

K-matter as Mach's principle realization

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Abstract

It is shown that if one takes into account Mach's principle in the form which follows from quantum theory and considers it as a complementary constraint between the parameters which characterize the energy density and geometry of the universe in addition to Einstein equations for a FRW universe, non-relativistic matter transforms into an analogue of K-matter. The exact solutions of the Einstein equations for the universe with such matter and cosmological constant are found. It is demonstrated that the Machian universe under consideration with a nonzero cosmological constant is equivalent to the open de Sitter universe. In the limit of zero cosmological constant such a universe evolves as a Milne universe, but in contrast to it, it contains matter with nonzero energy density. The possible application of proposed approach to the description of the present cosmological data is discussed. The problem of the age of the universe is considered as an example.

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1 Introduction

As is well-known, the standard Λ CDM model gives the satisfactory description of the most of the present cosmological data under the assumption of the existence of dark energy as the largest constituent of mass-energy in the universe. It is believed that a high level of fine-tuning is required in this model. Even if the smallness of cosmological constant and "coincidence problem" (an almost equal contribution of matter and dark energy to the total energy budget of the universe at the present epoch) are not problems in themselves [1], nevertheless one must take into account the phenomenological character of the Λ CDM model, as regards the choice of the form of the energy density and equation of state. It should not be ignored that there were some indications that specific cosmological observations differed from the predictions of the Λ CDM model at statistically significant level [2]. It makes the search for alternative models not as unreasonable as it might seem.

It was noticed that the models (called "coasting cosmology", since in such models the universe expands with constant velocity [3]) in which the scale factor R of the

universe depends on synchronous proper time t linearly, $R(t) \sim t$, agrees good enough with the cosmological observations [4, 5]. In particular, this approach does not suffer from the horizon problem. It is in concordance with the early universe nucleosynthesis constraints and fits to type Ia supernovae data. One can expect that such a scenario will agree with estimates of age of the universe in comparison to ages of old objects and provide the degree scale for the first acoustic peak of the cosmological microwave background. It is compatible with constraints from large scale structure formation and agrees with the physics of recombination as deduced from cosmic microwave background anisotropy. There are also expectations that the primordial lithium problem (a discrepancy between the primordial lithium predicted from the WMAP data and the stellar abundance determinations) [6] might be resolved under the assumption of a linear evolution of the scale factor [4, 5].

The linear dependence of a scale factor on time can be associated with a Milne model of the universe. This model is based on the assumptions that the universe is open ($k = -1$) and that the gravitational action of matter can be neglected (the energy density $\rho = 0$). It cannot be correct near the point of initial cosmological singularity, $t = 0$, since in this limit the energy density of matter tends to infinity and gravity cannot be neglected. One attempt to settle this problem was to consider a model of a universe (called “Dirac - Milne” universe by analogy of sea of positive and negative energy states proposed by Dirac) containing equal quantities of matter and antimatter under the assumption that antimatter is characterized by a negative gravitational mass [5].

The linear dependence of a scale factor R on time can be achieved in cosmological models in which the energy density $\rho \sim R^{-2}$ and the total pressure $p = -\frac{1}{3}\rho$. Such “coasting cosmologies” describe the universe dominated by exotic “K-matter” which may be related to cosmic strings [3, 7].

In the present paper we show that if one takes into account an additional constraint between the parameters which characterize the energy density and geometry of the universe in addition to Einstein equations for a Friedmann-Robertson-Walker (FRW) universe, non-relativistic matter with the energy density evolving as $\rho \sim R^{-3}$ transforms into an analogue of K-matter. This constraint can be interpreted as Mach’s principle [8] in Sciamia’s formulation [9, 10]. Being introduced to explain the inertial forces acting on a body via the quantity and distribution of matter in the whole universe, nowadays Mach’s principle has many definitions [11, 12]. Despite its simplified character, Sciamia’s linearized theory gives a specific mathematical relation between the parameters of the universe instead of general statement of Mach’s principle.

2 Quantum roots of Sciamia’s relation

Sciamia’s relation obtains a natural explanation in the framework of quantum isotropic cosmological model [13, 14]. Generally speaking, quantum theory adequately

describes properties of various physical systems. Its universal validity demands that the universe as a whole must obey quantum laws as well. Since quantum effects are not a priori restricted to certain scales, then one should not conclude in advance that they cannot have any impact on processes at scales larger than Planckian (more detailed arguments can be found, e.g., in Refs. [15]).

Quantum theory for a homogeneous and isotropic universe can be constructed on the basis of a Hamiltonian formalism with the use of material reference system as a dynamical system [13, 14]. Defining the time parameter or the “clock” variable, it is possible to pass from the Wheeler–DeWitt equation to the Schrödinger-type equation. The similar equations containing a time variable defined by means of coordinate condition were considered by a number of authors under the quantization of the FRW universe (see, e.g., Refs. [16]). Using the Schrödinger-type equation one can obtain equations of motion for the expectation values of a scale factor and its conjugate momenta. These equations pass into the equations of general relativity when the dispersion around the expectation values for a scale factor, matter fields and their conjugate momenta can be neglected.

Under this approach, in semi-classical limit, the equations of the theory are reduced to the form of Einstein equations for the FRW universe [14]. Such a quantum theory predicts that the following relation must hold for the expectation value of the scale factor R in the state $|M\rangle$ which describes the universe with the definite total amount of mass M much larger than Planck mass, $M \gg M_P$,

$$\langle M|R|M\rangle = GM, \quad (1)$$

G is the Newtonian gravitational constant (for details, see Refs. [14]). In classical limit, it appears to be possible to pass from the expectation value $\langle M|R|M\rangle$ to the classical value of the scale factor $R(t)$ which evolves in time in accordance with the Einstein equations for the FRW universe

$$\dot{R}^2 = \frac{8\pi G}{3} \rho R^2 + \frac{\Lambda}{3} R^2 - k, \quad \ddot{R} = -\frac{4\pi}{3}(\rho + 3p)R + \frac{\Lambda}{3} R, \quad (2)$$

where

$$\rho = \frac{M}{(4\pi/3)R^3} \quad (3)$$

is the energy density of matter with the mass M in the equivalent flat-space volume $(4\pi/3)R^3$, Λ is the cosmological constant,

$$p = -\rho - \frac{R}{3} \frac{d\rho}{dR} \quad (4)$$

is the isotropic pressure, and $k = +1, 0, -1$ for spatially closed, flat or open models. In semi-classical limit, the relation (1) takes the form of Sciama’s inertial force law which describes Mach’s principle [9, 10],

$$R = GM. \quad (5)$$

The same equality between the mass and “radius” of the universe was considered by Whitrow and Randall [17]. It is also similar to the relation valid for the Einstein universe (see, e.g., Ref. [18]).

For the present-day universe the radius of its observed part is estimated as $R_0 \sim 10^{28} \text{cm} \sim 10^{61}$ (in units of Planck length $l_P \sim 10^{-33} \text{cm}$), the mass-energy is $M_0 \sim 10^{56} \text{g} \sim 10^{80} \text{GeV} \sim 10^{61}$ (in units of Planck mass $m_P \sim 10^{19} \text{GeV}$), and the mean energy density equals to $\rho_0 \sim 10^{-29} \text{g cm}^{-3} \sim 10^{-122}$ (in units of Planck energy density $\rho_P \sim 10^{93} \text{g cm}^{-3}$). It means that nowadays $\rho_0 \sim G^{-1} R_0^{-2}$. Then from the definition of energy density $\rho_0 \sim M_0 R_0^{-3}$, it follows that the relation $R_0 \sim G M_0$ must hold. The same conclusion can be made from the exact equation (5). Since this equation must be true for an arbitrary chosen instant of time t , there arises the problem of mass increase, as interpreted from the point of view classical cosmology. Namely, it follows that total mass increases proportionally to a scale factor, $M(t) \sim R(t)$, if the gravitational constant G and velocity of light c are both constant. This difficulty can be resolved, in particular, if one supposes that the natural constants G or c change with time.

The questions arised in connection with these problems were discussed using different frameworks and for different purposes. According to Dirac’s large number hypothesis, the Newtonian constant G must depend on time, so that $G \sim t^{-1}$ and $R \sim t^{1/3}$ [19] or $G \sim t^{-1}$ and $R \sim t$ [20]. In the Brans-Dicke theory the constant G is related to the average value of some dynamical scalar field ϕ which is coupled to the mass density ρ of the universe, $\langle \phi \rangle \approx G^{-1}$, where $\langle \phi \rangle \sim \rho R^2$ [21, 22]. Models with varying speed of light were applied in order to solve the horizon, flatness, cosmological constant, and other cosmological problems (see, e.g., Refs. [23]). Matter creation processes in the context of the cosmological models and their influence on the evolution of the universe were studied in Refs. [24].

If we go back and consider the equation (5) as following from the relation (1), then we can interpret it in terms of quantum theory. In quantum model the state vector of isotropic universe is a superposition of all possible $|M\rangle$ - states which are not orthogonal between themselves, so that the inner product $\langle M_1 | M_2 \rangle \neq 0$, and the universe can transit spontaneously from the state with the mass M_1 to the state with the mass $M_2 \neq M_1$ with nonzero probability $P(1 \rightarrow 2) = |\langle M_1 | M_2 \rangle|^2$. For example, the probability of transition of the universe from the ground state (with respect to gravitational field) to any other state obey the Poisson distribution with the mean number of occurrences $n = \frac{1}{2}(M_2 - M_1)^2$ (for more details, see Refs. [14]). Then $R_1 \rightarrow R_2$, when $M_1 \rightarrow M_2$. If one would try to interpret this result in terms of the Newtonian cosmology, describing the universe as a flat Euclidean 3-space filled with a uniform matter with the energy density $\rho(t)$ (3), such a transition would correspond to the passage to the sphere of radius $R_2 > R_1$ which includes a mass $M_2 > M_1$.

3 FRW equations with Mach's principle

If one assumes that Mach's principle is a fundamental law of nature, it must be implemented into the classical field equations. One point of view is that Einstein's field equations need not to be modified, while Mach's principle should be considered as an additional condition. Such an approach was chosen by Wheeler who proposed to understand Mach's principle as a selection rule (boundary condition) of the solutions of the field equations [25]. The Brans-Dicke theory mentioned above uses another way in which the field equations are generalized to become Machian [21, 26].

Since in our approach Mach's principle in the form (5) follows from quantum theory in semi-classical limit, under classical description it can be introduced as an addition constraint and added to the classical field equations (2). With account of the constraint (5), the energy density of matter (3) takes the form of K-matter energy density with the corresponding equation of state,

$$\rho = \frac{3}{G} \frac{1}{4\pi R^2}, \quad p = -\frac{1}{3}\rho. \quad (6)$$

According to common classification (see, e.g. Ref. [27]), matter with such an equation of state can be attributed to strings, since it naturally appears in string cosmology. But in this approach it does not mean that the universe is string-dominated. The energy density and pressure in the form (6) arise as an effect of an additional constraint between the global geometry and the total amount of matter in the universe as a whole.

The field equations are reduced to the form

$$\dot{R}^2 = \frac{\Lambda}{3} R^2 + (2 - k), \quad \ddot{R} = \frac{\Lambda}{3} R. \quad (7)$$

Their solution is

$$R(t) = \sqrt{\frac{3(2-k)}{\Lambda}} \sinh\left(\sqrt{\frac{\Lambda}{3}}t\right), \quad R(0) = 0. \quad (8)$$

Expansion of this solution for small $|\frac{\Lambda}{3}t^2|$ yields

$$R(t) = \sqrt{2-k}t \left[1 + \frac{1}{6} \left(\frac{\Lambda}{3}t\right)^2 + \dots \right]. \quad (9)$$

From the Hubble expansion rate

$$H(t) = \frac{\dot{R}}{R} = \sqrt{\frac{\Lambda}{3}} \coth\left(\sqrt{\frac{\Lambda}{3}}t\right), \quad (10)$$

one obtains the expansion in the same limit

$$Ht = 1 + \frac{1}{3} \left(\frac{\Lambda}{3}\right) t^2 - \frac{1}{45} \left(\frac{\Lambda}{3}\right)^3 t^4 + \dots \quad (11)$$

If $\Lambda \neq 0$, the expressions for the scale factor (8) and the Hubble expansion rate (10) are equivalent to the respective expressions for the de Sitter model of the universe with $k = -1$.

In the limiting case $\Lambda = 0$ it appears that

$$R(t) = \sqrt{2 - kt}, \quad Ht = 1. \quad (12)$$

This solution formally coincides with the solution of Milne model of open universe ($k = -1$), $R(t) \sim t$. But in contrast to the Milne model, where the energy density of matter vanishes, $\rho = 0$, in the case under consideration the energy density of matter is nonzero,

$$\rho = \frac{3H^2}{4\pi G(2 - k)}. \quad (13)$$

For a spatially flat universe ($k = 0$) this density equals to the critical density, $\rho = \rho_c \equiv \frac{3H^2}{8\pi G}$.

The equation (13) can be rewritten in the Whitrow-Randall form [17],

$$G\rho t^2 = \frac{3}{4\pi} \frac{1}{n},$$

i.e. $G\rho t^2$ is an invariant determined by the parameter $n = 2 - k$ characterizing the geometry of the universe.

Introducing a dimensionless parameter K as in the model of K-matter,

$$K \equiv \frac{8\pi G}{3} \rho R^2, \quad (14)$$

and using (6), one finds that $K = 2$. This value agrees with the observational constraints on the parameter K obtained by Kolb [3] and Gott and Rees [7].

The calculations with the parameters for standard Λ CDM model give the same value of $H_0 t_0$ for the present-day universe as follows from Eq. (12). Really, using the WMAP 7-year data [28] for the age of the universe $t_0 = 13.75 \pm 0.13$ Gyr and the Hubble parameter $H_0 = 71.0 \pm 2.5$ km s⁻¹ Mpc⁻¹, one finds: $H_0 t_0 = 0.998 \pm 0.045$. At the same time, substituting the cosmological constant $\Lambda = (1.302 \pm 0.143) \times 10^{-56}$ cm⁻² which corresponds to the dark energy density parameter $\Omega_\Lambda = 0.734 \pm 0.029$ [28] into Eq. (10) with the corresponding age of the universe t_0 , one gets a somewhat excessive value: $H_0 t_0 = 1.233 \pm 0.029$. It is necessary to keep in mind, of course, that the use of the values of the parameters of Λ CDM model in these estimations of $H_0 t_0$ has only illustrative character, since Eqs. (8)-(11) were obtained under the different model assumptions.

In the model, where the scale factor depends on time linearly (12), the age of the universe and the Hubble expansion rate depend on the redshift z according to the simple laws

$$t = \frac{1}{(1+z)H_0}, \quad H = H_0(1+z). \quad (15)$$

For the present expansion rate measured by Hubble Space Telescope observations, $H_0 = 73.8 \pm 2.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [29], the age of the universe appears to be equal $t_0 = 13.26 \pm 0.43 \text{ Gyr}$. This value does not differ drastically from the value predicted by the WMAP 7-year data for the Λ CDM model, and it lies within the expected limit of 12 to 14 Gyr.

4 Conclusion remarks

In the coasting cosmological models considered without reference to Mach's principle or matter creation, it is assumed the existence of a specific form of matter, such as K-matter with the energy density, which decreases in expansion as R^{-2} , or such a matter, whose energy density can be neglected in the open universe (Milne universe). The incorporation of Mach's principle into the theory does not change the physical properties of matter itself (such as a perfect fluid in the form of dust with the corresponding equation of state), but it takes into account the constraint which reflects collective behavior of matter in the universe considered as a whole. The local properties of the matter are not affected by Mach's principle.

The horizon problem, the luminosity distance-redshift relation, the angular diameter distance-redshift relation, and the galaxy number count as a function of redshift in the model of the FRW universe with energy density $\rho \sim R^{-2}$ were studied by Kolb [3]. In the case of a K-dominated universe, kinematic tests limit the parameter K to be $K \gtrsim 1$. In the model which takes into account Mach's principle in the form (5) the universe behave as K-dominated with the parameter $K = 2$ which agrees with the analysis of Refs. [3, 7].

There is some indication that in the cosmological model where the scale factor linearly depends on time, the light element abundances, the position of the first acoustic peak of the CMB can be satisfactorily described [4, 5].

From the analysis of type Ia supernovae discovered by the Supernova Cosmology Project, it follows that the data are consistent with the model in which the mass density and cosmological-constant energy density vanish, $(\Omega_M, \Omega_\Lambda) = (0, 0)$ [30]. It means that the model characterized by linear dependence of the scale factor on time agrees well with the SNe Ia observations [4]. It was shown that the accelerating expansion of the present-day universe extracted from the observed luminosity of the type Ia supernovae can be explained by the theory which takes into account the feedback coupling between geometry and matter (Mach's principle) [31].

In the model which accounts for Mach's principle, an assumption of large amounts of dark energy in the universe is not required to explain cosmological observations. The cosmological model with the scale factor R which evolves in time according to the equation (9) with zero or small cosmological constant can be a good alternative to the standard cosmological model.

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