$

where $k = |\boldsymbol{k}|$ and

$$\zeta(\boldsymbol{k}) \equiv \int \zeta(\boldsymbol{x})e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}d^3x\,.$$

The reality condition $\zeta^*(\boldsymbol{k}) = \zeta(-\boldsymbol{k})$ demands that $\mathcal{P}_\zeta(\boldsymbol{k}) = \mathcal{P}_\zeta(-\boldsymbol{k})$. Now, the dependence of the power spectrum on the direction of the momentum vector can be parametrised as

$$\mathcal{P}_\zeta(\boldsymbol{k}) = \mathcal{P}_\zeta^{\rm iso}(k)[1 + g(\hat{\boldsymbol{d}}\cdot\hat{\boldsymbol{k}})^2 + \cdots], \tag{2}$$

where $\hat{\boldsymbol{d}}$ is the unit vector along the preferred direction, $\hat{\boldsymbol{k}} \equiv \boldsymbol{k}/k$ and the ellipsis denotes higher than quadratic order terms, which are negligible if $g < 1$. A similar parametrisation

[3] Note that ζ is indeed predominantly Gaussian because the bispectrum B_ζ is related to the power spectrum P_ζ roughly as $B_\zeta \sim f_{\rm NL}P_\zeta^2$ (see Eq. (15)), with the observations suggesting $\mathcal{P}_\zeta = 4.8 \times 10^{-5}$ for the spectrum [1].

can be assigned to higher order correlators, i.e. the bispectrum, trispectrum etc. (for the bispectrum see Sec. 3.2).

What are observations saying about g? In Ref. [6] it was found that $g = 0.29 \pm 0.03$ at the level of 9σ!. However, the preferred direction was too close to the ecliptic plane so the authors suspected some unknown systematic. Hence, this number can be considered only as an upper bound $g \lesssim 0.3$. The observations of the Planck satellite will strengthen this bound by at least an order of magnitude and reduce it to $g \lesssim 0.02$ if statistical anisotropy in the spectrum of the curvature perturbation is indeed not observed [7].

3 Vector fields and the curvature perturbation

Statistical anisotropy in the curvature perturbation cannot be generated if one considers the effects of scalar fields only, because the latter cannot introduce a preferred direction. In this paper we will study how vector boson fields can directly influence the curvature perturbation and generate statistical anisotropy.[4] We will investigate the contribution of vector fields to ζ through the so-called δN formalism [9].

According the δN formalism the curvature perturbation is the difference of the logarithmic expansion between uniform density and spatially flat slices of spacetime: $\zeta = \delta(\ln a) \equiv \delta N$, where a is the scale factor of the Universe and N corresponds to the elapsing e-folds of expansion. We will assume that N is influenced by both scalar and vector boson fields. For simplicity, we consider one of each of such fields, i.e. $N = N(\phi, \boldsymbol{A})$. Then, the curvature perturbation can be written as

$$\zeta = N_\phi \delta\phi + N_A^i \delta A_i + \frac{1}{2} N_{\phi\phi} (\delta\phi)^2 + \frac{1}{2} N_{\phi A}^i \delta\phi \delta A_i + \frac{1}{2} N_{AA}^{ij} \delta A_i \delta A_j + \cdots \,, \qquad (3)$$

where $N_\phi \equiv \frac{\partial N}{\partial \phi}$, $N_A^i \equiv \frac{\partial N}{\partial A_i}$, $N_{\phi\phi} \equiv \frac{\partial^2 N}{\partial \phi^2}$ $N_{\phi A}^i \equiv \frac{\partial^2 N}{\partial \phi \partial A_i}$ and $N_{AA}^{ij} \equiv \frac{\partial^2 N}{\partial A_i \partial A_j}$, with $i = 1, 2, 3$ labelling spatial components and Einstein summation over repeated indexes is assumed. Now, since the vector field has three degrees of freedom, at a flat slice of spacetime foliation, we define

$$\delta \boldsymbol{A}(\boldsymbol{k}, t) = \sum_\lambda \hat{\boldsymbol{e}}_\lambda(\hat{\boldsymbol{k}}) \delta A_\lambda(\boldsymbol{k}, t) \,, \qquad (4)$$

where $\lambda = L, R, \parallel$ denotes the three polarisations and the polarisation vectors can be defined as

$$\hat{e}_L \equiv \frac{1}{\sqrt{2}}(1, i, 0), \quad \hat{e}_R \equiv \frac{1}{\sqrt{2}}(1, -i, 0) \quad \text{and} \quad \hat{e}_\parallel \equiv (0, 0, 1) \,, \qquad (5)$$

where 'L', 'R' denote the left and right transverse polarisations respectively and '$\parallel$' denotes the longitudinal polarisation (if physical). Then, assuming approximately isotropic

[4] Indirectly, statistical anisotropy in ζ can also be generated by considering a mild anisotropisation of the inflationary expansion, due to the presence of a vector boson field condensate. In this case, it is the perturbations of the inflaton scalar field which are rendered statistically anisotropic [8].

expansion[5], the power-spectrum for each polarisation of the vector field perturbations is

$$\langle \delta A_\lambda(\boldsymbol{k}) \delta A_\lambda(\boldsymbol{k}') \rangle = (2\pi)^3 \delta(\boldsymbol{k}+\boldsymbol{k}') \frac{2\pi^2}{k^3} \mathcal{P}_\lambda(k) \,. \tag{6}$$

3.1 The spectrum

In Ref. [10], it was shown that the correlators of the perturbations of the vector field can be written as

$$\langle \delta A_i(\boldsymbol{k}) \delta A_j(\boldsymbol{k}') \rangle = (2\pi)^3 \delta(\boldsymbol{k}+\boldsymbol{k}') \frac{2\pi^2}{k^3} \left[T^+_{ij}(\hat{\boldsymbol{k}}) \mathcal{P}_+ + i T^-_{ij}(\hat{\boldsymbol{k}}) \mathcal{P}_- + T^{\parallel}_{ij}(\hat{\boldsymbol{k}}) \mathcal{P}_{\parallel} \right] , \tag{7}$$

where

$$T^+_{ij}(\hat{\boldsymbol{k}}) \equiv \delta_{ij} - \hat{k}_i \hat{k}_j \ , \quad T^-_{ij}(\hat{\boldsymbol{k}}) \equiv \varepsilon_{ijk} \hat{k}_k \quad \text{and} \quad T^{\parallel}_{ij}(\hat{\boldsymbol{k}}) \equiv \hat{k}_i \hat{k}_j \tag{8}$$

with δ_{ij} being the Kronecker's delta, and we have defined for the transverse spectra

$$\mathcal{P}_\pm \equiv \frac{1}{2} (\mathcal{P}_R \pm \mathcal{P}_L) \,, \tag{9}$$

denoting the parity even and odd polarisations. For a parity conserving theory we have $\mathcal{P}_- = 0$. Using the above, we obtain the power spectrum of the curvature perturbation as

$$\begin{aligned} \mathcal{P}_\zeta(\boldsymbol{k}) &= N_\phi^2 \mathcal{P}_\phi(k) + N_A^i N_A^j \left[T^+_{ij}(\hat{\boldsymbol{k}}) \mathcal{P}_+(k) + T^{\parallel}_{ij}(\hat{\boldsymbol{k}}) \mathcal{P}_{\parallel}(k) \right] = \\ &= N_\phi^2 \mathcal{P}_\phi + N_A^2 \left[\mathcal{P}_+ + (\mathcal{P}_{\parallel} - \mathcal{P}_+)(\hat{\boldsymbol{N}}_{\boldsymbol{A}} \cdot \hat{\boldsymbol{k}})^2 \right] , \end{aligned} \tag{10}$$

where $N_A \equiv |\boldsymbol{N_A}| = \sqrt{N_A^i N_A^i}$, $\hat{\boldsymbol{N}}_{\boldsymbol{A}} \equiv \boldsymbol{N_A}/N_A$.

From the above we see that the isotropic part of the spectrum is

$$\mathcal{P}_\zeta^{\rm iso}(k) = N_\phi^2 \mathcal{P}_\phi(k) + N_A^2 \mathcal{P}_+(k) \tag{11}$$

and the preferred direction is given by $\hat{\boldsymbol{d}} = \hat{\boldsymbol{N}}_{\boldsymbol{A}}$ (cf. Eq. (2)). The anisotropy parameter is

$$g = \frac{N_A^2 (\mathcal{P}_{\parallel} - \mathcal{P}_+)}{N_\phi^2 \mathcal{P}_\phi + N_A^2 \mathcal{P}_+} = \beta \frac{\mathcal{P}_{\parallel} - \mathcal{P}_+}{\mathcal{P}_\phi + \beta \mathcal{P}_+} \,, \tag{12}$$

where we have defined

$$\beta \equiv \frac{N_A^2}{N_\phi^2} \,, \tag{13}$$

which quantifies the relative contribution of the vector over the scalar field to the modulation of N. Notice that particle production becomes isotropic ($g = 0$) if $\mathcal{P}_+ = \mathcal{P}_{\parallel}$.

[5]This is in contrast to Ref. [8] where it is the anisotropy in the expansion which sources statistical anisotropy in ζ.

3.2 The bispectrum

Statistical anisotropy is also possible to manifest in higher order correlators of the curvature perturbation. In this paper we discuss only the bispectrum (for the trispectrum see Ref. [11]).

The bispectrum of the curvature perturbation is defined as

$$\langle\zeta(\boldsymbol{k})\zeta(\boldsymbol{k}')\zeta(\boldsymbol{k}'')\rangle = (2\pi)^3\delta(\boldsymbol{k}+\boldsymbol{k}'+\boldsymbol{k}'')B_\zeta(\boldsymbol{k},\boldsymbol{k}',\boldsymbol{k}'')\,. \tag{14}$$

The bispectrum B_ζ is a measure of the non-Gaussianity of the curvature perturbation since, for Gaussian ζ, B_ζ is exactly zero.

The curvature perturbation is generated due to the quantum fluctuations of suitable fields which are stretched to become classical perturbations during inflation. Since quantum fluctuations are Gaussian (which reflects their randomness) sizable non-Gaussianity in ζ is generated only if the process through which the perturbations of the relevant fields affect the Universe expansion and imprint their contribution to the curvature perturbation. If this process is significantly non-linear deviations from Gaussianity will be generated. This is why, the bispectrum is quantified by the so-called non-linearity parameter $f_{\rm NL}$, which can be defined as follows

$$B_\zeta(\boldsymbol{k_1},\boldsymbol{k_2},\boldsymbol{k_3}) = -\frac{6}{5}f_{\rm NL}\left[P_\zeta(k_1)P_\zeta(k_2)+P_\zeta(k_2)P_\zeta(k_3)+P_\zeta(k_3)P_\zeta(k_1)\right], \tag{15}$$

where $4\pi k^3 P_\zeta \equiv (2\pi)^3\mathcal{P}_\zeta$. The value of $f_{\rm NL}$ depends on the configuration of the three momentum vectors which are used to define the bispectrum. The most popular configurations are the "equilateral", for which $k_1=k_2=k_3$, and the "squeezed", for which $k_1=k_2\gg k_3$.

How does the contribution of a vector field affect the bispectrum of the curvature perturbation? In Ref. [12] it was shown that

$$B_\zeta = B_\phi + B_{\phi A} + B_A\;, \tag{16}$$

where

$$B_\phi = N_\phi^2 N_{\phi\phi}\left[\frac{4\pi^4}{k_1^3k_2^3}\mathcal{P}_\phi(k_1)\mathcal{P}_\phi(k_2)+\frac{4\pi^4}{k_2^3k_3^3}\mathcal{P}_\phi(k_2)\mathcal{P}_\phi(k_3)+\frac{4\pi^4}{k_3^3k_1^3}\mathcal{P}_\phi(k_3)\mathcal{P}_\phi(k_1)\right], \tag{17}$$

$$B_{\phi A} = -\frac{1}{2}N_\phi N_{\phi A}^i\left[\frac{4\pi^4}{k_1^3k_2^3}\mathcal{P}_\phi(k_1)\mathcal{M}_i(\boldsymbol{k_2}) + 5\text{ cyclic permutations}\right] \tag{18}$$

and

$$B_A = \frac{4\pi^4}{k_1^3k_2^3}\mathcal{M}_i(\boldsymbol{k_1})N_{AA}^{ij}\mathcal{M}_i(\boldsymbol{k_2})+\frac{4\pi^4}{k_2^3k_3^3}\mathcal{M}_i(\boldsymbol{k_2})N_{AA}^{ij}\mathcal{M}_i(\boldsymbol{k_3})+\frac{4\pi^4}{k_3^3k_1^3}\mathcal{M}_i(\boldsymbol{k_3})N_{AA}^{ij}\mathcal{M}_i(\boldsymbol{k_1}), \tag{19}$$

where

$$\boldsymbol{\mathcal{M}}(\boldsymbol{k}) \equiv \mathcal{P}_+(k)N_A\left[\hat{\boldsymbol{N}}_{\boldsymbol{A}} + p(k)\hat{\boldsymbol{k}}(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{N}}_{\boldsymbol{A}}) + iq(k)\hat{\boldsymbol{k}}\times\hat{\boldsymbol{N}}_{\boldsymbol{A}}\right] \tag{20}$$

and we have defined

$$p \equiv \frac{\mathcal{P}_{\parallel} - \mathcal{P}_{+}}{\mathcal{P}_{+}} \quad \text{and} \quad q \equiv \frac{\mathcal{P}_{-}}{\mathcal{P}_{+}} . \tag{21}$$

Using the above, we obtain $f_{\rm NL}$ in the equilateral and squeezed configurations respectively as follows

$$\frac{6}{5} f_{\rm NL}^{\rm eql} = \frac{\mathcal{B}_{\zeta}^{\rm eql}(\boldsymbol{k_1}, \boldsymbol{k_2}, \boldsymbol{k_3})}{3[\mathcal{P}_{\zeta}^{\rm iso}(k)]^2} \tag{22}$$

and

$$\frac{6}{5} f_{\rm NL}^{\rm sqz} = \frac{\mathcal{B}_{\zeta}^{\rm sqz}(\boldsymbol{k_1}, \boldsymbol{k_2}, \boldsymbol{k_3})}{2\mathcal{P}_{\zeta}^{\rm iso}(k_1)\mathcal{P}_{\zeta}^{\rm iso}(k_3)} , \tag{23}$$

where $\mathcal{B}_{\zeta}^{\rm eql} \equiv \frac{k^3}{2\pi^2} B_{\zeta}^{\rm eql}$ and $\mathcal{B}_{\zeta}^{\rm sqz} \equiv \frac{k_1^3 k_3^3}{4\pi^2} B_{\zeta}^{\rm sqz}$, with $k \equiv k_1 = k_2 = k_3$ in the equilateral configuration.

4 The Vector Curvaton Paradigm

For a vector field to directly affect the curvature perturbation in the Universe we need two ingredients. First, we need a mechanism to break the conformal invariance of the vector field and generate a superhorizon spectrum of vector field perturbations δA_μ. Second, we need a mechanism that will allow these perturbations to affect (or even generate) the curvature perturbation ζ. This can be done only if the vector field and/or its perturbations, in some way affect the Universe evolution.

In this section we focus on the second ingredient, i.e. on a mechanism for the generation of a contribution of the vector field perturbations to the curvature perturbation of the Universe; namely the Vector Curvaton mechanism. Thus, we assume that some other mechanism has produced the necessary surerhorizon spectrum of perturbations (as is discussed in Sec. 5) during inflation, which for the moment we take for granted.

A single vector field cannot play the role of the inflaton. The reason is straightforward. Inflation homogenises a vector field and a homogeneous vector field picks up a preferred direction in space.[6] Thus, if a homogeneous vector field dominated the Universe during inflation it would lead to excessive anisotropic stress, which would produce too much of a large-scale anisotropy and, therefore, will be in conflict with CMB observations. A huge number $\mathcal{N}$ of vector fields, randomly oriented, could avoid this problem [14]. Indeed, if this is the case then the statistical anisotropy produced is $g \propto 1/\sqrt{\mathcal{N}}$, which means that hundreds of vector fields are needed to satisfy the observational bounds. This not only implies the use of giant gauge groups but also requires the tuning of the initial conditions so that they are the same for all the fields. Another option is to consider a "triad" of orthogonally oriented vector fields (again with the same initial conditions) so that the excessive anisotropic stress is eliminated [15]. For the above reasons we will not consider vector fields as inflatons.

[6] Unless one tunes the spatial components of the vector field to zero by design [13].

If the vector field is not the inflaton, it needs to affect the Universe expansion in some other way, either at the end or after the end of inflation. There are a multitude of mechanisms which may allow a vector field to do that, mirroring the corresponding scalar field models. Prominent examples include the curvaton [16], the inhomogeneous end of inflation [17] and the modulated reheating [18] mechanisms. Historically, statistical anisotropy by vector field perturbations was first studied in the context of the inhomogeneous end of inflation mechanism [19],[7] using a particular model of hybrid inflation.[8] Here, however, we concentrate on the curvaton mechanism, which has the considerable advantage that it does not rely on an interaction of any kind between the vector field and the inflaton sectors. As is the case of the scalar curvaton, the vector curvaton is not a particular model but it can correspond to a multitude of realisations, hence we refer to the mechanism as a paradigm rather than a model. The vector curvaton mechanism was first introduced in the pioneering work in Ref. [22], which was the first article to consider the possibility that a vector boson field can contribute to the curvature perturbation in the Universe.

The idea of the curvaton assumes the existence of a spectator field during inflation, which has nothing to do with inflationary dynamics but it is light enough so that it manages to obtain a superhorizon spectrum of perturbations. After the end of inflation (possibly long afterwards), the curvaton becomes heavy and begins undergoing oscillations which allow it to come to dominate (or nearly dominate) the Universe before its decay. Owing to its perturbations, the density of the curvaton is perturbed throughout space so that its (near) domination occurs at different times at different locations. Thus, its effect to the evolution of the Universe is location depended, which is the reason why it can affect (or even generate) the curvature perturbation in the Universe. Note that, for a vector field to do this, it must avoid generating an excessive anisotropic stress at domination.

4.1 The setup

Consider a massive Abelian vector boson field, with Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 W_\mu W^\mu, \tag{24}$$

where $F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$ is the field strength tensor. Inflation homogenises the vector field so that $W_\mu = W_\mu(t)$. If $m \neq 0$ it is easy to show that the temporal component of the homogeneous vector field is zero, i.e. $W_t = 0$. If $m = 0$ then the field is gauge invariant and we can set $W_t = 0$ by virtue of a gauge choice. However, in this case the value of the spatial vector field $\boldsymbol{W}$ is not well defined because gauge invariance allows us to change it as $\boldsymbol{W} \to \boldsymbol{W} + \boldsymbol{C}$, where $\boldsymbol{C}$ is a constant vector of arbitrary magnitude. Thus, we will concentrate on the case $m \neq 0$ from now on, where gauge invariance is broken and the homogeneous "zero-mode" is well defined.

[7]For modulated reheating with vector fields see Ref. [21].

[8]For non-Gaussianity in this model see also [20].

The energy-momentum tensor for the vector field is

$$T_{\mu\nu} = \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} - F_{\mu\rho} F^{\rho}_{\nu} + m^2 \left(W_\mu W_\nu - \frac{1}{2} g_{\mu\nu} W_\rho W^\rho \right) , \tag{25}$$

where $g_{\mu\nu}$ is the metric tensor (negative signature is assumed). The above can be written as [22]

$$T^{\nu}_{\mu} = \mathrm{diag}(\rho_A, -p_\perp, -p_\perp, +p_\perp) , \tag{26}$$

where

$$\rho_A \equiv \rho_{\rm kin} + V_A \quad \text{and} \quad p_\perp \equiv \rho_{\rm kin} - V_A \tag{27}$$

with

$$\rho_{\rm kin} \equiv -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad \text{and} \quad V_A \equiv -\frac{1}{2} m^2 W_\mu W^\mu . \tag{28}$$

Notice that the energy-momentum tensor is similar to the one of a perfect fluid with the crucial difference that the pressure along the longitudinal direction is of opposite sign to the pressure along the transverse directions. This means that, if this pressure were not zero and the vector field dominated the Universe, it would give rise to significant anisotropic stress, which is the reason why a single vector field cannot play the role of the inflaton.

Using Eq. (24) one can obtain the equation of motion for the homogeneous vector field, which reads

$$\ddot{\boldsymbol{W}} + H \dot{\boldsymbol{W}} + m^2 \boldsymbol{W} = 0 , \tag{29}$$

where the dot denotes derivative with respect to the cosmic time t and $H \equiv \dot{a}/a$ is the Hubble parameter, i.e. the rate of the Universe expansion. At this point we need to stress that $\boldsymbol{W}$ is the *comoving* and not the physical vector field. Indeed, the mass term in Eq. (24) can be written as

$$\delta \mathcal{L}_m \equiv \frac{1}{2} m^2 W_\mu W^\mu = \frac{1}{2} m^2 (W_t^2 - a^{-2} W_i W_i) = -\frac{1}{2} m^2 |\boldsymbol{W}/a|^2 , \tag{30}$$

where we used that $W_t = 0$ and a spatially flat FRW metric $ds^2 = dt^2 - a^2 dx^i dx^i$. From the above it can deduced that the *physical* vector field has spatial components

$$\boldsymbol{A} \equiv \boldsymbol{W}/a . \tag{31}$$

In terms of the physical vector field, Eq. (29) is written as

$$\ddot{A} + 3H\dot{A} + (\dot{H} + 2H^2 + m^2) A = 0 , \tag{32}$$

where $A \equiv |\boldsymbol{A}|$. From Eqs. (28) and (31) one finds

$$\rho_{\rm kin} = \frac{1}{2} (\dot{A} + HA)^2 \quad \text{and} \quad V_A = \frac{1}{2} m^2 A^2 . \tag{33}$$

The solution of Eq. (32) is of the form [23]

$$A = t^{\frac{1}{2}\left(\frac{w-1}{w+1}\right)} \left[c_1 J_d(mt) + c_2 J_{-d}(mt) \right] , \tag{34}$$

where w is the barotropic parameter of the Universe, $d \equiv \frac{1+3w}{6(1+w)}$, c_1, c_2 are constants of integration and J_d denotes Bessel function of the the first kind with order d.

When the physical vector field is light $m \ll H \Leftrightarrow mt \ll 1$ the above solution can be approximated as [23]

$$A = \frac{2}{2w+1}\left(\frac{a}{a_{\rm end}}\right)^{\frac{1}{2}(3w-1)}\left(A_{\rm end} + \frac{\dot{A}_{\rm end}}{H_{\rm end}}\right), \tag{35}$$

where the subscript 'end' denotes the end of inflation. From the above solution it can be shown that [23]

$$\frac{V_A}{\rho_{\rm kin}} \simeq (mt)^2 \ll 1\,. \tag{36}$$

Thus, when the vector field is light its energy density is dominated by its kinetic density. Therefore, [23]

$$\rho_A \simeq \rho_{\rm kin} = \frac{1}{2}\left(\dot{A}_{\rm end} + H_{\rm end}A_{\rm end}\right)^2\left(\frac{a}{a_{\rm end}}\right)^{-4} \Rightarrow \rho_A \propto a^{-4}, \tag{37}$$

i.e. the light vector field scales as radiation with the Universe expansion.

When the physical vector field is heavy $m \gg H \Leftrightarrow mt \gg 1$ the solution in Eq. (34) becomes [23]

$$A = \sqrt{\frac{2}{\pi}}\,t^{-\frac{1}{1+w}}\left[c_1\cos\left(mt - \frac{1+2d}{4}\pi\right) + c_2\cos\left(mt - \frac{1-2d}{4}\pi\right)\right], \tag{38}$$

which shows that the vector field is undergoing rapid quasi-harmonic oscillations whose envelope is decreasing as $\|A\| \propto a^{-3/2}$. This is easy to understand since, for a heavy vector field, within a Hubble time one can ignore the friction term in Eq. (32) and write it as $\ddot{A} + m^2 A \simeq 0$. From Eqs. (33) and (38) it is straightforward to find

$$\rho_A = \frac{1}{\pi}\,t^{-\frac{2}{1+w}}\left[c_1^2 + c_2^2 + 2c_1c_2\cos(d\pi)\right] \Rightarrow \rho_A \propto a^{-3}, \tag{39}$$

where we used $a \propto t^{\frac{2}{3(1+w)}}$ in a spatially flat FRW Universe. Thus, we see that the density of the heavy oscillating vector field scales as pressureless matter with the Universe expansion.

But is it pressureless indeed? From Eqs. (27), (33) and (38) we readily obtain [23]

$$p_\perp = -\frac{1}{\pi}\,t^{-\frac{2}{1+w}}\left[c_1^2\sin(2mt - d\pi) + c_2^2\sin(2mt + d\pi) + 2c_1c_2\sin(2mt)\right] \Rightarrow \overline{p_\perp} = 0\,, \tag{40}$$

i.e. over a Hubble time (which corresponds to a large number of oscillations) the average transverse pressure is zero. Since the longitudinal pressure is $-p_\perp$ this is zero too. This means that the energy-momentum of the rapidly oscillating homogeneous vector field is that or pressureless *isotropic* matter (cf. Eq. (26)). Hence, the vector field can dominate the Universe without introducing excessive anisotropic stress. One way of understanding this is that, due to the harmonic oscillations which send $A \to -A$, the direction of the vector field is rapidly alternated, so that, over a Hubble time, there is *no net direction* and the vector field behaves as an approximately isotropic fluid.

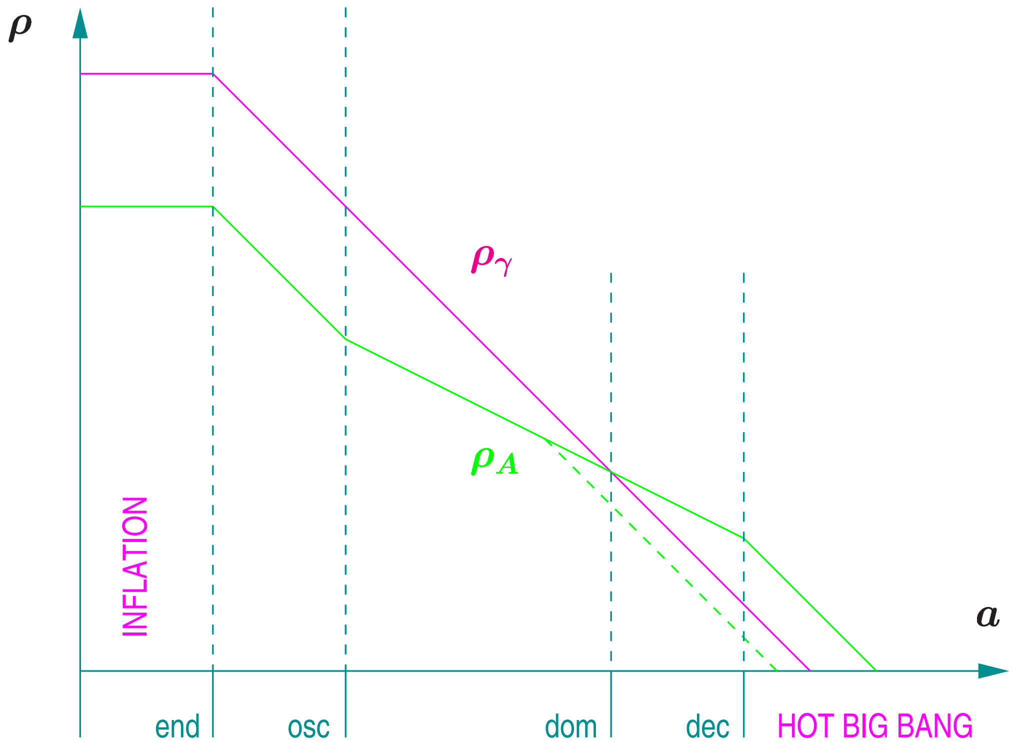

Figure 1: Log-log plot depicting the evolution of the density of the vector curvaton ρ_A and the background radiation due to inflationary reheating ρ_γ, from the end of inflation and until the onset of the hot big bang. During inflation, the vector field has a negligible contribution to the density of the Universe $\rho_A \ll \rho_{\rm inf}$. At the end of inflation (denoted by 'end'), the inflationary energy is given to a thermal bath of radiation $\rho_{\rm inf} \rightarrow \rho_\gamma$ (prompt reheating is assumed for simplicity). After the end of inflation $\rho_\gamma \propto a^{-4}$, which is mimicked by the vector field density while the vector field remains light, i.e. $\rho_A \propto a^{-4}$. Thus, the vector field density parameter $\Omega_A = \rho_A/\rho$ remains constant with $\Omega_A \ll 1$, where $\rho = \rho_\gamma + \rho_A$ is the density of the Universe ($\rho \simeq \rho_\gamma$ during this period). At some later time (denoted 'osc') the vector field becomes heavy and begins oscillating. From then on, it behaves as a pressureless isotropic fluid whose density scales as $\rho_A \propto a^{-3}$. Thus, its density parameter grows as $\Omega_A \propto a$. This allows the oscillating vector curvaton to dominate the Universe at some later moment (denoted by 'dom'), when $\Omega_A \simeq 1$. Afterwards, the vector curvaton decays into the thermal bath of the hot big bang (its decay is denoted by 'dec'). The slanted dashed line corresponds to the possibility when the vector curvaton decays before domination, so that $\Omega_A^{\rm dec} < 1$. In this case the hot big bang begins at the original inflationary reheating.

4.2 The vector curvaton scenario

We have seen that the density of a massive Abelian vector field homogenised by inflation scales as radiation when the field is light and as matter when it becomes heavy. After the end of inflation the energy density is eventually transferred into a newly formed thermal bath of relativistic particles. The density of this thermal bath is dominated by radiation ρ_γ. The homogeneous vector field is initially light so its density also scales as radiation. Therefore, the density parameter $\Omega_A \equiv \rho_A/\rho$ of our vector field remains constant, where $\rho \approx \rho_\gamma$ is the density of the Universe. During radiation domination the Hubble parameter reduces with time as $H(t) = 1/2t$ so that eventually the vector field becomes heavy and begins its coherent oscillations. From then on, its density scales like matter and its density parameter grows $\Omega_A \propto a$. Thus, the vector fiend has a chance to dominate (or nearly dominate) the Universe before its decay. When it does so it imposes its own curvature perturbation onto the Universe, according to the curvaton scenario [16], without introducing any anisotropic stress [22]. A schematic representation of the vector curvaton scenario is presented in Fig. 1.

The superhorizon perturbations of the vector field satisfy the same equation of motion as Eq. (32). The reason is that this equation is linear and that the gradient term is heavily diluted for superhorizon perturbations.[9] Hence, the perturbations follow the same behaviour as the homogeneous zero-mode (with $k = 0$). Thus, when the vector field becomes heavy they undergo quasi-harmonic oscillations too and their anisotropic stress is also eliminated.

The existence of these perturbations of the vector field implies that the density ρ_A is also perturbed and the field's (near) domination of the Universe occurs at slightly different times at different locations. This results in a difference (perturbation) in the timescale of the Universe history, which, according to the δN philosophy, results in a contribution to the curvature perturbation ζ. Since the density ρ_A is a scalar quantity, this is a *scalar* contribution to ζ (and not a vector contribution).

Let us now quantify the above. The curvature perturbation in the Universe is, in principle, the sum of the contribution of the vector field ζ_A and any preexisting curvature perturbation, already present in the radiation fluid ζ_γ. Then we can write [16]

$$\zeta = \zeta_\gamma + \zeta_A = (1 - \hat{\Omega}_A)\hat{\zeta}_\gamma + \hat{\Omega}_A\hat{\zeta}_A \ , \tag{41}$$

where $\hat{\Omega}_A \equiv \frac{3\Omega_A}{4-\Omega_A} \simeq \Omega_A$ and we have assumed that the vector field has already become heavy. In the above, $\hat{\zeta}_i$ corresponds to the curvature perturbation attributed to the i-th component of the Universe content, which, on a spatially flat slice of spacetime, is given by [16]

$$\hat{\zeta}_i \equiv -H\frac{\delta\rho_i}{\dot{\rho}_i} = \frac{1}{3}\frac{\delta\rho_i}{\rho_i + p_i} \ , \tag{42}$$

[9]In momentum space the gradient term is $\nabla^2 A \to (k/a)^2 A$ where k/a is the physical momentum scale, which is $k/a \ll H$ for superhorizon perturbations. Note that the mass of the vector field is much larger than H, when it is oscillating and the same is true for its perturbations.

where we used the continuity equation $\dot{\rho}_i + 3H(\rho_i + p_i) = 0$ for independent fluids. The above suggest that the contribution of the heavy vector field to ζ is

$$\zeta_A = \frac{1}{3}\hat{\Omega}_A \frac{\delta\rho_A}{\rho_A}\,. \tag{43}$$

At the onset of the oscillations $\rho_{\rm kin} \approx V_A$ so that $\rho_A = 2V_A = m^2\|A\|^2$. Thus, to first order we find[10]

$$\zeta_A = \frac{2}{3}\hat{\Omega}_A \frac{\|A_i\|\,\|\delta A_i\|}{\|A\|^2} = \frac{2}{3}\hat{\Omega}_A \frac{A_i \delta A_i}{A^2}\,. \tag{44}$$

From Eq. (3) we see that the contribution of the vector field to ζ to first order is $\zeta_A = N_A^i \delta A_i$. Comparing with the above we get

$$N_A^i = \frac{2}{3}\hat{\Omega}_A \frac{A_i}{A^2} \Rightarrow N_A^{ij} = \frac{2}{3}\hat{\Omega}_A \frac{\delta_{ij}}{A^2}\,. \tag{45}$$

Thus, to second order, the contribution of the vector curvaton to the curvature perturbation is [10]

$$\zeta_A = \frac{2}{3}\hat{\Omega}_A \frac{A_i \delta A_i}{A^2} + \frac{1}{3}\hat{\Omega}_A \frac{\delta A_i \delta A_i}{A^2}\,. \tag{46}$$

Note, however, that for $\delta A_\lambda / A \ll 1$, the one-loop correction (last term in the above) is negligible.

Let us now turn our attention to non-Gaussianity. With the above values of N_A^i and N_A^{ij} and Eqs. (22) and (23) it can be shown that [12]

$$\frac{6}{5} f_{\rm NL}^{\rm eql} = \beta^2 \mathcal{P}_+^2 \frac{3}{2\hat{\Omega}_A} \frac{\left(1 + \frac{1}{2}q^2\right) + \left[p + \frac{1}{8}\left(p^2 - 2q^2\right)\right] \hat{A}_\perp^2}{(\mathcal{P}_\phi + \beta\mathcal{P}_+)^2} \tag{47}$$

and

$$\frac{6}{5} f_{\rm NL}^{\rm sqz} = \beta^2 \mathcal{P}_+^2 \frac{3}{2\hat{\Omega}_A} \frac{1 + p\hat{A}_\perp^2}{(\mathcal{P}_\phi + \beta\mathcal{P}_+)^2}\,, \tag{48}$$

where p and q were defined in Eq. (21) and $\hat{A}_\perp$ is the projection of the unit vector $\hat{\boldsymbol{A}} = \boldsymbol{A}/A$ onto the plane of the three momentum vectors $\boldsymbol{k_1}$, $\boldsymbol{k_2}$ and $\boldsymbol{k_3}$ which are used to define the bispectrum (cf. Eq. (14)).

From Eqs. (47) and (48) is is evident that, in the vector curvaton scenario, one can write

$$f_{\rm NL} = f_{\rm NL}^{\rm iso}\left(1 + \mathcal{G}\hat{A}_\perp^2\right) \tag{49}$$

where $\mathcal{G}$ is the anisotropy parameter for non-Gaussianity. In analogy to g in Eq. (2), $\mathcal{G}$ quantifies statistical anisotropy in the bispectrum of ζ. If $\mathcal{G} \gg 1$, then non-Gaussianity is predominantly anisotropic, which means that $f_{\rm NL}$ should have a clear angular modulation on the sky. If non-Gaussianity is indeed observed and no such modulation is found then models which predict $\mathcal{G} \gg 1$ will be ruled out.

It is important to note above that the directions of statistical anisotropy in the spectrum and the bispectrum are correlated, since they are both determined by $\hat{\boldsymbol{A}}$. This is a smoking gun for the contribution of vector fields to ζ.

[10] The zero-mode and the perturbations begin oscillating simultaneously and in phase.

5 Particle production of vector fields

The vector curvaton mechanism can affect (or even generate) the curvature perturbation provided the vector field has, somehow, obtained a superhorizon spectrum of perturbations during inflation. In order to do this we need to have a mechanism which breaks the conformal invariance of the vector field.

A massless Abelian vector field is conformally invariant, which means that it is not affected by the Universe expansion (it perceives it as a conformal transformation to which it is insensitive). Hence, it does not undergo particle production during inflation. Consequently, its quantum fluctuations do not give rise to classical perturbations of the field as is done, for example, with minimally coupled massless scalar fields. Thus, one expects a light vector field to be approximately conformally invariant and the production of its perturbations to be suppressed. An explicit breakdown of the vector field conformality is, therefore, required. This is model dependent, which suggests that observations might be able to discern between models and provide insight on the underlying theory.

In this section we discuss two specific models which break the conformality of an Abelian vector field and have attracted considerable attention to date. Before going into these models however, let us discuss how we can use any such mechanism to obtain the spectra of perturbations of the vector field components.

Assume, for the moment, that we are indeed operating under a suitable mechanism that breaks the conformality of the vector field. The first step is to perturb the vector field around the homogeneous value as $A_\mu \to A_\mu(t) + \delta A_\mu(\boldsymbol{x}, t)$. Then we Fourier transform the perturbations as

$$\delta \boldsymbol{A}(\boldsymbol{k}, t) \equiv \int \delta \boldsymbol{A}(\boldsymbol{x}, t) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} d^3x$$

and find the equations of motion of the Fourier components $\delta\boldsymbol{A}(\boldsymbol{k}, t)$ for the given model.

The next step is to promote the vector field perturbations to quantum operators by expanding in terms of creation and annihilation operators

$$\delta\hat{\boldsymbol{A}}(\boldsymbol{x}, t) = \int \frac{d^3k}{(2\pi)^3} \sum_\lambda \left[\hat{\boldsymbol{e}}^\lambda \hat{a}_\lambda(\boldsymbol{k}) \delta\mathcal{A}(k, t) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{\boldsymbol{e}}^{\lambda *} \hat{a}^\dagger_\lambda(\boldsymbol{k}) \delta\mathcal{A}^*(k, t) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\right]. \tag{50}$$

The mode functions $\delta\mathcal{A}(k, t)$ and the Fourier components of the perturbations of the vector field satisfy the same equations of motion because the latter are linear. Thus, we need to solve these equations and find the the mode functions. To do this we need to employ the following boundary conditions, which are simply due to the fact that the perturbations begin as quantum fluctuations well within the horizon ($k/aH \to \infty$)

$$\delta\mathcal{A}_{L,R} \underset{k/aH\to\infty}{=} \frac{e^{ik/aH}}{\sqrt{2k}} \quad \text{and} \quad \delta\mathcal{A}_\parallel \underset{k/aH\to\infty}{=} \gamma \, \frac{e^{ik/aH}}{\sqrt{2k}}, \tag{51}$$

where $\gamma \equiv \frac{E}{m} = \sqrt{(\frac{k}{am})^2 + 1}$ is the Lorentz boost factor which takes us from the frame with $\boldsymbol{k} = 0$ (where all components of the vector field perturbation are equivalent) to the one of momentum $\boldsymbol{k}$. Apart from γ, we see that the vacuum boundary conditions are identical to the Bunch-Davis vacuum also employed for the particle production of scalar fields.

Once we solve the equations of motion and find the mode functions we can obtain the power spectra of the superhorizon ($k/aH \to 0$) perturbations using

$$\mathcal{P}_\lambda = \frac{k^3}{2\pi^2}|\delta\mathcal{A}_\lambda|^2_{k/aH\to 0} \; . \tag{52}$$

The typical value of the vector field perturbation is $\delta A_\lambda \sim \sqrt{\mathcal{P}_\lambda}$. Now, let us employ this method on two concrete models for the generation of a perturbation spectrum for the vector field during inflation.

5.1 Non-minimal coupling to gravity

This mechanism was first considered in Ref. [24] for the generation of a primordial magnetic field of superhorizon coherence. It was employed as a vector curvaton in Ref. [25] and also in Ref. [10].

Consider a massive Abelian vector field with a non-minimal coupling to gravity as follows

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(m^2 + \alpha R)W_\mu W^\mu, \tag{53}$$

where R is the scalar curvature and α is a constant. The non-minimal coupling corresponds to a contribution to the effective mass of the field such that

$$m_{\rm eff}^2 \equiv m^2 + \alpha R\,. \tag{54}$$

The equations of motion have been found and solved for the above theory during de Sitter inflation, providing the following exact solutions for the mode functions $\delta\mathcal{A}_\lambda$ of the perturbations. For the transverse components the solution is [25]

$$\delta\mathcal{A}_{L,R} = a^{-3/2}\sqrt{\frac{\pi}{H}}\frac{e^{i\frac{\pi}{2}(\nu-\frac{1}{2})}}{1-e^{i2\pi\nu}}\left[J_\nu\left(\frac{k}{aH}\right) - e^{i\pi\nu}J_{-\nu}\left(\frac{k}{aH}\right)\right], \tag{55}$$

where

$$\nu \equiv \sqrt{\frac{1}{4} - \left(\frac{m_{\rm eff}}{H}\right)^2}. \tag{56}$$

The above solution produces a scale invariant spectrum if $\nu = 3/2$. This can be achieved if $m \ll H$ and $\alpha \approx \frac{1}{6}$, because during de Sitter inflation $R = -12H^2$.[11] With this choice for m and α the solution for the longitudinal mode function is [10]

$$\delta\mathcal{A}_\parallel = \frac{1}{\sqrt{2}}\left[\left(\frac{k}{aH}\right) - 2\left(\frac{aH}{k}\right) + 2i\right]\frac{e^{ik/aH}}{\sqrt{2k}}\,. \tag{57}$$

[11] This theory is in effect a modified gravity theory but it can be shown that the Friedman Equation is not affected if $\alpha = \frac{1}{6}$ and also RW^2 is negligible compared to the Einstein-Hilbert action for $W_\mu W^\mu \ll m_P^2$.

The transverse solutions are the same because the theory is parity invariant. However, it is clear that there is striking difference between the transverse and the longitudinal solutions. Using Eq. (52), we can now find the power spectra for the components of the perturbation. We obtain

$$\mathcal{P}_+ = \left(\frac{H}{2\pi}\right)^2, \quad \mathcal{P}_- = 0 \quad \text{and} \quad \mathcal{P}_\| = 2\left(\frac{H}{2\pi}\right)^2, \tag{58}$$

i.e. $p = 1$ and $q = 0$ as expected (cf. Eq. (21)). Thus we find that particle production is anisotropic at a level of 100%. This means that the vector field contribution to ζ should be subdominant, for otherwise it would violate the observational constrains which do not allow statistical anisotropy above 30%. Hence, we have to assume that ζ is primarily due to some other source, presumably a light scalar field, and the contribution of the vector field is significant only at the level of generating significant statistical anisotropy.

From Eq. (12), we obtain for the anisotropy parameter

$$g = \frac{\beta}{1+\beta} \approx \beta \ll 1 \tag{59}$$

where we considered that $\mathcal{P}_\phi = (H/2\pi)^2$ and also that it is the scalar field which primarily modulates N so that $\beta \ll 1$ in Eq. (13).

Using that $p = 1$ and $q = 0$ in Eqs (47) and (48), we obtain

$$\frac{6}{5} f_{\rm NL}^{\rm eql} = 2\frac{\beta^2}{\Omega_A}\left(1 + \frac{9}{8}\hat{A}_\perp^2\right) \quad \text{and} \quad \frac{6}{5} f_{\rm NL}^{\rm sqz} = 2\frac{\beta^2}{\Omega_A}\left(1 + \hat{A}_\perp^2\right). \tag{60}$$

Thus, we see that $\mathcal{G} \sim 1$. This is because, in this theory, there is only one mass-scale involved, that is H. Dimensionless quantities, therefore, such as $\mathcal{G}$ or p are expected to be of order unity. In Ref. [12] it was shown that, whenever it is so, there is a clear prediction for the maximum non-Gaussianity, which provides a direct link with statistical anisotropy in the spectrum:

$$f_{\rm NL}^{\rm max} \sim 10^3 \left(\frac{g}{0.1}\right)^{3/2}. \tag{61}$$

From the above, it is evident that, through the vector curvaton mechanism, significant non-Gaussianity can be produced.

This theory was criticised in that it may suffer from instabilities such as ghosts [26]. However, it is not clear whether this is indeed so. In Ref. [26], it was shown that the modes of the longitudinal perturbations are ghosts but only when subhorizon. Given that these modes are subhorizon only for a limited time and also in view of the fact that the energy density of inflation is much larger than ρ_A, one may wonder whether these ghosts manage to destabilise the vacuum. A discussion on this issue can be found in Ref. [27].

5.2 Varying kinetic function and mass

Consider a massive Abelian vector boson field, with Lagrangian density

$$\mathcal{L} = -\frac{1}{4} f F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 W_\mu W^\mu, \tag{62}$$

where $f = f(t)$ is the kinetic function, which is approaching unity by the end of inflation so that, afterwards, the vector field is canonically normalised. The above theory does not suffer from instabilities (ghost free) [28, 29], which motivates the use of the Maxwell-type kinetic term even if the field is not a gauge boson. Note here that an Abelian massive vector field is renormalisable even if it is not a gauge boson [30].

The spatial components of the physical canonically normalised vector field in this case are [23]

$$\boldsymbol{A} = \sqrt{f}\,\boldsymbol{W}/a\,. \tag{63}$$

The mass of the physical, canonically normalised vector field is

$$M \equiv \frac{m}{\sqrt{f}}\,. \tag{64}$$

The massless version of this theory has been extensively considered, firstly for inflationary particle production for the generation of a primordial magnetic field [31] and more recently for the mild anisotropisation of inflation, which gives rise to statistical anisotropy in the curvature perturbation through anisotropic particle production of the inflaton [8]. Here we investigate if this vector field can play the role of the vector curvaton. This was first studied in Ref. [21].

The equations of motion for the mode functions for this model were obtained in Ref. [21]. They read

$$\left[\partial_t^2 + \left(H + \frac{\dot{f}}{f}\right)\partial_t + \left(\frac{k}{a}\right)^2 + \frac{m^2}{f}\right]\delta\mathcal{W}_{L,R} = 0 \tag{65}$$

and

$$\left[\partial_t^2 + \left(H + \frac{\dot{f}}{f}\right)\partial_t + \left(2H + 2\frac{\dot{m}}{m} - \frac{\dot{f}}{f}\right)\frac{\left(\frac{k}{a}\right)^2\partial_t}{\left(\frac{k}{a}\right)^2 + \frac{m^2}{f}} + \left(\frac{k}{a}\right)^2 + \frac{m^2}{f}\right]\delta\mathcal{W}_\parallel = 0\,, \tag{66}$$

where $\delta\mathcal{W}_\lambda = a\,\delta\mathcal{A}_\lambda/\sqrt{f}$, cf. Eq. (63). Using the above, particle production has been studied in Ref. [23]. It was found that, the transverse components obtain a scale invariant superhorizon spectrum of perturbations when

$$f \propto a^{-1\pm 3} \quad \text{and} \quad M_* \ll H_* \tag{67}$$

where the subscript '*' denotes the time of horizon exit. To obtain a scale-invariant spectrum for the longitudinal component one needs an additional condition on the time-dependence of $m(t)$, which reads [23]

$$m \propto a\,. \tag{68}$$

If the vector field is a gauge boson then f is the gauge kinetic function which is related with the gauge coupling as $f \sim 1/e^2$. This means that only the case when $f \propto a^{-4}$ is possible since, only then does the gauge field remain weakly coupled during inflation. The gauge kinetic function is one of the three fundamental functions which define a supergravity theory.[12] In supergravity, the gauge kinetic function is a holomorphic function of the scalar fields of the theory. Now, during inflation, supergravity corrections are expected to give masses $\sim H$ to the scalar fields [32]. This means that, during inflation, these scalar fields are fast-rolling down the slopes of the scalar potential, which would cause significant variation to the gauge kinetic function. Indeed, it is easy to show that $\dot{f}/f \sim H$ is natural to expect. Here we should note that, if f is modulated by the inflaton field then, under fairly general conditions, $f \propto a^{-4}$ is an attractor solution during inflation, which arises due to the backreaction of the vector field onto the roll of the inflaton down the inflationary scalar potential [33]. Thus, even though, originally, the inflaton may be also fast-rolling, the backreaction slows down its roll and allows slow-roll inflation to occur even with a relatively steep scalar potential. Indeed, it was shown in Ref. [33] that this is a neat way to overcome the infamous η-problem of inflation, while simultaneously obtaining $f \propto a^{-4}$ as an attractor solution.

Now let us discuss the behaviour of the mode functions $\delta\mathcal{A}_\lambda$ in more detail. Firstly, we define

$$x \equiv \frac{k}{aH} \quad \text{and} \quad z \equiv \frac{M}{3H}\,. \tag{69}$$

From the above and Eq. (64) it is evident that, if $f \propto a^2$ then $z =$ constant, while if $f \propto a^{-4}$ then $z \propto a^3$. Note that $x \propto a^{-1}$ always, while Eq. (67) requires that $z_* \ll 1$.

It turns out that there are three possible stages for the mode evolution [23]. When $x > 1 \gg z$, then the mode is still subhorizon and it is oscillating so that it can be matched to the boundary conditions in Eq. (51). As time passes x decreases and the mode becomes superhorizon. When $x, z \ll 1$ then the mode is found to undergo power-law evolution. The third possible stage has to do with the case $f \propto a^{-4}$ only, when z is growing in time. In this case we could finally reach the time when $z \gtrsim 1 \gg x$, when the superhorizon mode begins oscillating again. Since we are interested in superhorizon scales (they are the ones which can affect the curvature perturbation as observed in the CMB) we will consider the modes which are caught by the end of inflation either when $x, z \ll 1$, or when $z \gtrsim 1 \gg x$, which is possible only with $f \propto a^{-4}$.

5.2.1 Power-law regime

This is the case when $M < 3H \Leftrightarrow z < 1$ when inflation ends. The mode functions for the superhorizon modes are found to be [23]

$$\delta\mathcal{A}_{L,R} = \frac{i}{\sqrt{2k}}\left(\frac{H}{k}\right) \quad \text{and} \quad \delta\mathcal{A}_\parallel = -\frac{1}{\sqrt{2k}}\left(\frac{H}{k}\right)\frac{1}{z}\,. \tag{70}$$

Using this, the power spectra for the superhorizon perturbations of the vector field are

$$\mathcal{P}_+ = \mathcal{P}_{L,R} = \left(\frac{H}{2\pi}\right)^2 \quad \text{and} \quad \mathcal{P}_\parallel = \left(\frac{H}{2\pi}\right)^2\left(\frac{H}{3M}\right)^2, \tag{71}$$

[12] The other two are the Kähler potential and the superpotential.

where we used that the theory is parity invariant. Because $M < 3H$ in this regime we find that $\mathcal{P}_+ \ll \mathcal{P}_\parallel$, which means that particle production is strongly anisotropic. Therefore, if inflation ends in this power-law regime then the contribution of the vector curvaton to the curvature perturbation has to be subdominant (otherwise it would generate excessive statistical anisotropy), so that $\beta \ll 1$, which means that N is primarily modulated by a scalar field and not by our vector curvaton. In this case, therefore, the vector curvaton can only generate some statistical anisotropy in ζ. For the spectrum, the anisotropy parameter is (cf. Eq. (12))

$$g = \beta \frac{\mathcal{P}_\parallel}{\mathcal{P}_+} = \beta / z^2, \tag{72}$$

where $\mathcal{P}_\phi = \mathcal{P}_+$. Similarly, for the non-linearity parameter we obtain

$$\frac{6}{5} f_{\rm NL} = \beta^2 \frac{3}{2\hat{\Omega}_A} \left[1 + \left(p + \frac{1}{8} \kappa p^2 \right) \hat{A}_\perp^2 \right], \tag{73}$$

where $\kappa = 1$ $\{\kappa = 0\}$ for the equilateral {squeezed} configurations and we used Eqs. (47) and (48) and also that $\beta \ll 1$, $q = 0$ and $p = 1/z^2$ (cf. Eq. (21)). Since the vector curvaton must have a subdominant contribution to ζ we have $\Omega_A \ll 1$, which gives $\hat{\Omega}_A \to \frac{3}{4}\Omega_A$. Using this and the above, we find that the isotropic part of non-Gaussianity has

$$f_{\rm NL}^{\rm iso} = \frac{5}{3} \frac{\beta^2}{\Omega_A} = \frac{5}{3} \frac{g^2 z^4}{\Omega_A}, \tag{74}$$

while the anisotropy parameter for non-Gaussianity is

$$\mathcal{G} = p + \frac{1}{8} \kappa p^2 \gg 1, \tag{75}$$

i.e. non-Gaussianity is predominantly anisotropic. Thus, if non-Gaussianity is indeed observed and it does not feature a strong angular modulation this regime of this model will be ruled out.[13]

If $f \propto a^2$ then $M =$constant and, because of Eq. (67), $M = M_* \ll H$. Thus, in this case $\mathcal{P}_+, \mathcal{P}_\parallel =$constant and the above are the only possibility. If, however, $f \propto a^{-4}$ then $M \propto a^3$ and $\mathcal{P}_\parallel$ is decreasing in time. Yet, scale invariance is maintained because the amplitude of the modes when they exit the horizon is reduced in time in accordance to the reduction of the value of the spectrum. As a result, the piling of modes does not spoil the flatness of the superhorizon spectrum [23]. This case offers the possibility that M will reach and surpass H before the end of inflation. If this happens, then we are no more in the power-law regime.

5.2.2 Oscillatory regime

This regime corresponds to the possibility that $M \gg H \Leftrightarrow z \gg 1$ when inflation ends. Because of Eq. (67), when the cosmological scales exit the horizon we have $M_* \ll H$. Thus,

[13]Note here that $\mathcal{G} \gg 1$ because we have two scales into the model, H and M, in contrast to the non-minimally coupled model of the previous section, where we only had one scale H so that $\mathcal{G} \sim 1$.

this regime can be realised only if M is growing during inflation, which is possible only if $f \propto a^{-4}$. Note that, when $M \gtrsim H$, particle production stops and the perturbation of the field on scales that leave the horizon becomes essentially zero (exponentially suppressed). However, the superhorizon scales which left the horizon when the vector field was still light, retain their perturbations, which evolve as follows.

In this case, the mode functions are given by [23]

$$2\sqrt{\frac{H}{\pi}}\left(\frac{k}{H}\right)^{3/2}\delta\mathcal{A}_{L,R} = \frac{i}{\sqrt{z}}J_{1/2}(z)+\frac{1}{3}x^3\sqrt{z}J_{-1/2}(z) \tag{76}$$

$$2\sqrt{\frac{H}{\pi}}\left(\frac{k}{H}\right)^{3/2}\delta\mathcal{A}_{\|} = \frac{1}{3}x^3\sqrt{z}J_{1/2}(z)-\frac{1}{\sqrt{z}}J_{-1/2}(z)\,. \tag{77}$$

Considering superhorizon modes (i.e. $x\to 0$) and also that $z\gg 1$ in this regime, the above simplify to

$$\delta\mathcal{A}_{L,R} = \frac{i}{\sqrt{2H}}\left(\frac{H}{k}\right)^{3/2}\frac{\sin z}{z} \quad \text{and} \quad \delta\mathcal{A}_{\|} = -\frac{1}{\sqrt{2H}}\left(\frac{H}{k}\right)^{3/2}\frac{\cos z}{z}\,. \tag{78}$$

Thus, we see that the mode functions are undergoing rapid coherent oscillations since $z\gg 1$. Using these, we find that the average of the power spectra is

$$\overline{\mathcal{P}_+} = \overline{\mathcal{P}_\|} = \frac{1}{2}\left(\frac{H}{2\pi}\right)^2\left(\frac{H}{3M}\right)^2. \tag{79}$$

Thus, we see that we have isotropic particle production, which implies $p=q=0$ (cf. Eq. (21)). In this case we do not need the input of a scalar field to generate the curvature perturbation as ζ can indeed be fully produced by the vector field alone. To our knowledge this is the only model that manages to produce ζ without the direct involvement of a fundamental scalar field. Because we need no scalar field, $\beta\gg 1$. Then Eqs. (47) and (48) both reduce to

$$f_{\rm NL} = \frac{5}{4\hat{\Omega}_A}\,, \tag{80}$$

which is identical to the case of a scalar curvaton [16].

5.2.3 Borderline regime

It is interesting to briefly consider the so-called "borderline" regime, when $M\sim 3H \Leftrightarrow z\sim 1$ at the end of inflation. Again, this is realisable only if M is growing during inflation, i.e. when $f\propto a^{-4}$.

In this case one has

$$g = p = \frac{\mathcal{P}_\|}{\mathcal{P}_+} - 1 \equiv \frac{\delta\mathcal{P}}{\mathcal{P}_+}\,, \tag{81}$$

where $\delta\mathcal{P} \equiv \mathcal{P}_\parallel - \mathcal{P}_+$. Thus, we see that, to satisfy observational bounds $\mathcal{P}_\parallel \approx \mathcal{P}_+$ at least within 30%. If this is the case then it can be shown that [23]

$$f_{\rm NL} = \frac{5}{4\hat{\Omega}_A}\left(1 + g\hat{A}_\perp^2\right). \tag{82}$$

Therefore, statistical anisotropy in the spectrum and the non-Gaussianity have the same magnitude, i.e. $\mathcal{G} = g$. This is an interesting characteristic signature for this scenario.

5.2.4 The evolution of the zero mode

When $f \propto a^{-1\pm 3}$, it is straightforward to show that the equation of motion for the homogeneous physical vector field is

$$\ddot{\boldsymbol{A}} + 3H\dot{\boldsymbol{A}} + M^2\boldsymbol{A} = 0\,, \tag{83}$$

which looks identical to the Klein-Gordon equation for a minimally coupled massive scalar field.

If $f \propto a^2$ then $M =$constant$\ll 1$ and we have $\boldsymbol{A} \simeq$constant. This means that

$$\rho_A \simeq V_A \sim M_0^2 A_0^2 = \text{constant}\,, \tag{84}$$

where M_0 is the initial value of M ($M = M_0$, since M is constant) and A_0 is the initial value of $A = |\boldsymbol{A}|$.

Now, if $f \propto a^{-4}$ then $M \propto a^3$ and the solution to Eq. (83) is [23]

$$A = A_0\left(\frac{a}{a_0}\right)^{-3}\sqrt{2}\cos\left(z \pm \frac{\pi}{4}\right), \tag{85}$$

which means that the typical value of the vector field is $A \propto a^{-3}$. Using this, Eq. (33) gives

$$\rho_{\rm kin} = \left[A_0 M_0 \sin\left(z \pm \frac{\pi}{4}\right)\right]^2 \quad \text{and} \quad V_A = \left[A_0 M_0 \cos\left(z \pm \frac{\pi}{4}\right)\right]^2, \tag{86}$$

which results in

$$\rho_A = \rho_{\rm kin} + V_A = M_0^2 A_0^2 = \text{constant}\,. \tag{87}$$

Thus, we see that if $f \propto a^{-1\pm 3}$ then $\rho_A =$constant during inflation.

5.2.5 Curvaton physics

Let us briefly look into how the model parameters are constrained by the requirement that the vector field performs as a successful curvaton. As shown in Refs. [23, 28] such considerations impose the following constraint on the model parameters

$$\frac{H_*}{m_P} \sim \hat{\zeta}_A\sqrt{\Omega_A^{\rm dec}}\left(\frac{\max\{\Gamma_A; H_{\rm dom}\}}{\min\{m_A; H_*\}}\right)^{1/4}, \tag{88}$$

where 'dec' denotes the moment of the vector curvaton decay, 'dom' denotes the moment when the vector curvaton dominates the Universe (if it does not decay earlier than that), Γ_A is the decay rate of the vector curvaton and m_A is the final value of its mass at the end of inflation.

If the vector field is still light at the end of inflation, i.e. if $m_A \ll H_*$, we are in the anisotropic regime. Then we obtain [23]

$$\zeta \sim \frac{\Omega_A^{\rm dec} \hat{\zeta}_A}{\sqrt{g}} \, . \tag{89}$$

Using this and also the requirement that the curvaton decays before Big Bang Nucleosynthesis (BBN) (i.e. $\Gamma_A > T_{\rm BBN}^2/m_P$ where $T_{\rm BBN} \sim 1\,{\rm MeV}$ is the temperature at BBN) the constraint in Eq. (88) gives

$$H_* > \sqrt{g} \times 10^7\,{\rm GeV} \quad \text{and} \quad 10\,{\rm Tev} \lesssim m_A \ll H_* \, . \tag{90}$$

If the vector field becomes heavy by the end of inflation, i.e. if $m_A \gtrsim H_*$, then we are in the (almost) isotropic regime. In this case the vector curvaton alone can be responsible for ζ as we have discussed. Thus, we have [23]

$$\zeta = \zeta_A \sim \Omega_A^{\rm dec} \hat{\zeta}_A \Rightarrow \Omega_A^{\rm dec} H_* \gtrsim \zeta^2 m_P \, . \tag{91}$$

This results in the following bound

$$H_* \gtrsim 10^9\,{\rm GeV} \, . \tag{92}$$

The above corresponds to a relatively high scale of inflation which, in supergravity models, may result in gravitino overproduction. However, the latter can be avoided through the entropy release by the vector curvaton decay which can dilute the gravitinos.

Taking into account that the vector curvaton can decay at least through gravitational couplings, we find $\Gamma_A \geq m_A^3/m_P^2$. Using this it can be shown that the oscillations of the massive vector field cannot commence earlier than $N_{\rm osc}^{\rm max} \simeq 4.4$ e-folds of inflation. As a result, the range of m_A for which this regime can be realised is found to be [23]

$$1 < \frac{m_A}{H_*} < 10^6. \tag{93}$$

From Eqs. (90) and (93) we see that there is ample parameter space for both the possibilities (light or heavy vector curvaton) to be realised.

6 Conclusions

The high precision cosmological observations enable cosmologists to investigate beyond the "vanilla" predictions of inflation and thereby discriminate between inflation models. A new such observable is statistical anisotropy, which amounts to direction dependent patterns in the CMB (and possibly large scale structure too). Statistical anisotropy is within the reach of the forthcoming observations of the Planck satellite mission, which is expected to release its first cosmological data by the end of 2012. Currently, the latest CMB observations allow statistical anisotropy in the spectrum as much as 30% ($g \lesssim 0.3$). Planck will reduce this bound down to 2% if statistical anisotropy is not observed. It should be noted here that, even if the spectrum is weakly statistically anisotropic, the bispectrum can be predominantly anisotropic with $\mathcal{G} \gg 1$. This means that, if non-Gaussianity is indeed observed without a strong angular modulation of $f_{\rm NL}$ on the microwave sky, then all models which predict $\mathcal{G} \gg 1$ will be ruled out.

Vector boson fields are natural candidates for the generation of statistical anisotropy in the curvature perturbation, because they are expected to pick a preferred direction when homogenised by inflation. The vector curvaton paradigm offers a simple, generic and effective mechanism for the direct contribution of vector boson fields to the curvature perturbation. The mechanism assumes a Proca vector field, whose zero-mode begins oscillating when the field becomes heavy, after the end of inflation. As shown, the oscillating zero-mode behaves as a pressureless isotropic fluid and can (nearly) dominate the Universe without generating an excessive anisotropic stress. When doing so it imposes its own contribution to the curvature perturbation, in accordance to the curvaton mechanism. We should point out here that this perturbation is scalar in nature, because it is due to the perturbed value of the density of the vector field, which is a scalar quantity. A considerable advantage of the vector curvaton mechanism is that it does not rely on an interaction of any kind between the vector field and the inflaton sectors. This allows the vector field to correspond to physics at a much lower energy scale (e.g. TeV physics) than the scale of inflation, which may connect with observations in collider experiments such as the LHC.

The particle production process, through which the vector curvaton obtains a superhorizon spectrum of perturbations, is in general anisotropic. This is because the vector boson field has more than one degrees of freedom (three if massive), for which the efficiency of the particle production is in general different. Thus, the curvature perturbation contributed by a vector curvaton is, in general, statistically anisotropic. If particle production is indeed isotropic (at least at the level allowed by the observations) the vector curvaton mechanism can generate the curvature perturbation in the Universe from vector fields alone without directly involving any fundamental scalar fields. If, however, statistical anisotropy is indeed observed, then we *have* to go beyond scalar fields to explain the observations. This means that, the observation of statistical anisotropy in the CMB, may probe the gauge field content of theories beyond the standard model.

There are some related issues which we did not go into in this paper. For example, studies of the trispectrum in vector field scenarios [11], or of one-loop contributions [34]. Note also, the possibility that the vector curvaton is non-Abelian is investigated in Ref. [35], while the contribution to the curvature perturbation by P-forms can be found in Ref. [36].

The interest now is in finding realistic candidates in theories beyond the standard model, which can play the role of the vector curvaton. Examples can include the supermassive gauge bosons of grand unified theories [37][14] or the vector fluxes on probe branes in the context of DBI-inflation [39]. Another promising idea is investigating the possibility that the vector bosons associated with gauged axions can play the role of the vector curvaton. In this case, the generation of parity-violating statistical anisotropy is possible [40].

Acknowledgements

I would like to thank my collaborators Mindaugas Karčiauskas, David H. Lyth, Yeinzon Rodriguez-Garcia and Jacques M. Wagstaff. I am grateful to the University of Crete for the hospitality.

References

[1] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **192** (2011) 18.

[2] F. K. Hansen, A. J. Banday and K. M. Gorski, Mon. Not. Roy. Astron. Soc. **354** (2004) 641.

[3] K. Land and J. Magueijo, Phys. Rev. Lett. **95** (2005) 071301; Mon. Not. Roy. Astron. Soc. **378** (2007) 153.

[4] H. K. Eriksen, F. K. Hansen, A. J. Banday, K. M. Gorski and P. B. Lilje, Astrophys. J. **605** (2004) 14 [Erratum-ibid. **609** (2004) 1198].

[5] A. de Oliveira-Costa and M. Tegmark, Phys. Rev. D **74** (2006) 023005.

[6] N. E. Groeneboom and H. K. Eriksen, Astrophys. J. **690** (2009) 1807; N. E. Groeneboom, L. Ackerman, I. K. Wehus and H. K. Eriksen, Astrophys. J. **722** (2010) 452; D. Hanson and A. Lewis, Phys. Rev. D **80** (2009) 063004.

[7] A. R. Pullen and M. Kamionkowski, Phys. Rev. D **76** (2007) 103529.

[8] C. Pitrou, T. S. Pereira and J. P. Uzan, JCAP **0804** (2008) 004; M. a. Watanabe, S. Kanno and J. Soda, Phys. Rev. Lett. **102** (2009) 191302; T. R. Dulaney and M. I. Gresham, Phys. Rev. D **81** (2010) 103532; M. a. Watanabe, S. Kanno and J. Soda, Prog. Theor. Phys. **123** (2010) 1041; S. Kanno, J. Soda and M. a. Watanabe, JCAP **1012** (2010) 024; B. Himmetoglu, JCAP **1003** (2010) 023; A. E. Gumrukcuoglu, B. Himmetoglu and M. Peloso, Phys. Rev. D **81** (2010) 063528; M. a. Watanabe, S. Kanno and J. Soda, Mon. Not. Roy. Astron. Soc. **412** (2011) L83; S. Bhowmick and S. Mukherji, 1105.4455 [hep-th].

[14] These can be associated with the simultaneous generation of a primordial magnetic field [38].

[9] A. A. Starobinsky, JETP Lett. **42** (1985) 152 [Pisma Zh. Eksp. Teor. Fiz. **42** (1985) 124]; M. Sasaki and E. D. Stewart, Prog. Theor. Phys. **95** (1996) 71; D. H. Lyth, K. A. Malik and M. Sasaki, JCAP **0505** (2005) 004.

[10] K. Dimopoulos, M. Karciauskas, D. H. Lyth and Y. Rodriguez, JCAP **0905** (2009) 013.

[11] C. A. Valenzuela-Toledo and Y. Rodriguez, Phys. Lett. B **685** (2010) 120.

[12] M. Karciauskas, K. Dimopoulos and D. H. Lyth, Phys. Rev. D **80** (2009) 023509.

[13] S. Koh and B. Hu, 0901.0429 [hep-th].

[14] A. Golovnev, V. Mukhanov and V. Vanchurin, JCAP **0806** (2008) 009; A. Golovnev and V. Vanchurin, Phys. Rev. D **79** (2009) 103524.

[15] M. C. Bento, O. Bertolami, P. V. Moniz, J. M. Mourao and P. M. Sa, Class. Quant. Grav. **10** (1993) 285; C. Armendariz-Picon, JCAP **0407** (2004) 007; A. Maleknejad and M. M. Sheikh-Jabbari, 1102.1513 [hep-ph]; 1102.1932 [hep-ph].

[16] S. Mollerach, Phys. Rev. D **42** (1990) 313; A. D. Linde and V. F. Mukhanov, Phys. Rev. D **56** (1997) 535; D. H. Lyth and D. Wands, Phys. Lett. B **524** (2002) 5; T. Moroi and T. Takahashi, Phys. Lett. B **522** (2001) 215 [Erratum-ibid. B **539** (2002) 303]; K. Enqvist and M. S. Sloth, Nucl. Phys. B **626** (2002) 395.

[17] D. H. Lyth, JCAP **0511** (2005) 006; L. Alabidi and D. Lyth, JCAP **0608** (2006) 006.

[18] G. Dvali, A. Gruzinov and M. Zaldarriaga, Phys. Rev. D **69** (2004) 023505; Phys. Rev. D **69** (2004) 083505; L. Kofman, astro-ph/0303614; K. Enqvist, A. Mazumdar and M. Postma, Phys. Rev. D **67** (2003) 121303.

[19] S. Yokoyama and J. Soda, JCAP **0808** (2008) 005.

[20] M. Shiraishi and S. Yokoyama, 1107.0682 [astro-ph.CO].

[21] K. Dimopoulos, Phys. Rev. D **76** (2007) 063506.

[22] K. Dimopoulos, Phys. Rev. D **74** (2006) 083502.

[23] K. Dimopoulos, M. Karciauskas and J. M. Wagstaff, Phys. Rev. D **81** (2010) 023522.

[24] M. S. Turner and L. M. Widrow, Phys. Rev. D **37** (1988) 2743.

[25] K. Dimopoulos and M. Karciauskas, JHEP **0807** (2008) 119.

[26] B. Himmetoglu, C. R. Contaldi and M. Peloso, Phys. Rev. D **79** (2009) 063517; Phys. Rev. Lett. **102** (2009) 111301; Phys. Rev. D **80** (2009) 123530.

[27] M. Karciauskas and D. H. Lyth, JCAP **1011** (2010) 023.

[28] K. Dimopoulos, M. Karciauskas and J. M. Wagstaff, Phys. Lett. B **683** (2010) 298.

[29] S. M. Carroll, T. R. Dulaney, M. I. Gresham and H. Tam, Phys. Rev. D **79** (2009) 065011; T. R. Dulaney, M. I. Gresham and M. B. Wise, Phys. Rev. D **77** (2008) 083510 [Erratum-ibid. D **79** (2009) 029903].

[30] J. M. Cornwall, D. N. Levin and G. Tiktopoulos, Phys. Rev. D **10** (1974) 1145.

[31] M. Giovannini, Phys. Rev. D **64** (2001) 061301; K. Bamba and J. Yokoyama, Phys. Rev. D **69** (2004) 043507; Phys. Rev. D **70** (2004) 083508; O. Bertolami and R. Monteiro, Phys. Rev. D **71** (2005) 123525; J. M. Salim, N. Souza, S. E. Perez Bergliaffa and T. Prokopec, JCAP **0704** (2007) 011; K. Bamba and M. Sasaki, JCAP **0702** (2007) 030; J. Martin and J. Yokoyama, JCAP **0801** (2008) 025; K. Bamba and S. D. Odintsov, JCAP **0804** (2008) 024; K. Bamba, C. Q. Geng and S. H. Ho, JCAP **0811** (2008) 013; S. Kanno, J. Soda and M. a. Watanabe, JCAP **0912** (2009) 009.

[32] M. Dine, L. Randall and S. D. Thomas, Nucl. Phys. B **458** (1996) 291; Phys. Rev. Lett. **75** (1995) 398; D. H. Lyth and T. Moroi, JHEP **0405** (2004) 004.

[33] K. Dimopoulos and J. M. Wagstaff, Phys. Rev. D **83** (2011) 023523.

[34] C. A. Valenzuela-Toledo, Y. Rodriguez and D. H. Lyth, Phys. Rev. D **80** (2009) 103519.

[35] N. Bartolo, E. Dimastrogiovanni, S. Matarrese and A. Riotto, JCAP **0910** (2009) 015; JCAP **0911** (2009) 028; Adv. Astron. **2010** (2010) 752670; K. Murata and J. Soda, JCAP **1106** (2011) 037; M. Karciauskas, 1104.3629 [astro-ph.CO].

[36] C. Germani and A. Kehagias, JCAP **0903** (2009) 028; JCAP **0911** (2009) 005; T. S. Koivisto and N. J. Nunes, Phys. Rev. D **80** (2009) 103509; T. S. Koivisto, D. F. Mota and C. Pitrou, JHEP **0909** (2009) 092; T. S. Koivisto and N. J. Nunes, Phys. Lett. B **685** (2010) 105.

[37] Jacques M. Wagstaff, K. Dimopoulos and G. Lazarides, in preparation.

[38] K. Dimopoulos, 0806.4680 [hep-ph].

[39] K. Dimopoulos, Y. Zavala and D. Wills, in preparation.

[40] K. Dimopoulos and M. Karciauskas, in preparation.