The Illustration For The Application Of Asset Share In Premium

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Abstract

With a case of whole life insurance, on the base of the actuarial theory of life insurance, this paper accounts for the theoretical foundation of the pricing method of asset share, establishes an intact theoretical framework, and simulates the process that the insurance company ascertains gross premium. In addition, for a real policy, the intact cash flow table is made out. This paper has valuable reference for further systematic study.

Keywords：premium; asset share; surplus; profit; cash flow table
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The calculation of premium is the kernel of actuarial theory of life insurance, and more crucial to product development. But now insurance market is filled with various insurances, and the additive method of net premium can't meets requirements of the insurance company. So the insurance industry increasingly advocates the pricing method of asset share which is to be the mainstream. In this context, this paper applies asset share to test premium, makes an exploratory study on its theoretical foundations, and simulates the process that the insurance company ascertains gross premium with the pricing method of asset share.

With a case of whole life insurance, the paper firstly demonstrates the calculation of predetermined asset share and empirical asset share, whose difference implies potential profit; then analyzes the surplus; finally makes a case study on a practical whole life insurance policy particularly in terms of one marginal equilibrium year index in profit targets, evaluates the reasonability of the premium, works out relevant adjustment of premium, makes out the intact cash flow table and simulates the process that the insurance company ascertains gross premium.

Remark: In this paper the notations in the formulas are consistent with literature [1].

1. The theory of asset share

***Definition* 1** Predetermined asset share is defined as asset on per valid policy reappraised by insurer, on the base of predetermined mortality, interest rate, surrender rate and expenses rate.

To the x-year-old man applying one unit whole life insurance policy, assuming death compensation, surrender value are given at the end of the year, according to the expenses and receipts equilibrium principle, we have

$$ {}_{t+1}AS\ \ p_{x+t}^{(t)} = [{}_tAS + G(1-c_t) - e_t](1+i) - q_{x+t}^{(1)} - q_{x+t}^{(2)}\ {}_{t+1}CV \qquad t = 0,1,2,3..... \qquad (1) $$

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***Definition* 2** Because life insurance is usually a long contract, the change during the contract is difficultly predicted, the insurer takes conservative value to predetermined levels. Empirical asset share is defined as the asset due to practical empirical cost factor, also called empirical adjusted asset share.

Let ${}_{t+1}\hat{AS}$ be the asset share of the year $t+1$ by empirical cost factors on the base of ${}_{t}AS$.

$\hat{i}_{t+1}$, $\hat{q}^{(1)}_{x+t}$, $\hat{q}^{(2)}_{x+t}$, $\hat{c}_t$, $\hat{e}_t$ denote separately the corresponding effective value.

$$ {}_{t+1}AS\ \hat{p}^{(t)}_{x+t} = [{}_{t}AS + G(1-c_t) - e_t](1+\hat{i}) - \hat{q}^{(1)}_{x+t} - \hat{q}^{(2)}_{x+t}\ {}_{t+1}CV \qquad t = 0,1,2,3..... \qquad (2) $$

therefore, we can deduce

$$ {}_{t+1}\hat{AS} - {}_{t+1}AS = ({}_{t}AS + G)(\hat{i}_{t+1} - i) + \qquad ① $$

$$ (Gc_t + e_t)(1+i) - (G\hat{c}_t + \hat{e}_t)(1+\hat{i}_{t+1}) + \qquad ② $$

$$ q^{(1)}_{x+t}(1 - {}_{t+1}AS) - \hat{q}^{(1)}_{x+t}(1 - {}_{t+1}\hat{AS}) + \qquad ③ $$

$$ q^{(2)}_{x+t}({}_{t+1}CV - {}_{t+1}AS) - \hat{q}^{(2)}_{x+t}({}_{t+1}CV - {}_{t+1}\hat{AS}) \qquad ④ $$

From the above ,the difference between predetermined asset share and empirical asset share is made up of four parts: ①the difference of interest; ② the difference of expenses; ③the difference of mortality; ④the difference of surrender. Obviously we can see the potential profit of the insurer.

2. The analysis of surplus

Comparing the calculation of empirical asset share and predetermined asset share, we can find the insurance premium is very conservative .As time goes on, operating profit can be predicted, which leads to the surplus, i.e. balance amount between asset and liability.

Let $A(t), L(t), U(t)$ be respectively the asset, the amount of liability and the amount of surplus at the end of the year t. According to the asset balance principle, we have

$$ A(t) = L(t) + U(t) \qquad (3) $$

For the sake of analysis, the definitions and computing formulas of net revenue and profit are given as follows:

***Definition* 3** Net revenue is defined as the benefit derived at the insurance year t for each unit valid policy at the end of each insurance year t, also called current profit and loss, generally denoted by $S(t)$.

Then

$$S(t) = U(t) - \frac{U(t-1)}{p_{x+t}^{(t)}} \tag{4}$$

***Definition* 4** Profit is defined as the profit of the year t, which is equal to the value deducting the interest income of surplus from net revenue, and can be considered as the loss and profit which current business bring, generally denoted by $Q(t)$.

Then

$$Q(t) = S(t) - U(t-1)\frac{i_t}{p_{x+t}^{(t)}} \tag{5}$$

Net revenue containing more interest income is susceptible to the past operation, so profit can better account for the current operating.

When reserve is the only liability of the insurance company, surplus is the balance between asset share and reserve at the end of the year, then

$$U(t) = {}_tAS - {}_tV_x \tag{6}$$

$$Q(t) = {}_tAS - {}_tV_x - ({}_{t-1}AS - {}_{t-1}V_x)\frac{1+i_t}{p_{x+t}^{(t)}} \tag{7}$$

3. The test of asset share in gross premium

For insurance companies, there are a number of factors to take into account when designing gross premium. Once the insurer sets the gross premium, he has to continuously monitor the influence of this particular deal on the company's cash flow. By doing this, the insurer can determine whether or not the premium meets its target profit. The asset share provides the analytical basis for the insurance company. If the asset share generated by the experimental gross premium can't meet the company's goal, the gross premium will be raised, or particular policy interest (e.g. the death payment or cash value) will have to be reduced. On the contrary, if the asset share exceeds the expected, the insurer can cut down on the premium to attract more customers. For dividend-paying insurance and non-dividend paying insurance, the asset share can be adjusted by revising the experimental dividend and other non-contractual factors. Usually, the test of asset share in gross premium costs at best 20 or 30 years.

In short, to the test, we should first choose an experimental premium according to the fundamental factors comprising gross premium, and then evaluate whether it meets the targets of the insurance company. If not, we will adjust the factors and eventually reach the gross premium.

3.1 The test of target profit

For the insurer, the major business objective is to make profits. Therefore, we will have to consider the reasonableness of gross premium in realization of target profit margin, if not, work out its adjusted formula by the calculation of asset share.

Several profit margin indexes are used in the test of asset share. Here we take one of them:

marginal equilibrium year, i.e. the first year that surplus and accumulated profit is greater than 0, which is generally limited within 5 to 10 year, denoted by n.

For the sake of analysis, we will provide the theoretical hypothesis: the reserve is the only liability item for the insurance company. The target profit then becomes: the asset share of the nth year exceeds the reserve.

The following is the process of asset share testing gross premium on the marginal equilibrium year.

① Set the marginal equilibrium year and adjust the premium.

Choose the target of asset share to be $K > {}_nV_x$. Under certain insurance loading and cash value, we can calculate the gross premium G, and reach the asset share ${}_nAS$ at the end of the nth year according to the previously designed experimental gross premium H. The calculations are as follows:

$$K = \sum_{t=0}^{n-1} \frac{[G(1-c_t)-e_t]l_{x+t}^{(t)}(1+i)^{n-t} - [d_{x+t}^{(1)} + d_{x+t\ t+1}^{(2)} CV](1+i)^{n-t-1}}{l_{x+n}^{(t)}} \tag{8}$$

$${}_nAS = \sum_{t=0}^{n-1} \frac{[H(1-c_t)-e_t]l_{x+t}^{(t)}(1+i)^{n-t} - [d_{x+t}^{(1)} + d_{x+t\ t+1}^{(2)} CV](1+i)^{n-t-1}}{l_{x+n}^{(t)}} \tag{9}$$

Deduct equation (8) from equation (7) from left side and right side, we have:

$$K - {}_nAS = \sum_{t=0}^{n-1} \frac{(G-H)(1-c_t)l_{x+t}^{(t)}(1+i)^{n-t}}{l_{x+n}^{(t)}}$$

When K is certain, we adjust the gross premium to be

$$G = H + \frac{(K - {}_nAS)_n p_x^{(t)} n^{\,n}}{\sum_{t=0}^{n-1}(1-c_t)_t p_x^{(t)} n^{\,t}} \tag{10}$$

The second item of Equation (9) can be considered as the adjusted value based on experimental gross premium.

② Analytical process

If the designed gross premium is reasonable, the initial asset shares will be less than the insurance reserve. This is because the actual expenses in the first few years tend to be relatively higher, and the number will decrease after several years. In this case, we can adjust the gross premium according to the target asset share value by adding insurance loading or deducting cash value from the gross premium.

If the initial asset shares greatly exceed the insurance reserve, the most possibility is that the actual expenditure is far less than the expected. In this circumstance, we will have to adjust the gross premium by adding cash value or deducting insurance loading from the gross premium.

③ Approximate the real value through repeated iteration.

Repeat step one and two, calculate the adjusted gross premium based on the target asset share, then adjust the insurance loading and cash value using computer software, in this way until ascertain the gross premium.

The above discussion can be considered as the epitome of the insurer using the asset share to test gross premium. Of course, in the real world, the insurance company will take into account other comprehensive factors when setting final gross premium.

3.2 Example Analysis

The following section will illustrate the process of asset share testing gross premium by a real example of whole-life insurance policy. We calculate the actual asset share, monitor the influence of experimental gross premium on the company's cash inflows and outflows, analyze whether or not it meets the company's target, and eventually explain validly the result.

Suppose company ABC is working on developing a kind of whole life insurance: the target customer is 35-year old man; the insurance amount will be 10,000 RMB; the premium will be paid out at the beginning of each insurance year.

Expected interest rate: $i = 0.06$;

Insurance loading:

(1) The first year is c_1 equal to 30% of gross premium, and the succeeding year is c_2 equal to 8% of gross premium;

(2) The fixed-fee is e_1 equal to 10 RMB for the first year, and e_2 equal to 4 RMB for the succeeding years.

Cash value: according to the company's empirical data is ${}_tCV = {}_{35}V_x - 8 \quad t = 1,2,3.....20$;

The number of deaths is provided by the experimental life-span table;

The number of policy-holders canceling the contract is provided by the Linton A failure rate.

Suppose the surrendering action happens at the end of the insurance year, we can therefore have the effective insurance statistics.

To simulate the process of the asset share testing gross premium, Suppose:

The insurance amount is paid out at the end of year that the policy-holder died, and the surrender value is paid out at the end of year of cancellation.

The nation's minimum legal standard of insurance reserve and cash value is considered when calculating insurance reserve and cash value.

1) Actuarial process

a. Deduce the experimental gross premium according to the company's hypothesis:

Present value of insurance amount is $10000A_{35} = 1287.194$

Experimental gross premium satisfies

$$H = 10000\ A_{35} + H \times 30\% + 10 + H \times 8\% \times a_{35} + 4 \times a_{35}$$

therefore $H = 97.18$

annual average net premium is $P = 10000 A_{35} / \ddot{a}_{35} = 83.62$

insurance loading in the first year is $97.18 \times 30\% + 10 = 39.15$

insurance loading from the second year on is $97.18 \times 8\% + 4 = 11.77$

net premium in the first year is $P_1 = 97.18 - 97.18 \times 30\% - 10 = 58.03$

net premium from the second year on is $P_2 = 97.18 - 97.18 \times 8\% - 4 = 85.41$

b. Illustration of insurance reserve:

Theoretical reserve is given by the formula ${}_tV_x = A_{x+t} - P_x \ddot{a}_{x+t} \quad t = 1,2,3.....20$ [1].

To legal reserve, we first consider that (refer to literature [11])

Modified net premium in the first year is $a = \{1 - \min(m, 0.35)\} \times H$

Where m denotes predetermined expense rate in the first year, for this policy which is

$(97.18 \times 30\% + 10) / 97.18 = 0.40$.

legal net premium from the second year on is $b = P + (P - a) / a_{35}$

then legal reserve is ${}_tV_L = 10000\ A_{35+t} - b\ \ddot{a}_{35+t} \qquad t = 1,2,3.....20$

So we can get

Table 1 Comparison table between minimum legal reserve and theoretical reserve

Year	1	2	3	4	5	6	7	8	9	10
L	46.85	118.66	193.58	271.70	353.09	437.83	526.01	617.68	712.92	811.79
T	41.34	113.20	188.16	266.32	347.76	432.55	520.77	612.49	707.79	806.71
Year	11	12	13	14	15	16	17	18	19	20
L	914.34	1020.61	1130.65	1244.49	1362.13	1483.60	1608.88	1737.80	1870.80	2007.37
T	909.31	1015.65	1125.75	1239.65	1357.36	1478.89	1604.24	1733.39	1866.31	2002.95

In table 1, “L” symbolizes legal reserve and “T” symbolizes theoretical reserve;

c. Illustration of cash value

Actual cash value by the company is ${}_tCV_A = {}_{35}V_x - 8 \quad t = 1,2,3.....20$

Minimum legal cash value is

${}_tCV_L = r \times \max(s,\ 0)$

where $r = 0.80 + t \times 0.01 \quad t = 1,2,3.....20$

s is reserve at the end of insurance year.

So we can have

Table 2 Comparison table between minimum legal cash value and actual cash value

Year	1	2	3	4	5	6	7	8	9	10
L	37.95	97.30	160.67	228.23	300.13	376.54	457.63	543.56	634.50	730.61
A	33.34	105.20	180.16	258.32	339.76	424.75	512.77	604.49	699.79	798.71
Year	11	12	13	14	15	16	17	18	19	20
L	832.05	938.96	1051.50	1169.82	1294.02	1424.26	1560.61	1703.64	1852.09	2007.37
A	901.31	1007.65	1117.75	1231.65	1349.36	1470.89	1596.24	1725.39	1858.31	1994.95

In table 2, “L” symbolizes minimum legal cash value and “A” symbolizes actual cash value；

According to the above analysis, we can draw the cash flow table 3 (refer to page 9 and 10)

2) Illustration of the cash flow table 3

E＝B－C－D;H＝the previous year of N; I＝B$\times$97.18; J＝B×F; K＝C$\times$10000;

L＝D×G; M＝(H＋I－J)$\times$0.06; N＝H＋I－J－K－L+M; O＝N/E; Q＝O－P

R＝Q－（Q’s previous year）×B/E; S＝R－（Q’s previous year）$\times$0.06 ×B/E

T＝R－（R’s previous year）$\times$1.06 ×B/E

3) Result analysis

a. Item N demonstrates gross premium is sufficient for the death payment, surrender value and all other expenses.

b. Item G is the actual cash value set by the insurance company, which is higher than the minimum legal cash value from the table 2. These demonstrate the actual cash value is quiet reasonable, which is consistent with the company's own operating requirements and is also not against minimum government requirement.

c. Item P is not an actual insurance reserve but the legal insurance reserve, refer to Table 1, the legal reserve is higher than the theoretical reserve. In the initial design of the gross premium, the insurance company should reach the standards of the supervision organization, and then take further amendment to insurance reserve, finally makes out actual reserve. Therefore, it is reasonable and desirable to illustrate the process of asset share testing gross premium by adopting legal insurance reserve.

d. Item Q shows that surplus of each valid insurance policy after one year of operation，and reflects the changes of assets. If the items of asset changing can’t meet the target of the insurance company or the standard of supervision organizations, the insurance company can alter the expected ratio in order to reach the target. However, high surplus for the policy appears at the first year, which means gross premiums may be high and the insurance loading is excessive.

e. Item U is the capital possessed by the valid policy when surrender occurred. At that time, the

insured can only receive cash value not whole capital amount. The amount deducting cash value from asset share still belong to the valid insurance policy, which is a source of cash to insurance companies. Smaller U means less dependence on the surrender, so changes of surrender rate will not cause serious negative effects to the insurance company. Vice versa, larger U shows that the heavier dependence on the surrender, changes of surrender rate might adversely affect insurers. U for this policy has become increasingly dependent on surrender rate, which has to be modified.

4) Conclusion

The asset share analysis illustrates the influence of gross premium on cash flow. According to this, we determine that the gross premium is too high and requires adjustment.

The following adjusts the premium according to the marginal profit index: Let n be 5:

The target asset share $K = 353.09 = {}_5V_{\text{L}}$;

Now we have the asset share based on the experimental gross premium: ${}_5AS = 455.90$

According to formula (9), the adjusted premium: $G = 80.04$

The change of premium turns out to be 17.14 RMB. The gross premium after adjustment amounts to 80.04 RMB. From the perspective of marginal equilibrium year, the premium meets the requirement. Later, we have to modify the corresponding additional premium based on the adjusted gross premium. We repeat step one and continue to make the cash flow table and then compare the relationship between asset share and insurance reserve. Then we make adjustments with regard to the marginal equilibrium year and repeat the process, until finally reach the reasonable gross premium.

In the real world, the insurance companies have to take into account other profit targets when designing gross premium. And they would need to rely on computer software to perform substantial amount of tests before they come up with the appropriate gross premium.

4. Conclusion

On the base of primary actuarial theory, the paper makes an intact framework that asset share tests gross premium, simulates the pricing process of effective life insurance product (the pricing method of asset share). The method is quite complex and highly flexibility, which requires the professional skills of the insurer and computer proficiency. And the theoretical knowledge concerning insurance actuarial practice will be necessary. In the real world settings, this is traditionally accomplished by actuaries and other professional people.

To practical life insurance policy, this paper can help make a decision on the primary reasonability of premium, which is valuable reference for further systematic study.

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论资产份额在保费中的应用

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摘 要 本文以寿险精算理论为基础，以终身寿险为例，通过用资产份额测试总保费的合理性，阐明了资产份额定价法的理论基础，建立了一套完整的理论框架，模拟出保险公司确定总保费的过程,并以实际保单为例，得出了完整的现金流量表。本文的研究对进一步系统学习，具有一定的参考价值。

关键词 保费；资产份额；盈余；利润；现金流量表

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Cash flow table 1

A	B	C	D	E	F	G	H	I	J	K
Insurance year	Year –beginning number	Number of deaths	Number of surrender	Year-end valid number	Annual loading	Cash value	Year –beginning fund	Premium income	Expense payment	Death payment
1	100000	99	9990	89911	39.15	33.34	0.00	9718000.00	3915000.00	990000
2	89911	98	5389	84424	11.77	105.20	4828113.40	8737550.98	1058252.47	980000
3	84424	106	4216	80102	11.77	180.16	11710933.82	8204324.32	993670.48	1060000
4	80102	113	3520	76469	11.77	258.32	18237328.36	7784312.36	942800.54	1130000
5	76469	122	3054	73293	11.77	339.76	26429885.28	7431257.42	900040.13	1220000
6	73293	131	2634	70528	11.77	424.75	32681141.68	7122613.74	862658.61	1310000
7	70528	142	2252	68134	11.77	512.77	38848771.12	6853911.04	830114.56	1420000
8	68134	155	1971	66008	11.77	604.49	44990163.61	6621262.12	801937.18	1550000
9	66008	170	1778	64060	11.77	699.79	51116608.08	6414657.44	776914.16	1700000
10	64060	186	1597	62277	11.77	798.71	57215385.82	6225350.80	753986.20	1860000
11	62277	205	1490	60582	11.77	901.31	63312415.57	6052078.86	733000.29	2050000
12	60582	224	1388	58970	11.77	1007.65	69444391.93	5887358.76	713050.14	2240000
13	58970	246	1292	57432	11.77	1117.75	75457204.38	5730704.60	694076.90	2460000
14	57432	270	1200	55962	11.77	1231.65	81419329.00	5581241.76	675974.64	2700000
15	55962	296	1113	54553	11.77	1349.36	87326091.89	5438387.16	658672.74	2960000
16	54553	325	1085	53143	11.77	1470.89	93170317.01	5301460.54	642088.81	3250000
17	53143	353	1056	51734	11.77	1596.24	98853554.42	5164436.74	625493.11	3530000
18	51734	383	1027	50324	11.77	1725.39	104380418.49	5027510.12	608909.18	3830000
19	50324	414	998	48912	11.77	1858.31	109724985.06	4890486.32	592313.48	4140000
20	48912	447	969	47496	11.77	1994.95	114869954.00	4753268.16	575694.24	4470000

Cash flow table 2

A	M	N	O	P	Q	R	S	T	U
Insurance year	Surrender payment	Interest income	Year-end funds	Asset share	Insurance reserve	Surplus	Net revenue	Profit	Asset share-Cash value
1	333066.60	348180.00	4828113.40	53.70	46.85	6.85	6.85	6.85	20.36
2	566922.80	750444.71	11710933.82	138.72	118.66	20.06	12.76	12.32	33.52
3	759554.56	1135295.26	18237328.36	227.68	193.58	34.10	12.96	11.69	47.52
4	909286.40	1504730.41	26429885.28	345.63	271.70	73.93	38.21	36.07	87.31
5	1037627.04	1977666.15	32681141.68	445.90	353.09	92.81	15.67	11.05	106.14
6	1118791.50	2336465.81	38848771.12	550.83	437.83	112.99	16.55	10.76	126.08
7	1154758.04	2692354.06	44990163.61	660.32	526.01	134.31	17.34	10.33	147.55
8	1191449.79	3048569.31	51116608.08	774.40	617.68	156.72	18.09	9.77	169.91
9	1244226.62	3405261.08	57215385.82	893.15	712.92	180.23	18.75	9.06	193.36
10	1275539.87	3761205.03	63312415.57	1016.63	811.79	204.84	19.44	8.32	217.92
11	1342951.90	4205849.68	69444391.93	1146.29	914.34	231.95	21.38	8.75	244.98
12	1398618.20	4477122.03	75457204.38	1279.59	1020.61	258.98	20.69	6.39	271.94
13	1444133.00	4829629.92	81419329.00	1417.66	1130.65	287.01	21.10	5.15	299.91
14	1477980.00	5179475.77	87326091.89	1560.45	1244.49	315.96	21.41	3.74	328.80
15	1501837.68	5526348.38	93170317.01	1707.89	1362.13	345.76	21.63	2.18	358.53
16	1595915.65	5869781.32	98853554.42	1860.14	1483.60	376.54	21.61	0.32	389.25
17	1685629.44	6203549.88	104380418.49	2017.64	1608.88	408.76	21.96	-1.25	421.40
18	1771975.53	6527941.17	109724985.06	2180.37	1737.80	442.57	22.36	-2.85	454.98
19	1854593.38	6841389.47	114869954.00	2348.50	1870.80	477.70	22.36	-4.97	490.19
20	1933106.55	7142851.68	119787273.04	2522.05	2007.37	514.68	22.74	-6.78	527.10