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Application of the Titius–Bode Rule to the 55 Cancri System: Tentative Prediction of a Possibly Habitable Planet

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Abstract

Following the notion that the Titius–Bode rule (TBR) may also be applicable to some extra-solar planetary systems, although this number could be relatively small, it is applied to 55 Cancri, which is a G-type main-sequence star currently known to host five planets. Based on a detailed calculational process that incorporates a generalized version of the TBR by considering four different metrics for the assessment of planetary distances, we tentatively predict four hypothetical planets (or more generally: planetary positions) around 55 Cancri, located approximately at 0.085, 0.41, 1.50 and 2.95 AU from the star. The planet at 1.50 ± 0.05 AU (if existing) appears to be located at the outskirts of the stellar habitable zone, which is affected by stellar parameters as well as the adopted model for the outer boundary of circumstellar habitability. The existence of this planet is fully consistent with previous detailed orbital stability simulations based on the five known planets. For example, Raymond et al. (2008) [ApJ 689, 478] argued that additional planets would be possible between 55 Cnc f and 55 Cnc d, including planets at locations near 1.50 and 2.95 AU. Specifically, if two additional planets are assumed to exist between 55 Cnc f and 55 Cnc d, the domains of stability would be given as 1.3–1.6 and 2.2–3.3 AU. As part of our study, we also consider possible changes in our results in reply to the finding that 55 Cnc e may be located at 0.0156 rather than at 0.038 AU. We find that none of our conclusions is impacted by the acclaimed change of position for 55 Cnc e. Moreover, we compute the distance of the next possible outer planet of the 55 Cnc system, which if existing is predicted to be located between 10.9 and 12.6 AU. This data is again consistent with the results from previously obtained stability models, which argue that any orbitally stable planet in the 55 Cnc system exterior to 55 Cnc d would be located beyond 10 AU.

Key words: astrobiology — methods: statistical — stars: individual (55 Cnc) — stars: planetary systems

1. Introduction

One of the more controversial aspects in the study of the Solar System as well as studies of selected extrasolar planetary systems is the application of the Titius–Bode rule (TBR). Historically, the TBR was first derived and applied to the Solar System where it played a significant role in the search for new planetary objects (e.g., Nieto 1972). The discovery of Uranus by Herschel in 1781 and the largest object of the Mars–Jupiter asteroid belt, Ceres, by Piazzi in 1801 appeared to confirm the applicability and significance of the TBR. The TBR historically given by the formula

$$r_n = 0.4 + 0.3 \times 2^n, \quad n = -\infty, 0, 1, 2, 3, ... \tag{1}$$

(in AU) is, however, entirely inapplicable to Neptune and to the former Solar System planet Pluto, which is still considered to be an appropriate representative of the steadily increasing number of Kuiper Belt Objects (e.g., Jewitt & Luu 2001).

A considerable amount of previous discussion revolves around the interpretation of the TBR for the Solar System, including its well-known insufficiency, see, e.g., Graner & Dubrulle (1994), Hayes & Tremaine (1998), Lynch (2003), and Neslušan (2004) for previous studies largely based on statistical analyses. For example, Lynch (2003) argued that it is not possible to conclude unequivocally that laws (or rules) of Titius–Bode type are, or are not, significant, a conclusion that also appears to be consistent with the work by Hayes & Tremaine (1998). On the other hand, the conclusions by Lynch (2003) were criticized by Neslušan (2004), who argued that based on their approach the corresponding sequence of TBR distances matches the power law by chance with a very low probability.

If no appropriate physical explanation for the TBR is identifed, it is typically argued that the TBR may be merely a rule of chance (“numerology”), fueled by the psychological tendency to identify patterns where none exist (Newman et al. 1994). On the other hand, there are previous detailed astrophysical and astrodynamical studies that potentially point to a physical background of the TBR, at least for the Solar System and (by implication) for a selected number of extrasolar planetary systems as well. For example, White (1972) argued that jet streams may develop in a rotating gaseous disk at discrete orbital distances given by a geometric progression. This result appears to be broadly consistent with work by Schmitz (1984), who pursued an analytic analysis of the structure, stability, and form of marginally stable axisymmetric perturbations of idealized rotating gas clouds. His study shows that the radial parts of the expansion solutions obtained from the governing differential yield simple oscillating functions with TBR-type features.

A similar approach was undertaken by Graner & Dubrulle

Table 1. Stellar Parameters

Quantity	Value	Reference
Spectral Type	G8 V	Gonzalez (1998)
Distance[a]	12.5 ± 0.13 pc	ESA (1997)
Apparent Magnitude V	5.96	SIMBAD website[b]
RA Coordinate	08 52 35.811	SIMBAD website[b]
DEC Coordinate	+28 19 50.95	SIMBAD website[b]
Effective Temperature	5280 K	see text
Radius[c]	$0.925 \pm 0.023\ R_\odot$	Ribas et al. (2003)
Mass	$0.92 \pm 0.05\ M_\odot$	Valenti & Fischer (2005)
Age[d]	5 ± 3 Gyr	Fischer et al. (2008)
Metallicity [Fe/H]	$+0.31 \pm 0.04$	Valenti & Fischer (2005)

[a]Data from the *Hipparcos Catalogue*.
[b]See `http://simbad.u-strasbg.fr`
[c]Alternative value: $1.15 \pm 0.035\ R_\odot$ (Baines et al. 2008).
[d]A more stringent determination, which is 4.5 ± 1 Gyr, has been given by Donahue (1998) and Henry et al. (2000).

Table 2. Planetary System[a]

Planet	Distance (AU)		Period (d)		$M \sin i\ (M_\mathrm{J})$		Discovery
55 Cnc e	0.038	± 0.0000010	2.81705	± 0.0001	0.034	± 0.0036	2004
55 Cnc b	0.115	± 0.0000011	14.65162	± 0.0007	0.824	± 0.007	1996
55 Cnc c	0.240	± 0.000045	44.3446	± 0.007	0.169	± 0.008	2002
55 Cnc f	0.781	± 0.007	260.00	± 1.1	0.144	± 0.04	2007
55 Cnc d	5.77	± 0.11	5218	± 230	3.835	± 0.08	2002

[a]Data given by Fischer et al. (2008) and references therein.

(1994), who argued that a Titius–Bode type law emerges automatically as a consequence of the scale invariance and rotational symmetry of the protoplanetary disc, and that such geometrical relationships are a generic characteristic of a broad range of physical systems. Li et al. (1995) investigated the nonlinear development and evolution of density disturbances in nebular disks on the basis of one of their previous studies. Their analysis showed that the perturbed density is unstable with respect to self-modulation, leading to the formation of density field localization and collapse. In the case of a self-similar collapse, the perturbed density was found to increase with time and to form steady rings with very large but limit amplitudes. The distances of these rings were found to follow a geometric progression akin to the TBR.

Another approach invoking statistical numerical experiments was adopted by Hayes & Tremaine (1998). They chose to fit randomly selected artificial planetary systems to Titius–Bode type laws considering a distance rule inspired by the Hill stability of adjacent planets. They did not identify a heightened significance of the TBR, except that its meaning is that stable planetary systems tend to be spaced in a regular manner. However, they indicate that their method could be used to identify potentially unstable planetary systems, especially if applied in conjunction with long-term orbit integrations. A further contribution to the interpretation of the TBR was made by Nottale (1996) and Nottale et al. (1997) based on an approach of scale relativity and quantization of the Solar System. It attempts to describe the Solar System in terms of fractal trajectories governed by a Schrödinger-like equation. The physical background and justification of this approach, however, remain highly uncertain. A controversial but testable aspect of this theory is that it proposes the existence of one or two small planets between the Sun and Mercury, which to date have no support through observations. On the other hand, these studies point to the general possibility of "empty" orbits, which may also be of interest to other applications of the TBR.

In the following, we will apply a generalized version of the TBR, without subscribing to a specific physical interpretation, to the planetary system of 55 Cancri (= ρ^1 Cnc = HD 75732 = HIP 43587 = HR 3522; G8 V) (see Table 1), which is the fourth star other than the Sun after 51 Peg, 70 Vir, and 47 UMa (not counting the pulsar PSR 1257+12 as well as other controversial cases) that was identified hosting a planet. A previous effort to apply the TBR to 55 Cnc has been pursued by Poveda & Lara (2008), which will also be discussed in our paper. 55 Cnc is known to host five planets, named 55 Cnc b to 55 Cnc f, which were discovered between 1996 and 2007 by Butler et al. (1997), Marcy et al. (2002), McArthur et al. (2004), and Fischer et al. (2008); note that all planets were detected using the radial velocity method. The approximate positions of the planets are 0.038, 0.115, 0.240, 0.781 5.77 AU, respectively, and their masses ($M_p \sin i$) range from 0.034 to 3.835 M_J (see Table 2). However, there is still a controversy regarding the location of the innermost planet 55 Cnc e, which

Table 3. Stellar Effective Temperature

$T_{\rm eff}$ (K)	Reference
5150 ± 75	Gonzalez (1998)
5336 ± 90	Fuhrmann et al. (1998)
5250 ± 70	Gonzalez & Vanture (1998)
5243 ± 93	de Strobel et al. (2001)
5338 ± 53	Ribas et al. (2003)
5279 ± 62	Santos et al. (2004)
5234 ± 44	Fischer & Valenti (2005)

based on a re-analysis of existing data may actually be positioned at 0.0156 AU (Dawson & Fabrycky 2010).

Also note that the detection of the outermost planet 55 Cnc d constitutes the first exosolar planet discovered orbiting farther than 4 AU from its host star, indicating that Jupiter-type planets at Jupiter-like distances in exoplanetary systems may not be rare. Moreover, there is an ongoing discussion about the principal possibility of additional planets in the 55 Cnc system, particularly in the gap between 0.8 and 5 AU. This speculation is fueled by detailed orbital stability studies, including studies involving putative Earth-mass planets (von Bloh et al. 2003; Rivera & Haghighipour 2007; Raymond et al. 2008; Ji et al. 2009). If such a planet exists, it would also possibly imply the presence of a potentially habitable planet if appropriate conditions are met (e.g., Lammer et al. 2009).

The star 55 Cnc is a middle-aged main-sequence star with a mass of approximately 0.95 $M_\odot$ (Valenti & Fischer 2005), with a stellar effective temperature of about 5250 K (see Table 3). From the *Hipparcos* parallax of 79.8 ± 0.84 mas (ESA 1997), a luminosity of 0.61 ± 0.04 $L_\odot$ (see Table 1) can be deduced, which is consistent with its spectral type G8 V (Gonzalez 1998); see also discussion by Marcy et al. (2002). The age of the star can be obtained from the strength of the chromospheric Ca II H+K emission, indicating an age of 4.5 ± 1 Gyr (Donahue 1998; Henry et al. 2000); see also Baliunas et al. (1997) for previous work. Another element of our work consists in the assessment of 55 Cnc's circumstellar habitability, which will be pursued following the approach by Kasting et al. (1993) and Underwood et al. (2003); additionally, it will consider the possible extension of the outer boundary of habitability as discussed by, e.g., Forget & Pierrehumbert (1997), Mischna et al. (2000), and Halevy et al. (2009).

Our paper is structured as follows. After the introduction of historical aspects of the TBR and 55 Cnc itself, as done, we describe our methods and results. Emphasis will be placed on the calculational process concerning the TBR based on different metrics. In addition, we comment on the possibility of a habitable planet in the 55 Cnc system as implied by the TBR. Thereafter, we consider other relevant studies, particularly those regarding constraints from orbital stability simulations. We also comment on a previous application of the TBR to the 55 Cnc system given by Poveda & Lara (2008). Finally, we present our summary and conclusions.

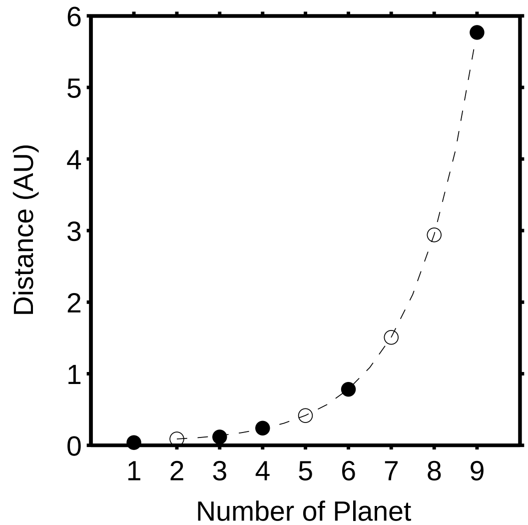

Fig. 1. Distances of observed (full circles) and predicted (open circles) planets (or planetary positions) in the 55 Cnc system. The predicted planets have been computed following the TBR based on Metric 1 (see Table 2).

2. Methods and Results

2.1. Computation of the TBR Parameters

The centerpiece of our study consists in applying the TBR to the planetary system of 55 Cnc. Akin to the application of the TBR to the Solar System, we select as approach

$$a_i^{\rm TB} = \begin{cases} A & \text{if} \quad i = 1 \\ A + B \cdot Z^{i-2} & \text{if} \quad i > 1 \end{cases} \qquad (2)$$

Here A, B, and Z constitute free parameters, which need to be determined based on the known positions of the previously discovered five planets. First, we will use the original version of data given by Fischer et al. (2008) and references therein. Based on the identified spacings of the known planets, 55 Cnc e, 55 Cnc b, 55 Cnc c, 55 Cnc f, and 55 Cnc d, are identified as $i = 1$, 3, 4, 6, and 9, respectively.

This version of the TBR is consistent with its original version customarily applied to the Solar System. Note that B indicates the compactness of the system (i.e., the higher B, the less dense the overall planetary distribution), whereas Z indicates the progression of the relative spacing of the planets. For the Solar System, the free parameters of the above given equation take the following values: $A = 0.4$, $B = 0.3$, and $Z = 2.0$. Note that based on the work by Dawson & Fabrycky (2010), there is a controversy regarding the location of the innermost planet 55 Cnc e, which based on a re-analysis of existing data may actually be positioned at 0.0156 AU instead of 0.038 AU. In order to accomodate this possibility, we also consider a revised version of the TBR where the position of the innermost planet is changed to $a_1^{\rm TB} = A/Z$. Apparently, there is also the possibility of alternate approaches, which if selected would obviously render different versions of the proposed TBR for the 55 Cnc planetary system.

The main aspect of our study is to determine and optimize

Table 4. Titius–Bode Parameters

Metric	A	B	Z
Original Data			
Metric 1	0.038 $\pm$ 0.007	0.049 $\pm$ 0.001	1.975 $\pm$ 0.007
Metric 2	0.038 $\pm$ 0.008	0.049 $\pm$ 0.001	1.977 $\pm$ 0.008
Metric 3	0.033 $\pm$ 0.003	0.050 $\pm$ 0.001	1.969 $\pm$ 0.005
Metric 4	0.037 $\pm$ 0.007	0.044 $\pm$ 0.003	2.011 $\pm$ 0.022
DF10 Data			
Metric 1	0.032 $\pm$ 0.003	0.049 $\pm$ 0.001	1.973 $\pm$ 0.006
Metric 2	0.031 $\pm$ 0.001	0.050 $\pm$ 0.001	1.971 $\pm$ 0.006
Metric 3	0.031 $\pm$ 0.002	0.050 $\pm$ 0.001	1.969 $\pm$ 0.006
Metric 4	0.031 $\pm$ 0.001	0.047 $\pm$ 0.001	1.991 $\pm$ 0.004

Table 5. Distances of Real and Hypothetical Planets (Original Data[a])

Planet	Observed[b]	Titius–Bode Rule[b]			
		Metric 1	Metric 2	Metric 3	Metric 4
55 Cnc e	0.038	0.038	0.038	0.033	0.037
... (HPL 1)	...	0.087	0.087	0.083	0.082
55 Cnc b	0.115	0.135	0.134	0.131	0.126
55 Cnc c	0.240	0.229	0.228	0.226	0.216
... (HPL 2)	...	0.415	0.414	0.414	0.397
55 Cnc f	0.781	0.782	0.780	0.784	0.760
... (HPL 3)	...	1.507	1.506	1.512	1.491
... (HPL 4)	...	2.940	2.940	2.946	2.961
55 Cnc d	5.77	5.770	5.775	5.770	5.916

[a]Data given by Fischer et al. (2008) and references therein.
[b]Distances in units of AU.

Table 6. Distances of Real and Hypothetical Planets (Modified Data[a])

Planet	Observed[b]	Titius–Bode Rule[b]			
		Metric 1	Metric 2	Metric 3	Metric 4
55 Cnc e	0.016	0.016	0.016	0.016	0.015
... (HPL 1)	...	0.082	0.081	0.081	0.078
55 Cnc b	0.115	0.130	0.129	0.130	0.125
55 Cnc c	0.240	0.224	0.224	0.225	0.218
... (HPL 2)	...	0.411	0.411	0.413	0.403
55 Cnc f	0.781	0.779	0.781	0.783	0.771
... (HPL 3)	...	1.506	1.509	1.511	1.504
... (HPL 4)	...	2.940	2.945	2.945	2.965
55 Cnc d	5.77	5.770	5.774	5.768	5.872

[a]Data of 55 Cnc e as given by Dawson & Fabrycky (2010).
[b]Distances in units of AU.

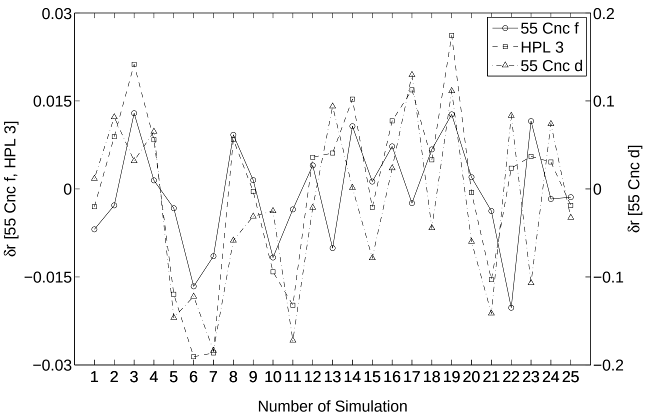

Fig. 2. Distance variations δr for 55 Cnc f, 55 Cnc d, and the hypothetical planet 3 for a series of 25 statistical simulations. For 55 Cnc f and 55 Cnc d, these variations are due to the observational uncertainties, which are 0.007 and 0.11 AU, respectively (see Table 2). For HPL 3, the distance variations were computed following the TBR based on Metric 1 (see Table 2).

the parameters A, B, and Z based on the positions of the five known planets of the 55 Cnc system. This process requires the choice of a metric that will also entail the computation of a combined residual for the five known planets. In case of the present study, four different metrics have been selected, which are: absolute linear deviation (Metric 1), relative linear deviation (Metric 2), absolute quadratic deviation (Metric 3), and relative quadratic deviation (Metric 4); see the Appendix for further information.

Table 4 gives detailed information about the TBR parameters A, B, and Z, including their respective uncertainty bars, which are due to the observationally given distance uncertainties for 55 Cnc b to e (see Table 1). Based on the distance parameters by Fischer et al. (2008), it is found that $A \simeq 0.037$, $B \simeq 0.048$, and $Z \simeq 1.98$. There is very little impact through the particular choice of metric. For example, the spacing parameter Z is found to vary between 1.977 ± 0.008 (Metric 2) and 2.011 ± 0.022 (Metric 4). Furthermore, the distance uncertainties for 55 Cnc b to e, given by the observations, have also a relatively small effect on the values of A, B, and Z; note that in case of Z, the associated uncertainty is 1% or less. If the revised data by Dawson & Fabrycky (2010) are selected, there is a virtually no effect regarding B and Z; for A, now given as $A \simeq 0.031$, there is obviously a change, which however is entirely uniform (within its uncertainty bar) regarding the adopted metrics.

2.2. *Applications and Tests*

The derivation of the TBR parameters A, B, and Z can be used to "predict" additional planets (or more generally: planetary positions) in the 55 Cnc system (see Table 5 and 6). Specifically, four hypothetical planets (or planetary positions) are identified: 0.085, 0.41, 1.50 and 2.95 AU. As expected from the previous discussion, the influence of the different selections of metrics as well as the uncertainty about position of 55 Cnc e, is only of very minor importance. Figure 1 offers a detailed depiction of the observed and predicted planets. Note the very high quality of the fit.

Before discussing further results of our study, we want to present the findings from a detailed test about the convergence of the TBR Parameters, notably Z. Once again, the TBR parameters A, B, and Z are obtained by trying to fit the positions of the known planets 55 Cnc b to 55 Cnc e while using the TBR. This requires the choice of a metric; additionally, it also requires the choice of a precision parameter regarding the fitting of the TBR parameters that must be attained before the process of optimizing A, B, and Z is discontinued. An important control parameter is the aggragated residual of the difference in the positions between the observed and predicted (fitted) distances of the five known planets in the 55 Cnc system.

Our results are given in Table 7. As expected, it is found that the higher the selected precision, the more accurate are the obtained values for A, B (not shown), and Z. However, the aggragated residual will never approach zero, although it will attain a relatively small value (i.e., between 10^{-2} and 5×10^{-4}), especially if Metric 3 is selected. This result is certainly as expected. It is due to the fact that the TBR (if applicable) must be viewed as a "rule" rather than a (physical) "law". There is no way that the TBR as proposed (or any modification) will be able to precisely represent the positions of the currently known five planets in the 55 Cnc system (just like encountered for the Solar System). Surely, similar limitations would apply to any hypothetical planet, if existing.

Next we focus on the case of the hypothetical planet HPL 3 (see Table 5 and 6), positioned close to 1.50 AU; see Table 8 for details. Concerning the four different metrics, the position of HPL 3 is found to range between 1.491 ± 0.051 (Metric 4) and 1.512 ± 0.031 (Metric 3), if the data by Fischer et al. (2008) are adopted and between 1.504 ± 0.025 (Metric 4) and 1.511 ± 0.028 (Metric 3) for the data by Dawson & Fabrycky (2010). Note again that the uncertainty in the position of HPL 3 is virtually inconsequential. This hypothetical planet is of potentially high relevance because it may be located in 55 Cnc's habitable zone (HZ). Therefore, it will be discussed in more detail below.

Considering the potential importance of HPL 3, if existing, we also investigated the distance variation δr due to the measurement uncertainties for 55 Cnc f and 55 Cnc d (see Table 1) as published; here the uncertainty measurement is most substantial for 55 Cnc d (both in absolute and relative units). The results are given in Fig. 2; note that the implemented version of the TBR has been based on Metric 1. Various positions for 55 Cnc f and 55 Cnc d were assumed using a Monte–Carlo approach (Press et al. 1989) considering their observationally deduced distance measurements. For the sake of curiosity, we obtained a total of 25 simulations. For our series of simulations, the absolute variation δr for HPL 3 was less than 0.03 AU.

Another application of the TBR as consists in the computation of the distance of the next possible outer planet of the 55 Cnc system, i.e., located beyond 55 Cnc d, which is at a distance of approximately 5.77 AU. This hypothetical planet is predicted to be located between 10.9 and 12.6 AU (see Table 9). More precisely, its position is predicted in the range between 11.33 ± 0.42 (Metric 3) and 11.86 ± 0.73 (Metric 4), if the data by Fischer et al. (2008) are adopted and between 11.33 ± 0.43

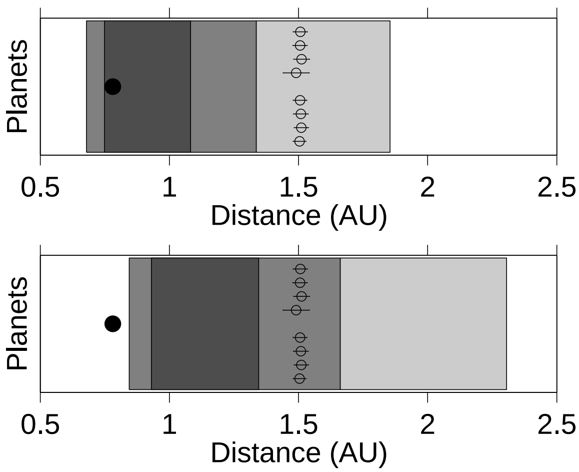

Fig. 3. Distances of 55 Cnc f (full circle) and the hypothetical planet 3 (open circles) in relationship to the HZ of 55 Cnc, where dark gray, medium gray and light gray denote the conservative, general, and extreme HZ, respectively (see Table 10). The position of HPL 3 was computed following the TBR based on Metric 1, 2, 3, and 4 (from top to bottom) noting that the first set of 4 is based on the computations according to the data by Fischer et al. (2008) and references therein (see Table 1), whereas the second set of 4 is based on computations considering the modified position of 55 Cnc e as derived by Dawson & Fabrycky (2010). The depicted uncertainty bars are 3σ, and are mostly due to the small observational uncertainties of 55 Cnc f and 55 Cnc d. The top figure shows the HZ of 55 Cnc based on $R = 0.925\ R_\odot$ (Ribas et al. 2003), whereas the bottom figure assumes $R = 1.15\ R_\odot$ (Baines et al. 2008), implying a relatively high stellar luminosity of 0.90 $L_\odot$.

(Metric 3) and 11.66 ± 0.39 (Metric 4) in response to the data by Dawson & Fabrycky (2010).

2.3. *The Habitable Zone of 55 Cnc*

In the following we discuss the possible location of a newly predicted planet in the HZ of 55 Cnc. The extent of 55 Cnc's HZ can be calculated following the formalism by Underwood et al. (2003) based on previous work by Kasting et al. (1993). Underwood et al. (2003) supplied a polynomial fit depending on the stellar luminosity and the stellar effective temperature that allows to calculate the extent of the conservative and the generalized HZ. Noting that 55 Cnc is less luminous than the Sun, it is expected that its HZ is less extended than the solar HZ, for which the limits of the generalized HZ have been given as 0.84 and 1.67 AU, respectively (Kasting et al. 1993).

The luminosity of 55 Cnc has been deduced as 0.61 ± 0.04 $L_\odot$ based on the stellar distance and apparent magnitude; note that this value is fully consistent with the standard values for the stellar effective temperature and luminosity (see Table 1). However, if the stellar radius is $1.15 \pm 0.035\ R_\odot$, obtained by Baines et al. (2008) based on new interferometric measurements, instead of $R = 0.925\ R_\odot$ (Ribas et al. 2003), the revised stellar luminosity is thus given as 0.90 $L_\odot$. This increased luminosity is, however, difficult, though not impossible, to reconcile with the other parameters of 55 Cnc, such as its apparent magnitude, spectral type and distance.

Based on the standard values for the stellar parameters of 55 Cnc, the limits of the conservative HZ are given as 0.75 and 1.08 AU, whereas the limits of generalized HZ are given as 0.68 and 1.34 AU (see Fig. 3 and Table 10). Surely, larger limits are attained if the higher value for the stellar luminosity is adopted. The underlying definition of habitability is based on the assumption that liquid surface water is a prerequisite for life, a key concept that is also the basis of ongoing and future searches for extrasolar habitable planets (e.g., Catanzarite et al. 2006; Cockell et al. 2009). The numerical evaluation of these limits is based on an Earth-type planet with a $CO_2/H_2O/N_2$ atmosphere.

We point out that concerning the outer edge of habitability, even less conservative limits have been proposed in the meantime (e.g., Forget & Pierrehumbert 1997; Mischna et al. 2000). They are based on the assumption of relatively thick planetary CO_2 atmospheres and invoke strong backwarming that is further enhanced by the presence of CO_2 crystals and clouds. This type of limits can be as large as 2.4 AU in case of the Sun; however, they depend on distinct properties of the planetary atmosphere, and are therefore subject to additional studies and controversies (e.g., Halevy et al. 2009). Concerning 55 Cnc, this type of outer limit can be as large as 1.85 AU for the standard stellar parameters and up to 2.31 AU for the increased value of the stellar luminosity. These limits can be applied to the hypothetical planet close to 1.50 AU (see Table 8). Note that the position of this hypothetical planet is not significantly affected by the choice of metric or by the type of data set, i.e., original data versus data by Dawson & Fabrycky (2010), selected. Based on the standard parameters for 55 Cnc, this planet would be located in the extremely extended HZ. If the increased value of the stellar luminosity of 0.90 $L_\odot$ is assumed, it would be located in the generalized HZ. A depiction of the different scenarios is given in Fig. 3.

Another important aspect regarding the circumstellar habitability of 55 Cnc concerns the planet 55 Cnc f. It is a hot Neptune with a mass of about 15% of Jupiter's mass; therefore, it is almost certainly unfit to host life. 55 Cnc f is lo-

Table 7. Convergence of Titius–Bode Parameters

Metric	Precision	Z	Residual
Metric 1	10^{-1}	1.9	4.48×10^{-1}
	10^{-2}	1.98	3.75×10^{-2}
	10^{-3}	1.975	3.22×10^{-2}
	10^{-4}	1.9760	3.11×10^{-2}
Metric 2	10^{-1}	2.0	4.62×10^{-1}
	10^{-2}	1.98	2.22×10^{-1}
	10^{-3}	1.977	2.18×10^{-1}
	10^{-4}	1.9760	2.17×10^{-1}
Metric 3	10^{-1}	2.1	7.32×10^{-2}
	10^{-2}	1.98	5.65×10^{-4}
	10^{-3}	1.969	4.80×10^{-4}
	10^{-4}	1.9710	4.77×10^{-4}
Metric 4	10^{-1}	2.0	6.03×10^{-2}
	10^{-2}	2.01	2.14×10^{-2}
	10^{-3}	2.011	2.11×10^{-2}
	10^{-4}	2.0113	2.11×10^{-2}

Table 8. Distance of the Hypothetical Planet 3

Metric	Distance (AU)	
...	Original Data	DF10 Data
Metric 1	1.507 ± 0.028	1.506 ± 0.027
Metric 2	1.506 ± 0.028	1.509 ± 0.029
Metric 3	1.512 ± 0.031	1.511 ± 0.028
Metric 4	1.491 ± 0.051	1.504 ± 0.025

Table 9. Distance of an Hypothetical Exterior Planet

Metric	Distance (AU)	
...	Original Data	DF10 Data
Metric 1	11.36 ± 0.44	11.35 ± 0.45
Metric 2	11.38 ± 0.44	11.35 ± 0.44
Metric 3	11.33 ± 0.42	11.33 ± 0.43
Metric 4	11.86 ± 0.73	11.66 ± 0.39

Table 10. Habitable Zone of 55 Cancri

Description	Distance (AU)	
...	Standard	Alternative
HZ-i (general)	0.68	0.84
HZ-i (conservative)	0.75	0.93
HZ-o (conservative)	1.08	1.35
HZ-o (general)	1.34	1.66
HZ-o (extreme)	1.85	2.31

cated at or near the inner edge of the 55 Cnc's HZ, especially if the standard value for the 55 Cnc's luminosity is adopted. However, 55 Cnc f may be an appropriate object to host habitable moons. Previous studies for habitable moons of extrasolar planets have been given by, e.g., Williams et al. (1997) and Donnison (2010).

3. Consideration of Other Studies

3.1. *Constraints from Orbital Stability Simulations*

Any planet proposed to exist in a star–planet system by the means of the TBR or otherwise needs to pass the test of orbital stability. Previously, detailed simulations of orbital stability for the system of 55 Cnc have been given by, e.g., Raymond et al. (2008) taking into account the five known planets of the 55 Cnc system. Raymond et al. (2008) presented a highly detailed study with particular focus on the effects of mean motion resonances due to the planets 55 Cnc f and 55 Cnc d. They showed that additional planets could exist between these two known planets, including hypothetical planets located near 1.50 and 2.95 AU. Specifically, if two additional planets are assumed to exist between 55 Cnc f and 55 Cnc d, the domains of stability are given as 1.3–1.6 and 2.2–3.3 AU. This is an important finding regarding the hypothetical, potentially habitable planet HPL 3, proposed to be located close to 1.50 AU (see Table 8).

A subsequent study of orbital stability was given by Ji et al. (2009). They also considered the impact of mean motion resonances and identified various unstable locations. However, they identied a wide region of orbital stability between 1.0 and 2.3 AU, a potential homestead of habitable terrestrial planets. These potential planets were also identied to have a relatively low orbital eccentricity. Furthermore, the orbital stability simulations by Raymond et al. (2008) and Ji et al. (2009) are also lending indirect support to the principal possibility of an exterior planet, which, if existing, should be located between 10.9 and 12.6 AU from the star (see Table 9).

Although orbital stability for the hypothetical planet near 1.50 AU appears to be warranted, it is still important to gauge if such a planet could have formed in the first place noting that the existence of close-in giant planet(s) have been viewed to have an adverse effect (Armitage et al. 2003). Subsequent studies by Raymond et al. (2005, 2006) have pointed out that the formation and habitability of terrestrial planets in the presence of close-in giant planets is indeed possible. Raymond et al. (2006) gave detailed simulations also encompassing the case of 55 Cnc. They found that assuming that the giant planets formed and migrated quickly, terrestrial planets may be able to form from a second generation of planetesimals. For 55 Cnc, objects with masses up to 0.6 $M_\oplus$ and, in some cases, substantial water content are able to form; these objects are found in orbit in 55 Cnc's HZ. This type of result is also consistent with findings from the quantitative numerical program by Wetherill (1996), who demonstrated that habitable terrestrial planets can form for a wide range of main-sequence stars, noting that the number and radial distribution of those planets is relatively insensitive to stellar mass.

3.2. *Previous Application of the TBR to 55 Cnc*

A previous attempt to predict additional planets in the 55 Cnc system based on the TBR has been given by Poveda & Lara (2008). This effort occurred at a time when all currently known planets 55 Cnc b to 55 Cnc f were already discovered. Poveda & Lara (2008) assumed a "law" of the form of

$$a_n^{\rm TB} \ = \ \alpha \cdot e^{\lambda n} \qquad (3)$$

(in AU) with α and λ as free parameters. Taking into account the planets 55 Cnc e, 55 Cnc b, 55 Cnc c, and 55 Cnc f, located between 0.038 and 0.781 AU, and represented by $n = 1$ to 4, α and λ were deduced as 0.0148 and 0.9781, respectively. This modified TBR is very successful to also "predict" the outermost planet 55 Cnc d at 5.77 AU, counted as $n = 6$, which however was well known when Poveda & Lara (2008) proposed the new law.

Making 55 Cnc d also part of the fit results in a small adjustment of the parameters α and λ, which then take the values of 0.0142 and 0.9975, respectively. The main aspect of the work by Poveda & Lara (2008) is to predict a new planet for $n = 5$ and possibly also an external planet for $n = 7$ as well. The corresponding distances of these hypothetical planets are thus given as 2.08 and 15.3 AU; the corresponding orbital periods of the the planets are approximately 3.1 and 62 years, respectively. None of these putative planets are expected to be located in the 55 Cnc's HZ. Moreover, the work by Poveda & Lara (2008) assumes the planet 55 Cnc e to be located at 0.038 AU rather than 0.0156 AU as argued by Dawson & Fabrycky (2010); obviously, Poveda & Lara published their work before the results by Dawson & Fabrycky became available. Thus, it is unclear if or how the "law" by Poveda & Lara (2008) could be altered to accomodate the finding by Dawson & Fabrycky (2010) if applicable.

Alike for the present work, it is up to future observational efforts to verify or falsify the existence of any of these putative planets. Moreover, the effort by Poveda & Lara (2008) will assist in gauging the applicability of the TBR outside of the Solar System, and help to discriminate between different versions of the TBR if found applicable to the 55 Cnc system.

4. Summary and Conclusions

The goal of our study was to provide applications of the TBR to the 55 Cancri star–planet system. This system may constitute a suitable candidate system for assessing the potential relevance of the TBR beyond the Solar System. 55 Cancri is a G8 V main-sequence star, i.e., a star of somewhat smaller mass and lower effective temperature than the Sun, and is currently known to host five planets. As part of this study, a calculational process was pursued that incorporates a generalized version of the TBR while considering four different metrics for the computation of the planetary distances, which is applied to the observed planets to tentatively propose the possibility of additional planets (or planetary orbital positions). The method also takes into account the observationally deduced uncertainties of the distances (i.e., semi-major axes) of the five planets known to date. The study arrived at the following results:

1. If the TBR is found to be applicable to the 55 Cnc system, a conjecture that needs to be checked in the context of future observations, it may contain four more planets (or "belts"), located between 55 Cnc e and 55 Cnc d, the currently known innermost and outermost planet, respectively. The projected distances of these putative planets are: 0.085, 0.41, 1.50, and 2.95 AU from the star.
2. This analysis is based on an implementation of the TBR with A, B, and Z as free parameters, which were obtained through careful statistical fitting considering the five known planets of the 55 Cnc system. Based on the parameters by Fischer et al. (2008), we obtain $A \simeq 0.037$, $B \simeq 0.048$, and $Z \simeq 1.98$. The parameter B indicates the compactness of the planetary system, which is much higher than for the Solar System as also indicated by the conventional version of Solar System's TBR ($B = 0.3$)[1]. The parameter Z indicates the relative geometrical spacing of the planetary system; it is virtually identical to that of the Solar System ($Z = 2.0$).
3. The values for A, B, and Z are not considerably affected by the statistical uncertainties in the positions of the known planets in the 55 Cnc system, the choice of metric regarding the statistical fitting of the TBR, and the possible change in data for the innermost planet 55 Cnc e (Dawson & Fabrycky 2010); see below. The revised data by Dawson & Fabrycky would have a miniscule effect on B and Z; for A, the effect would be somewhat larger but consistent (i.e., uniform regarding the various metrics).
4. The planet at 1.50 ± 0.05 AU (if existing) appears to be located at the outskirts of the stellar HZ. The final verdict of potential habitability partially depends on the stellar parameters of 55 Cnc, i.e., its luminosity. It also depends on the definition of stellar HZ, considering that the outer boundary of circumstellar habitability is considerably affected by a large variety of assumptions (e.g., Lammer et al. 2009), particularly the adopted atmospheric model of the putative planet (e.g., Forget & Pierrehumbert 1997; Mischna et al. 2000; Halevy et al. 2009).
5. The existence of the potentially habitable planet as predicted would be also consistent with previous orbital stability simulations. For example, Raymond et al. (2008) argued that additional planets could exist between 55 Cnc f and 55 Cnc d, including planets located near 1.50 and 2.95 AU. Specifically, if two additional planets are assumed to exist between 55 Cnc f and 55 Cnc d, the domains of stability are given as 1.3–1.6 and 2.2–3.3 AU. Similar results were obtained by Ji et al. (2009).
6. We computed the orbital distance of the next possible outer planet of the 55 Cnc system, which if existing would be located beyond 10 AU, as shown in the context of detailed orbital stability studies by Raymond et al. (2008). Depending on the selected metric in the application of the TBR, our study predicts that if existing this planet should be located between 10.9 and 12.6 AU from the star. Therefore, this planet would in principle be possible based on orbital stability considerations.
7. Based on a re-analysis of previous radial velocity measurements Dawson & Fabrycky (2010) argue that 55 Cnc e, the innermost planet of the 55 Cnc system, may be located at 0.0156 rather than at 0.038 AU. We adopted a minor modification of the employed TBR, and found that the main frame of our results remains unaltered. Specifically, no significant change was found for the possibly habitable planet at 1.50 AU and the next possible outer planet in the 55 Cnc system, located beyond 10 AU.

In summary, it should be pointed out that the existence of a potentially habitable planet near 1.50 AU in the 55 Cnc system appears in principle possible, noting that this type is consistent with previous simulations of orbital stability (Raymond et al. 2008; Ji et al. 2009) as well as previous studies of terrestrial planet formation in the presence of a hot, close-in Jupiter-type planets pursued by Raymond et al. (2005, 2006) and others. Clearly, additional observational and theoretical studies are required to verify or falsify the existence of the tentatively proposed additional planets in the 55 Cnc star–planet system.

[1] The increased compactness of the planetary system of 55 Cnc compared to the Solar System is already implied by the known five planets, which inspired coining the term "packed planetary systems" hypothesis (e.g., Raymond et al. 2005).

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APPENDIX: Application of Metrics

The central part of this study is to compute the parameters A, B, and Z of the Titius–Bode rule. This is done by minimizing the distance parameter δr_i for the five observed planets of the 55 Cnc system, known as 55 Cnc a to 55 Cnc e, which correspond to i = 1, 3, 4, 6, and 9 (not in this sequence). Therefore, we utilize different metrics given as

$$\delta r_i = \left| r_{pl,i} - r_{pl,i}^{\rm TB} \right| \tag{A1}$$

$$\delta r_i = \left| \frac{r_{pl,i} - r_{pl,i}^{\rm TB}}{r_{pl,i}} \right| \tag{A2}$$

$$\delta r_i = \left(r_{pl,i} - r_{pl,i}^{\rm TB} \right)^2 \tag{A3}$$

$$\delta r_i = \left(\frac{r_{pl,i} - r_{pl,i}^{\rm TB}}{r_{pl,i}} \right)^2 . \tag{A4}$$

These metrics are referred to as Metric 1, Metric 2, Metric 3, and Metric 4, respectively. Here $r_{pl,i}$ denotes the distance (semi-major axis) of planet i given by observation, whereas $r_{pl,i}^{\rm TB}$ denotes the distance value obtained through the application of the TBR.