Safety of Minkowski Vacuum

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Abstract

We give a simple argument suggesting that in a consistent quantum field theory tunneling from Minkowski to a lower energy vacuum must be impossible. Theories that allow for such a tunneling also allow for localized states of negative mass, and therefore, should be inconsistent.

The idea that we may be living in a false vacuum was a source of a lot of imagination about the possible fate of our Universe. Our vacuum is Minkowskian with a great accuracy. Thus, as a first step would be desirable to better understand the prospects of tunneling from the Minkowaski vacuum to a lower energy state.

A systematic study of this issue was pioneered by Coleman and De Luccia [1], who showed, that the tunneling goes through the formation of a vacuum bubble, and has non-zero probability as long as the size of the critical bubble is finite. They also pointed out that in certain cases, although the lower vacuum does exist, tunneling never takes place, because the effective size of the critical bubble is infinite. This mechanism is referred to as Coleman-De Luccia suppression.

In this short note, we wish to point out that in any consistent quantum field theory tunneling from a Poincare-invariant (Minkowski) vacuum should never happen. This means that the lower energy vacuum (if it exists) must be above the Coleman-De Luccia bound. If this is not the case, the theory is inconsistent. Thus, one way or the other, a Poincare-invariant Minkowski vacuum should be stable by consistency of the theory.

The argument for this stability is pretty simple. In order to prove it, let us imagine the opposite, and get convinced that we will encounter an inconsistency. So let us imagine, that we are in a Minkowski vacuum from which tunneling to a lower energy vacuum is possible. This means that a critical bubble of the true vacuum has a finite size. Such a bubble of course has a zero total energy. However, the finiteness of its size means that (by continuity) the same potential admits also other configurations that have negative total energy. For any smooth scalar potential that admits a zero energy bubble of finite size, the negative energy bubble can always be prepared by deforming the zero energy one. Of course, such a bubble will not be a static solution of the equations of motion, but there is no need for this. If such configuration exists, it will represent a localized object of a *negative* (ADM [2]) mass. Any theory admitting such objects is a disaster. This is obvious already from the fact that from large distance point of view such a negative-mass bubble will look as a negative-mass particle, and should share responsibility for all the trouble that such particles cause.

To a reader that is still not convinced that having a negative mass objects is a killer for a quantum field theory defined on a Poincare-invariant background, we offer to perform the following thought experiment.

Consider a decay of the Minkowski vacuum into a negative-energy bubble (bubble$_-$) plus some positive-energy wave-packet (bubble$_+$),

$$\text{vacuum} \rightarrow \text{bubble}_- + \text{bubble}_+ . \tag{1}$$

The role of the bubble$_+$ can be played by any positive-energy excitation in the theory, in particular by a positive energy lump of the same scalar field that allows interpolation between Minkowski and AdS vacua. The problem is, that in an interacting field theory that admits localized negative energy states such a process is impossible to forbid. Once not forbidden, it has an infinite rate, because due to Poincare-invariance the two lumps can be produced at an arbitrary relative momenta.

It is the simplest to understand the physical meaning of this infinity from the point of view of a large-distance observer that is observing the process at distances larger than a characteristic size of the lumps. For such an observer process is seen as a pair-creation of negative and positive mass particles, with four-momenta p_μ and q_μ. By Poincare-invariance of the background, the rate of pair-creation, Γ, can only depend on invariant scalar products p^2, q^2 and pq. However because of four-momentum conservation, which implies $p = -q$, all invariants are the same and equal to particle mass^2 ($\equiv m^2$) . Thus, $\Gamma(p^2) = \Gamma(m^2)$ is momentum-independent. Consequently, the total rate obtained by integrating over all final relative momenta ($p - q = 2p$) is infinite.

The infinity appears because the pair has opposite masses and thus the same velocities. This infinity is physical. Such a divergent summation over the relative momentum can never take place in theories where there are no negative energy states, since the final momenta are restricted by energy conservation/positivity.

Another way to see that the divergent summation over the relative momentum is unavoidable, is to introduce a Poincare-violating preferred frame parameterized by an infinitesimal vector ϵ_μ and then take the limit $\epsilon \rightarrow 0$. The role of such a vector is to set a "soft" reference frame. The role of such a frame can be played e.g., by an uniform density of a spectator scalar particle of a positive mass ϵ, which can decay into the above pair of positive and negative energy states. An uniform density of such particles then sets the preferred frame described by vector ϵ_μ. Such uniform density can be taken to be arbitrarily small and respectively its effect on the decay can be made as small as one wishes. The vacuum decay is recovered in the zero density limit.

In the presence of such a frame, the integration over relative momentum is an obvious necessity. Of course, for any non-zero ϵ_μ the rate now can also depend on the the products $p_\mu \epsilon^\mu$, but this dependence disappears in the limit $\epsilon = 0$.

We thus see, that the only consistent Poincare-invariant vacua are the exactly stable ones. Lower energy vacua either should not exist or be above the Coleman-De Luccia bound.

An immediate consistency check for our claim is to note that in supergravity the Minkowski vacua, which are known to allow only positive energy states [3], are also stable under tunneling [4]. But our arguments show that the stability of Minkowski vacuum is a matter of consistency regardless of supersymmetry. A consistent theory cannot be formulated on a metastable Poincare-invariant vacuum. Since our argument is based solely on the Poincare-invariance, the only possible loophole would be to explicitly break it. This possibility is beyond our interest (although see [5]).

Above reasoning straightforwardly applies and restricts other possible forms of scalar potentials that interpolate between the Minkowskli and AdS vacua with or without barriers.

Regarding our own vacuum, although it is not exactly Minkowskian the same reasoning should apply, due to tiny difference. Even if tunneling can take place it will be rather soft. So the vacuum stability is not on the list of things that our civilization has to worry about.

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[5] Such a Poincare-violation may be ”self-imposed” due to finite life-time of the vacuum (private communications with S. Mukhanov). This would be consistent with our results.