

Dynamical vs spectator models of (pseudo-)conformal Universe

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Abstract

We discuss two versions of the conformal scenario for generating scalar cosmological perturbations: a spectator version with a scalar field conformally coupled to gravity and carrying negligible energy density, and a dynamical version with a scalar field minimally coupled to gravity and dominating the cosmological evolution. By making use of the Newtonian gauge, we show that (i) no UV strong coupling scale is generated below M_{Pl} due to mixing with metric perturbations in the dynamical scenario, and (ii) the dynamical and spectator models yield identical results to the leading *non-linear* order. We argue that these results, which include potentially observable effects like statistical anisotropy and non-Gaussianity, are characteristic of the entire class of conformal models. As an example, we reproduce, within the dynamical scenario and working in comoving gauge, our earlier result on the statistical anisotropy, which was originally obtained within the spectator approach.

1 Introduction

Recently, an idea of attributing the flatness of the scalar spectrum of the primordial cosmological perturbations to approximate conformal symmetry, rather than approximate de Sitter symmetry of inflationary theory, has attracted some interest [1, 2, 3] (see Ref. [4] for earlier work). In the simplest version of the conformal scenario, one assumes that the gravity effects are totally negligible and considers a theory of two fields ϕ and θ of conformal weights 1 and 0, respectively¹, with the Lagrangian (mostly negative signature)

$$L_{\phi,\theta} = L_\phi + \frac{1}{2}\phi^2 (\partial_\mu\theta)^2 \quad (1)$$

¹The conformal weight of ϕ may be different from 1, it is important only that this weight is non-zero [3].

in effectively Minkowski space-time. In other words, this version assumes that the field ϕ (as well as θ) is a spectator whose dynamics does not affect the space-time metric. An example is a theory of complex scalar field conformally coupled to gravity, whose energy density is negligible compared to the total energy density in the Universe, with ϕ and θ identified with the modulus and phase of that complex field [1].

The background field ϕ_c is assumed to be spatially homogeneous and evolving non-trivially; then conformal invariance implies that

$$\phi_c(t) = -\frac{\sqrt{2}}{\sqrt{\lambda t}}, \quad t < 0, \quad (2)$$

where λ is a dimensionless constant and one assumes that $\lambda \ll 1$ for canonically normalized ϕ . The notations here are chosen in such a way that $\phi_c(t)$ is a solution in a theory with negative quartic potential,

$$L_\phi = \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi), \quad V(\phi) = -\frac{\lambda}{4} \phi^4. \quad (3)$$

In this background, the field θ starts off in the WKB regime which holds as long as $k|t| \gg 1$ (short wavelengths), where k is the spatial momentum of a θ -mode. One naturally considers the initial vacuum state. Fluctuations of θ freeze out at late times, when $k|t| \ll 1$ (large wavelength regime). These fluctuations have flat power spectrum and are thought of as precursors of the adiabatic perturbations. The conversion of θ -perturbations into adiabatic ones occurs at much later stage via, e.g., curvaton [5] or modulated decay [6] mechanism.

A peculiar feature of this scenario is the existence of the perturbations of the field ϕ itself. In spectator models, these perturbations have *red* power spectrum in the large wavelength regime [1, 3]. Interaction of the field θ with perturbations of ϕ leads to potentially observable effects, such as statistical anisotropy and specific forms of non-Gaussianity [7, 8, 9, 10]. These effects are quite generic, since both the properties of ϕ -perturbations and their interaction with θ are dictated by conformal invariance. The latter point is discussed, within the spectator approach, in Refs. [3, 10].

Instead of considering the spectator version of the conformal scenario, it is of interest to study dynamical (pseudo-)conformal models, i.e., treat the fields ϕ , θ as the only matter fields relevant at the early epoch. In this class of models, the rolling field $\phi_c(t)$ determines the homogeneous evolution of the Universe, while the perturbations of ϕ come together with metric perturbations. One model of this sort has been proposed by Hinterbichler and Khoury [3], who considered a theory with the action

$$S = -\frac{M_{Pl}^2}{2} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} L_{\phi\theta}, \quad (4)$$

where the scalar Lagrangian is given by Eqs. (1), (3). In the Hinterbichler–Khoury model, the rolling scalar field (2) has super-stiff equation of state, $p_c \gg \rho_c > 0$, and the Universe

contracts. So, this model is in a certain sense a reincarnation of the ekpyrosis scenario [11]. Another model is Galilean Genesis [2], in which the field ϕ is conformal Galileon [12]. In the latter model, pressure is negative and violates null energy condition, space-time is initially Minkowskian, the energy density increases in time, expansion speeds up, and eventually transition to the hot epoch occurs in some way (see Ref. [13] for the discussion of the last, “defrosting” stage). Clearly, both of these models are interesting alternatives to the inflationary scenario.

In dynamical conformal models, the properties of the perturbations $\varphi = \phi - \phi_c$ and associated metric perturbations are somewhat subtle. While the power spectrum of φ is red in the absence of gravity (in the large wavelength regime), the power spectrum of the curvature perturbations ζ is *blue* in dynamical models [2, 3] (see also Ref. [14]). This feature has lead Hinterbichler and Khoury [3] to argue, on the basis of power-counting, that the theory with the scalar Lagrangian (3) has UV strong coupling scale

$$\Lambda^{(1)} = \frac{\phi_c^3}{M_{Pl}^2}, \quad (5)$$

which is time-dependent and small at early times. They also argued that adding the field θ with the Lagrangian written in (1) yields another UV strong coupling scale

$$\Lambda^{(2)} = \lambda^{1/4} \phi_c. \quad (6)$$

With so low strong coupling scales, self-consistency of the model would imply extremely strong constraints on the self-coupling parameter λ [3].

Another side of the subtlety with φ -perturbations is that their mixing with metric is apparently important for all scales, at least in the large wavelength regime. So, one may doubt that the results of Refs. [7, 8, 9, 10], obtained within the spectator approach, are valid in dynamical conformal models as well.

In this note we intend to clarify these issues, making use of the Hinterbichler–Khoury model as an example. Concerning strong coupling in UV, we recall that naive power counting may or may not give correct results depending on the gauge choice. A famous example is given by non-Abelian gauge theories with the Higgs mechanism, where power counting in the unitary gauge suggests the UV strong coupling scale of the order of the Higgs expectation value, while power counting in R_ξ gauge shows that there is no strong coupling in UV at all². In the Hinterbichler–Khoury model, as well as in Galilean Genesis, the energy density is small at relevant times, and space-time is almost flat. It is well known that in such a situation it is appropriate to make use of the Newtonian gauge. We employ the Newtonian gauge in this note and show that metric perturbations are in fact small³, non-linear gravitational effects

²Modulo the Landau pole, which is another story.

³This has been pointed out in Ref. [2] in the context of Galilean Genesis.

are suppressed, and no UV strong coupling scale is generated below M_{Pl} . Hence, the UV scales (5) and (6) are actually not there. As a consequence, one does not have to impose strong constraints on the self-coupling λ .

Likewise, in the Newtonian gauge the main non-linear effects are due to the interaction of θ with perturbations φ , while the effects coming from the interaction of θ with metric perturbations are small. This shows that the analysis of Refs. [7, 8, 9, 10] does apply to dynamical conformal models. As an illustration of this general conclusion, we rederive one of the results of Ref. [7] (statistical anisotropy) within the dynamical model and in the gauge $\varphi = 0$, where the gravitational potential Ψ coincides with ζ and has blue power spectrum. This derivation also sheds some light on the cancellation of infrared effects, which was somewhat surprising in the spectator approach [7].

This paper is organized as follows. We introduce the Hinterbichler–Khoury model in Section 2. In Section 3 we discuss the strong coupling issue. The leading non-linear effects are considered in Section 4. We conclude in Section 5.

2 The model

We consider the model with the action (4) where the scalar Lagrangian is given by (1), (3). We first recall the properties of the homogeneous, spatially flat background at early times. As $t \rightarrow -\infty$, the gravitational effects on the background field ϕ_c are negligible, and ϕ_c rolls down negative quartic potential according to (2). The pressure is

$$p_c = \dot{\phi}_c^2 = \frac{2}{\lambda t^4},$$

while the energy density is small, $\rho_c \ll p_c$. Upon integrating the Raychaudhuri equation one finds the Hubble parameter

$$H = \frac{1}{3\lambda t^3 M_{Pl}^2}. \quad (7)$$

The Universe contracts, and matter in it has super-stiff equation of state. This regime persists as long as⁴

$$\lambda t^2 M_{Pl}^2 = \frac{2M_{Pl}^2}{\phi_c^2} \gg 1. \quad (8)$$

In this paper we consider exclusively early times, when the inequality (8) holds. It is worth noting that we can consistently set the scale factor equal to 1 wherever it enters without time derivative(s), and identify conformal time and cosmic time.

⁴As ϕ_c approaches M_{Pl} , the effect of the cosmological expansion on the evolution of ϕ_c becomes important ($H\dot{\phi}_c$ becomes comparable to $\ddot{\phi}_c$), energy density catches up with pressure, and the regime (1), (7) terminates.

We are primarily interested in perturbations $\varphi = \phi - \phi_c$ and associated metric perturbations. To simplify the formulas below, we partially fix the gauge by setting the longitudinal component of spatial metric perturbation equal to zero, so that

$$ds^2 = (1 + 2\Phi)dt^2 + 2\partial_i U dt dx^i - (1 + 2\Psi)\delta_{ij} dx^i dx^j . \quad (9)$$

We recall that in a general theory of one scalar field with canonical kinetic term, the gauge invariant combination (prime denotes the derivative with respect to conformal time)

$$\frac{v}{a} = \varphi - \frac{a\phi'_c}{a'}\Psi \quad (10)$$

is canonically normalized and obeys the equation

$$v'' - \Delta v - \frac{z''}{z}v = 0 , \quad (11)$$

where

$$z = \frac{a^2\phi'_c}{a'} .$$

In the case at hand $a = 1$ and $z = \dot{\phi}_c/H$, so the last term in the left hand side of Eq. (11) is negligible (it is of order $(\lambda t^4 M_{Pl}^2)^{-1} \cdot v$, see Ref. [3]). Hence, the field v is a free scalar field in effectively Minkowski space-time. The variable ζ , the curvature perturbation at comoving hypersurfaces, is related to v by

$$\zeta = -\frac{a'}{a^2\phi'_c} \cdot v = -\frac{H}{\dot{\phi}_c} \cdot v .$$

Both v and ζ have blue power spectrum [2, 3].

3 Strong coupling scales and absence thereof

3.1 Spurious scales $\Lambda^{(1)}$ and $\Lambda^{(2)}$: gauge $\varphi = 0$

To see the spurious appearance of the UV scales (5) and (6), let us choose the gauge $\varphi = 0$. In this gauge, the gravitational potential Ψ coincides with ζ , so our discussion parallels that of Hinterbichler and Khoury [3], who worked with the gauge-invariant variable ζ . Let us consider short wavelength modes, $k|t| \gg 1$. Then in the gauge $\varphi = 0$ one has from (10)

$$\Psi \sim \frac{1}{\sqrt{\lambda} M_{Pl}^2 t} v .$$

The constraint equations in this gauge are

$$\Delta U = -3\Psi' + \frac{a}{a'}\Delta\Psi + \frac{a^3}{a'}\frac{\rho_c - p_c}{2M_{Pl}^2}\Phi = -3\dot{\Psi} + \frac{1}{H}\Delta\Psi - \frac{p_c}{2M_{Pl}^2 H}\Phi, \quad (12)$$

$$\Phi = \frac{a}{a'}\Psi' = \frac{1}{H}\dot{\Psi}, \quad (13)$$

where we specified to the model at hand by writing the second expressions. We find from (13) that

$$h_{00} = 2\Phi \sim \sqrt{\lambda}t^2kv \quad (14)$$

and from (12) we see that $\partial_i U = h_{0i}$ has two large contributions

$$h_{0i} \sim \sqrt{\lambda}t^2kv \quad (15)$$

and

$$h_{0i} \sim \sqrt{\lambda}tv. \quad (16)$$

The naive power counting suggests the UV scale $\Lambda^{(1)}$ in the following way. There are cubic terms in the gravitational Lagrangian

$$L_{int} = M_{Pl}^2 h \partial h \partial h. \quad (17)$$

With h estimated by (16) this gives

$$L_{int} = M_{Pl}^2 (\sqrt{\lambda}t)^3 v \partial v \partial v = \frac{M_{Pl}^2}{\phi_c^3} v \partial v \partial v.$$

This can indeed be (mis)understood as the evidence for the UV scale $\Lambda^{(1)}$ given by (5). Perturbations estimated according to (14), (15) naively yield strong coupling scale which is even lower than $\Lambda^{(1)}$.

Likewise, the scale $\Lambda^{(2)}$ pops up in this gauge as follows. The interaction term in the θ sector reads

$$L_{int}^{h\chi\chi} = h \partial \chi \partial \chi, \quad (18)$$

where $\chi = \phi_c \theta$ is canonically normalized. The estimates (14), (15) give

$$L_{int}^{h\chi\chi} = \sqrt{\lambda}t^2 \partial v \partial \chi \partial \chi \sim \frac{1}{\sqrt{\lambda}\phi_c^2} \partial v \partial \chi \partial \chi.$$

This can be viewed as an indication of the strong coupling scale $\Lambda^{(2)} = \lambda^{1/4}\phi_c$.

3.2 No strong coupling below M_{Pl} : Newtonian gauge

Using the gauge $\varphi = 0$ is, however, not appropriate for studying the UV properties of the theory. In this gauge, metric perturbations are huge at early times. The reason is that the field ϕ fluctuates, but carries very little energy. Space-time is very close to Minkowskian, but hypersurfaces of constant φ are embedded in it in a cumbersome way.

Much clearer is the Newtonian gauge, in which $U = 0$. In this gauge

$$\Psi = -\Phi ,$$

and Φ obeys

$$\Delta\Phi = \frac{1}{2M_{Pl}^2}\phi'_c \frac{z}{a} \frac{\partial}{\partial\eta} \left(\frac{v}{z}\right) . \quad (19)$$

So, metric perturbation is small in this gauge at early times,

$$\Phi \sim \frac{1}{M_{Pl}^2\sqrt{\lambda}t^2}k^{-1}v \sim \frac{\sqrt{\lambda}\phi_c^2}{M_{Pl}^2}k^{-1}v , \quad (20)$$

where we again assume $k|t| \gg 1$. By plugging this into (17) we see that no strong coupling scale is generated in UV due to the interaction of φ with metric perturbations. This shows that there is actually no strong coupling in the ϕ sector of the theory at momenta below M_{Pl} , i.e., the scale $\Lambda^{(1)}$ is spurious.

The final point concerns the θ sector. The interaction term with metric perturbation is given by (18). In the Newtonian gauge this gives

$$L_{int}^{h\chi\chi} = \frac{1}{M_{Pl}^2\sqrt{\lambda}t^2}\partial^{-1}v\partial\chi\partial\chi = \frac{\sqrt{\lambda}\phi_c^2}{M_{Pl}^2}\partial^{-1}v\partial\chi\partial\chi .$$

The dimensionless coupling here is small provided that the inequality (8) holds. So, the scale $\Lambda^{(2)}$ is spurious too.

4 Leading interactions of θ

As discussed in detail in Refs. [7, 8, 9, 10] within the spectator approximation, potentially interesting effects are due to the interaction of the field θ with perturbations φ at the cubic level. In the dynamical Hinterbichler–Khoury model, the relevant terms in the action are of the zeroth and first order in perturbations of ϕ and metric,

$$S_\theta^{(0)} = \int d^4x \frac{1}{2}a^2\phi_c^2\eta^{\mu\nu}\partial_\mu\theta\partial_\nu\theta ,$$

and

$$S_\theta^{(1)} = \int d^4x \frac{1}{2} a^2 \phi_c^2 \left[\left(-\Phi + 3\Psi + 2\frac{\varphi}{\phi_c} \right) \theta'^2 - \left(\Phi + \Psi + 2\frac{\varphi}{\phi_c} \right) \partial_i \theta \partial_i \theta + 2\partial_i U \theta' \partial_i \theta \right], \quad (21)$$

respectively, where we still partially fix the gauge according to (9). In the spectator model, one has $\Psi = \Phi = U = 0$, while the perturbation φ coincides with v in the short wavelength regime $k|t| \gg 1$ and is given by [1]

$$\varphi = -\frac{3}{k^2 t^2} v \quad (22)$$

in the large wavelength regime $k|t| \ll 1$. To see that the results of the analysis in the spectator approximation are valid in the dynamical model as well, it is again convenient to use the Newtonian gauge.

4.1 Interactions in the Newtonian gauge

In the first place, let us check that the perturbations φ are the same in the Newtonian gauge as in the spectator approximation. In the short wavelength regime, one finds from (20) that

$$\frac{a\phi_c'}{a'} \Psi \sim \frac{1}{kt} v \ll v,$$

so that φ is indeed equal to v . In the large wavelength regime Eq. (19) gives

$$\Phi = -\Psi = \frac{1}{2M_{Pl}^2} \frac{\sqrt{2}}{\sqrt{\lambda t^3 k^2}} v. \quad (23)$$

Therefore, we find from (10) that the leading part of φ is indeed given by (22). As a cross check, it is straightforward to see that the metric perturbations (given by (20) and (23) in short and long wavelength limits, respectively) give contributions to the field equation for φ which are suppressed at least by $(\lambda M_{Pl}^2 t^2)^{-1}$. Hence, gravity does not modify φ in the Newtonian gauge.

Now, in the Newtonian gauge one always has

$$\Phi \ll \frac{\varphi}{\phi_c}.$$

Indeed, it follows from (20) that in the short wavelength limit the gravitational potential is doubly suppressed,

$$\frac{\Phi}{\varphi/\phi_c} = \frac{\Phi\phi_c}{v} \sim \frac{1}{\lambda M_{Pl}^2 t^2} \frac{1}{kt},$$

while it follows from (22) and (23) that in the large wavelength limit

$$\frac{\Phi}{\varphi/\phi_c} \sim \frac{1}{\lambda M_{Pl}^2 t^2}.$$

Hence, the gravitational potentials are subdominant in the action (21), as compared to φ . This shows that calculations in the spectator approximation give correct results in the dynamical model, modulo corrections suppressed by $(\lambda M_{Pl}^2 t^2)^{-1}$.

4.2 Statistical anisotropy in $\varphi = 0$ gauge

The statistical anisotropy in the power spectrum of θ , and hence in the resulting adiabatic perturbations, is due to the interaction of the field θ with those modes of φ and metric which are still superhorizon today. The analysis of this effect has been performed in Ref. [7] in the spectator approximation. Here we repeat this analysis in the gauge $\varphi = 0$ to see that the results of the spectator approximation are indeed reproduced, and also get better understanding of the cancellation of infrared effects.

It will become clear shortly that as long as $\phi_c \ll M_{Pl}$, our analysis is valid for both the spectator model of Ref. [1] and the dynamical Hinterbichler–Khouri model. For the time being, we specify to the dynamical scenario.

We are interested in the propagation of the field θ in perturbed background in the case when the wavelength of θ is much shorter than the wavelength of φ and metric perturbations. Indeed, the relevant adiabatic perturbations, and hence the modes of θ , are subhorizon today, while the perturbations of φ and metric, responsible for the statistical anisotropy, are still superhorizon. So, we consider φ and metric perturbations in the large wavelength regime. In the gauge $\varphi = 0$ we find from (10) that

$$\Psi = -\frac{1}{3\sqrt{2}\sqrt{\lambda}tM_{Pl}^2}v ,$$

while the constraint equations (12) and (13) give

$$\Phi = \frac{\sqrt{\lambda}}{\sqrt{2}}tv , \tag{24}$$

$$\Delta U = -\frac{3}{\sqrt{2}}\sqrt{\lambda}v , \tag{25}$$

where we made use of the fact that the terms with Ψ in (12) are small compared to the last term. The result (25) can be understood as follows. In the Newtonian gauge (as well as in the spectator approximation), large wavelength perturbation (22) of the field ϕ can be viewed as inhomogeneous time-shift of the background field (2) (cf. Refs. [1, 2]),

$$\delta t(\mathbf{x}) = -\frac{3\sqrt{\lambda}}{\sqrt{2}}\frac{1}{\Delta}v(\mathbf{x}) .$$

Gauge transformation to the gauge $\varphi = 0$ corresponds to the change of the time coordinate $t \rightarrow t + \delta t$. This induces

$$U(\mathbf{x}) = \delta t(\mathbf{x}) , \tag{26}$$

which is precisely the result (25). Note that since v is a free canonically normalized massless field in effectively Minkowski space-time, the power spectra of both φ and δt in the Newtonian gauge are red, but the power spectrum of metric perturbation $h_{0i} = \partial_i U$ is flat in the gauge $\varphi = 0$, while Φ and Ψ have blue power spectra in that gauge. This explains why the power law infrared enhancement of φ , visible in the spectator approximation, is actually irrelevant. Note also that Eq. (26) shows that the form of U is common to the dynamical and spectator models, so the analysis below and the result (30) are valid in both of these models.

Non-vanishing $U(\mathbf{x})$ yields the anisotropic last term in the action (21), We are going to keep at most two derivatives of U , and study anisotropy proportional to $\partial_i \partial_j U$: since we are interested in very large wavelengths of the perturbed background, higher derivatives are suppressed. For the canonically normalized field $\chi = \phi_c \theta$ we get, omitting isotropic terms,

$$S_\theta^{(1)} = \int d^4x \, 2\partial_i U \dot{\chi} \partial_i \chi .$$

Therefore, the field equation reads, again modulo isotropic corrections,

$$\ddot{\chi} - \Delta \chi - \frac{2}{t^2} \chi + 2\partial_i U \partial_i \dot{\chi} = 0 . \quad (27)$$

Let us search for the solution in the following form:

$$\chi = e^{i\mathbf{kx} - ikt - ikU(\mathbf{x})} \left(1 - \frac{i}{(k - k_i \partial_i U)t} \right) + e^{i\mathbf{kx} - ikU(\mathbf{x})} F^{(2)}(k, t) , \quad (28)$$

where $F^{(2)} \propto \partial_i \partial_j U$ and $F^{(2)}$ tends to zero as $t \rightarrow -\infty$. Note that in the asymptotic past, this solution is a plane wave, $\chi = e^{i\mathbf{kx} - ikt}$, modulo the time-shift (26). By substituting the Ansatz (28) into Eq. (27), we obtain the equation for $F^{(2)}$:

$$\ddot{F}^{(2)} + k^2 F^{(2)} - \frac{2}{t^2} F^{(2)} = 2k_i k_j \partial_i \partial_j U e^{-ikt} \frac{1}{k^2 t} .$$

Up to notations, this equation coincides with the anisotropic part of Eq. (4.5) of Ref. [7]. As $t \rightarrow 0$, the imaginary part of its solution is

$$\text{Im } F^{(2)} = \frac{\pi}{2} \frac{1}{k^2 t} \cdot \frac{k_i k_j}{k^2} \partial_i \partial_j U .$$

Thus, in the large wavelength regime the mode function of the field $\theta = \chi/\phi_c$ is

$$\theta = -\frac{i}{q} e^{i\mathbf{q}\mathbf{x}} \left(1 - \frac{\pi}{2k} \cdot \frac{k_i k_j}{k^2} \partial_i \partial_j U \right) , \quad (29)$$

where $q_i = k_i - \partial_i U$, and we omitted irrelevant isotropic corrections, as well as real part of $F^{(2)}$. The latter does not contribute to the statistical anisotropy to the linear order in

U (and hence in $\sqrt{\lambda}$), which comes from the interference of the two terms in parentheses in (29). So, the power spectrum of θ , and hence of the adiabatic perturbations ζ , contains anisotropic part,

$$\mathcal{P}_\zeta = \mathcal{P}_\zeta^{(0)} \left(1 - \frac{\pi}{k} \cdot \frac{k_i k_j}{k^2} \partial_i \partial_j U \right), \quad (30)$$

where $\mathcal{P}_\zeta^{(0)}$ is isotropic flat power spectrum. This is the result of Ref. [7]. Since we consider modes of U which are still superhorizon today, the tensor $\partial_i \partial_j U$ is constant throughout the visible Universe, and accounting for modes of U with the momenta below H_0 only, we obtain from Eq. (25) the estimate for its strength, $\partial_i \partial_j U \sim \sqrt{\lambda} H_0$.

5 Conclusions

To summarize, in this note we have made two simple observations. First, we have argued that mixing of scalar field(s) with metric in dynamical (pseudo-)conformal models does not introduce new UV strong coupling scales. Although we have explicitly considered the Hinterbichler–Khouri model, this conclusion appears generic. As another example, metric perturbations do decouple [2] in the Galilean Genesis model in the Newtonian gauge in the limit $M_{Pl} \rightarrow \infty$. A phenomenological consequence of our first observation is that one does not have to impose strong constraints on the parameters of dynamical models to ensure self-consistency; the self-coupling λ of the Hinterbichler–Khouri model (and its analog Λ^3/f^3 in the Galilean Genesis, see Ref. [10]) need not be particularly small. Since these parameters govern the non-linear effects, such as statistical anisotropy and non-Gaussianity, we conclude that these effects may have observable strengths.

Our second observation is that the spectator approximation does give correct results in dynamical models, provided the background space-time is sufficiently flat (which is likely to be a pre-requisite for the flat scalar spectrum in dynamical conformal models). A qualification is that this applies to the field θ and its interactions with perturbations of ϕ and metric: curvature perturbations generated directly by perturbations of the rolling field ϕ and their mixing with metric are entirely different story [2, 3] — and they are negligible anyway. A consequence of our second observation is that potentially observable effects discussed in Refs. [7, 8, 9, 10] are inherent in the entire class of both spectator and dynamical conformal models.

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