AUTOEQUIVALENCES OF THE TENSOR CATEGORY OF $U_q\mathfrak{g}$-MODULES

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Abstract. We prove that for $q \in \mathbb{C}^*$ not a nontrivial root of unity the cohomology group defined by invariant 2-cocycles in a completion of $U_q\mathfrak{g}$ is isomorphic to $H^2(P/Q;\mathbb{T})$, where P and Q are the weight and root lattices of $\mathfrak{g}$. This implies that the group of autoequivalences of the tensor category of $U_q\mathfrak{g}$-modules is the semidirect product of $H^2(P/Q;\mathbb{T})$ and the automorphism group of the based root datum of $\mathfrak{g}$. For $q = 1$ we also obtain similar results for all compact connected separable groups.

In a previous paper [6] we showed that if G is a compact connected group then the cohomology group defined by invariant unitary 2-cocycles on $\hat{G}$ is isomorphic to $H^2(\widehat{Z(G)};\mathbb{T})$ and we conjectured that for semisimple Lie groups a similar result holds for the q-deformation of G. In the present note we will prove that this is indeed the case using the technique from our earlier paper [5], where we considered symmetric cocycles and were inspired by the proof of Kazhdan and Lusztig of equivalence of the Drinfeld category and the category of $U_q\mathfrak{g}$-modules [2]. For $q = 1$ this gives an alternative proof of the main results in [6, Section 2] and allows us, at least in the separable case, to extend those results to non-unitary cocycles relying neither on ergodic actions nor on reconstruction theorems. At the same time this proof is less transparent than that in [6] and, as opposed to [6], relies heavily on the structure and representation theory of compact Lie groups.

We will follow the notation and conventions of [5]. Let G be a simply connected semisimple compact Lie group, $\mathfrak{g}$ its complexified Lie algebra. Fix a Cartan subalgebra and a system $\{\alpha_1, \dots, \alpha_r\}$ of simple roots. The weight and root lattices are denoted by P and Q, respectively. For $q \in \mathbb{C}^*$ not a nontrivial root of unity consider the quantized universal enveloping algebra $U_q\mathfrak{g}$. Denote by $\mathcal{C}_q(\mathfrak{g})$ the tensor category of admissible finite dimensional $U_q\mathfrak{g}$-modules, and by $\mathcal{U}(G_q)$ the endomorphism ring of the forgetful functor $\mathcal{C}_q(\mathfrak{g}) \to \mathcal{V}ec$.

An invertible element $\mathcal{E} \in \mathcal{U}(G_q \times G_q)$ is called a 2-cocycle on $\hat{G}_q$ if

$$(\mathcal{E} \otimes 1)(\hat{\Delta}_q \otimes \iota)(\mathcal{E}) = (1 \otimes \mathcal{E})(\iota \otimes \hat{\Delta}_q)(\mathcal{E}).$$

A cocycle is called invariant if it commutes with elements in the image of $\hat{\Delta}_q$. The set of invariant 2-cocycles forms a group under multiplication, which we denote by $Z^2_{G_q}(\hat{G}_q;\mathbb{C}^*)$. Cocycles of the form $(a \otimes a)\hat{\Delta}_q(a)^{-1}$, where a is an invertible element in the center of $\mathcal{U}(G_q)$, form a subgroup of the center of $Z^2_{G_q}(\hat{G}_q;\mathbb{C}^*)$. The quotient of $Z^2_{G_q}(\hat{G}_q;\mathbb{C}^*)$ by this subgroup is denoted by $H^2_{G_q}(\hat{G}_q;\mathbb{C}^*)$.

The center of $\mathcal{U}(G_q)$ is identified with functions on the set P_+ of dominant integral weights. By [5, Proposition 4.5] a function on P_+ is a group-like element of $\mathcal{U}(G_q)$ if and only if it is defined by a character of P/Q. Therefore the Hopf algebra of functions on P/Q embeds into the center of $\mathcal{U}(G_q)$. Hence every 2-cocycle c on P/Q can be considered as an invariant 2-cocycle $\mathcal{E}_c$ on $\hat{G}_q$. Explicitly, if U and V are irreducible $U_q\mathfrak{g}$-modules with highest weights μ and η, then $\mathcal{E}_c$ acts on $U \otimes V$ as multiplication by $c(\mu, \eta)$. We can now formulate our main result.

Theorem 1. *The homomorphism $c \mapsto \mathcal{E}_c$ induces an isomorphism*

$$H^2(P/Q;\mathbb{T}) \cong H^2_{G_q}(\hat{G}_q;\mathbb{C}^*).$$

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In particular, if $\mathfrak{g}$ is simple and $\mathfrak{g}\not\cong\mathfrak{so}_{4n}(\mathbb{C})$ then $H^2_{G_q}(\hat G_q;\mathbb{C}^*)$ is trivial, and if $\mathfrak{g}\cong\mathfrak{so}_{4n}(\mathbb{C})$ then $H^2_{G_q}(\hat G_q;\mathbb{C}^*)\cong\mathbb{Z}/2\mathbb{Z}$.

The last statement follows from the fact that for simple Lie algebras the group P/Q is cyclic unless $\mathfrak{g}\cong\mathfrak{so}_{4n}(\mathbb{C})$, in which case $P/Q\cong\mathbb{Z}/2\mathbb{Z}\times\mathbb{Z}/2\mathbb{Z}$, see e.g. Table IV on page 516 in [1].

Note that for $q>0$ the same result holds for unitary cocycles. This easily follows by polar decomposition, see [5, Lemma 1.1].

In the proof of the theorem we will assume that $q\ne 1$, the case $q=1$ is similar and for unitary cocycles is also proved by a different method in [6].

Our first goal will be to construct a homomorphism $H^2_{G_q}(\hat G_q;\mathbb{C}^*)\to H^2(P/Q;\mathbb{T})$. For every $\mu\in P_+$ fix an irreducible $U_q\mathfrak{g}$-module V_μ with highest weight μ and a highest weight vector ξ_μ. Recall [5, Section 2] that for $\mu,\eta\in P_+$ there exists a unique morphism

$$T_{\mu,\eta}\colon V_{\mu+\eta}\to V_\mu\otimes V_\eta \ \text{ such that }\ \xi_{\mu+\eta}\mapsto\xi_\mu\otimes\xi_\eta.$$

The image of $T_{\mu,\eta}$ is the isotypic component of $V_\mu\otimes V_\eta$ with highest weight $\mu+\eta$. Hence if $\mathcal{E}$ is an invariant 2-cocycle then it acts on this image as multiplication by a nonzero scalar $c_\mathcal{E}(\mu,\eta)$. As in the proof of [5, Lemma 2.2], identity $(T_{\mu,\eta}\otimes\iota)T_{\mu+\eta,\nu}=(\iota\otimes T_{\eta,\nu})T_{\mu,\eta+\nu}$ implies that $c_\mathcal{E}$ is a two-cocycle on P_+. Furthermore, the cohomology class $[c_\mathcal{E}]$ of $c_\mathcal{E}$ in $H^2(P_+;\mathbb{C}^*)$ depends only on the class of $\mathcal{E}$ in $H^2_{G_q}(\hat G_q;\mathbb{C}^*)$, since if $a\in\mathcal{U}(G_q)$ is a central element acting on V_μ as multiplication by a scalar $a(\mu)$ then the action of $(a\otimes a)\hat\Delta_q(a)^{-1}$ on the image of $T_{\mu,\eta}$ is multiplication by $a(\mu)a(\eta)a(\mu+\eta)^{-1}$. Thus the map $\mathcal{E}\mapsto c_\mathcal{E}$ defines a homomorphism $H^2_{G_q}(\hat G_q;\mathbb{C}^*)\to H^2(P_+;\mathbb{C}^*)$.

Given a cocycle on P/Q, we can consider it as a cocycle on P and then get a cocycle on P_+ by restriction. Thus we have a homomorphism $H^2(P/Q;\mathbb{T})\to H^2(P_+;\mathbb{C}^*)$. It is injective since the quotient map $P_+\to P/Q$ is surjective and a cocycle on P/Q is a coboundary if it is symmetric.

Lemma 2. *For every invariant 2-cocycle $\mathcal{E}$ on $\hat G_q$ the class of $c_\mathcal{E}$ in $H^2(P_+;\mathbb{C}^*)$ is contained in the image of $H^2(P/Q;\mathbb{T})$.*

Proof. Consider the skew-symmetric bi-quasicharacter $b\colon P_+\times P_+\to\mathbb{C}^*$ defined by

$$b(\mu,\eta)=c_\mathcal{E}(\mu,\eta)c_\mathcal{E}(\eta,\mu)^{-1}.$$

It extends uniquely to a skew-symmetric bi-quasicharacter on P. To prove the lemma it suffices to show that the root lattice Q is contained in the kernel of this extension. Indeed, since $H^2(P/Q;\mathbb{T})$ is isomorphic to the group of skew-symmetric bi-characters on P/Q, it then follows that there exists a cocycle c on P/Q such that the cocycle $c_\mathcal{E}c^{-1}$ on P_+ is symmetric. Then by [4, Lemma 4.2] the cocycle $c_\mathcal{E}c^{-1}$ is a coboundary, so $c_\mathcal{E}$ and the restriction of c to P_+ are cohomologous.

To prove that Q is contained in the kernel of b, recall [5, Section 2] that for every simple root α_i and weights $\mu,\eta\in P_+$ with $\mu(i),\eta(i)\ge1$ we can define a morphism

$$\tau_{i;\mu,\eta}\colon V_{\mu+\eta-\alpha_i}\to V_\mu\otimes V_\eta\ \text{ such that }\ \xi_{\mu+\eta-\alpha_i}\mapsto[\mu(i)]_{q_i}\xi_\mu\otimes F_i\xi_\eta-q_i^{\mu(i)}[\eta(i)]_{q_i}F_i\xi_\mu\otimes\xi_\eta.$$

The image of $\tau_{i;\mu,\eta}$ is the isotypic component of $V_\mu\otimes V_\eta$ with highest weight $\mu+\eta-\alpha_i$. Since the element $\mathcal{E}$ is invariant, it acts on this image as multiplication by a nonzero scalar $c_i(\mu,\eta)$. As in the proof of [5, Lemma 2.3], consider now another weight ν with $\nu(i)\ge1$. The isotypic component of $V_\mu\otimes V_\eta\otimes V_\nu$ with highest weight $\mu+\eta+\nu-\alpha_i$ has multiplicity two, and is spanned by the images of $(\iota\otimes T_{\eta,\nu})\tau_{i;\mu,\eta+\nu}$ and $(\iota\otimes\tau_{i;\eta,\nu})T_{\mu,\eta+\nu-\alpha_i}$, as well as by the images of $(T_{\mu,\eta}\otimes\iota)\tau_{i;\mu+\eta,\nu}$ and $(\tau_{i;\mu,\eta}\otimes\iota)T_{\mu+\eta-\alpha_i,\nu}$. We have

$$[\eta(i)]_{q_i}(T_{\mu,\eta}\otimes\iota)\tau_{i;\mu+\eta,\nu}-[\nu(i)]_{q_i}(\tau_{i;\mu,\eta}\otimes\iota)T_{\mu+\eta-\alpha_i,\nu}=[\mu(i)+\eta(i)]_{q_i}(\iota\otimes\tau_{i;\eta,\nu})T_{\mu,\eta+\nu-\alpha_i}. \qquad (1)$$

Apply the element

$$\Omega:=(\mathcal{E}\otimes1)(\hat\Delta_q\otimes\iota)(\mathcal{E})=(1\otimes\mathcal{E})(\iota\otimes\hat\Delta_q)(\mathcal{E})$$

to this identity. The morphisms $(T_{\mu,\eta}\otimes\iota)\tau_{i;\mu+\eta,\nu}$, $(\tau_{i;\mu,\eta}\otimes\iota)T_{\mu+\eta-\alpha_i,\nu}$ and $(\iota\otimes\tau_{i;\eta,\nu})T_{\mu,\eta+\nu-\alpha_i}$ are eigenvectors of the operator of multiplication by Ω on the left with eigenvalues $c_{\mathcal{E}}(\mu,\eta)c_i(\mu+\eta,\nu)$, $c_i(\mu,\eta)c_{\mathcal{E}}(\mu+\eta-\alpha_i,\nu)$ and $c_i(\eta,\nu)c_{\mathcal{E}}(\mu,\eta+\nu-\alpha_i)$, respectively. Since the morphisms $(T_{\mu,\eta}\otimes\iota)\tau_{i;\mu+\eta,\nu}$ and $(\tau_{i;\mu,\eta}\otimes\iota)T_{\mu+\eta-\alpha_i,\nu}$ are linearly independent, by applying Ω to (1) we conclude that these three eigenvalues coincide. In particular,

$$c_i(\mu,\eta)c_{\mathcal{E}}(\mu+\eta-\alpha_i,\nu)=c_i(\eta,\nu)c_{\mathcal{E}}(\mu,\eta+\nu-\alpha_i).$$

Applying this to $\eta=\nu=\mu$ we get

$$b(2\mu-\alpha_i,\mu)=1.$$

Since b is skew-symmetric, this gives $b(\alpha_i,\mu)=1$. The latter identity holds for all $\mu\in P_+$ with $\mu(i)\ge 1$. Since every element in P can be written as a difference of two such elements μ, it follows that α_i is contained in the kernel of b. □

Therefore the map $\mathcal{E}\mapsto c_{\mathcal{E}}$ induces a homomorphism $H^2_{G_q}(\hat G_q;\mathbb{C}^*)\to H^2(P/Q;\mathbb{T})$. Clearly, it is a left inverse of the homomorphism $H^2(P/Q;\mathbb{T})\to H^2_{G_q}(\hat G_q;\mathbb{C}^*)$ constructed earlier. Thus it remains to prove that the homomorphism $H^2_{G_q}(\hat G_q;\mathbb{C}^*)\to H^2(P/Q;\mathbb{T})$ is injective.

Assume $\mathcal{E}$ is an invariant 2-cocycle such that the cocycle $c_{\mathcal{E}}$ on P_+ is a coboundary. Then the considerations in [5, Section 2] following Lemma 2.2 apply and show that replacing $\mathcal{E}$ by a cohomologous cocycle we may assume that

$$\mathcal{E}T_{\mu,\eta}=T_{\mu,\eta}\quad\text{and}\quad\mathcal{E}\tau_{i;\nu,\omega}=\tau_{i;\nu,\omega}\tag{2}$$

for all $\mu,\eta\in P_+$, $1\le i\le r$ and $\nu,\omega\in P_+$ such that $\nu(i),\omega(i)\ge 1$. Therefore to prove injectivity it suffices to show the following result.

Proposition 3. *If $\mathcal{E}$ is an invariant 2-cocycle on $\hat G_q$ with property (2) then $\mathcal{E}=1$.*

By [5, Corollary 4.4] the result is true under the additional assumption that $\mathcal{E}$ is symmetric, that is, $\mathcal{R}_\hbar\mathcal{E}=\mathcal{E}_{21}\mathcal{R}_\hbar$ for an R-matrix $\mathcal{R}_\hbar\in\mathcal{U}(G_q\times G_q)$, which depends on the choice of a number $\hbar\in\mathbb{C}$ such that $q=e^{\pi i\hbar}$. We will show that this assumption is automatically satisfied for any $\hbar$.

The results of [5, Section 4] up to (but not including) Lemma 4.3 apply to any invariant cocycle satisfying (2). To formulate these results recall some notation.

For every weight $\mu\in P_+$ fix an irreducible $U_q\mathfrak{g}$-module $\bar V_\mu$ with lowest weight $-\mu$ and a lowest weight vector $\bar\xi_\mu$. For $\lambda\in P$ and $\mu,\eta\in P_+$ such that $\lambda+\mu\in P_+$ there exists a unique morphism

$$\operatorname{tr}^\eta_{\mu,\lambda+\mu}\colon \bar V_{\mu+\eta}\otimes V_{\lambda+\mu+\eta}\to\bar V_\mu\otimes V_{\lambda+\mu}\quad\text{such that}\quad \bar\xi_{\mu+\eta}\otimes\xi_{\lambda+\mu+\eta}\mapsto\bar\xi_\mu\otimes\xi_{\lambda+\mu}.$$

Using these morphisms define an inverse limit $U_q\mathfrak{g}$-module

$$M_\lambda=\varprojlim_\mu \bar V_\mu\otimes V_{\lambda+\mu}.$$

Denote by $\operatorname{tr}_{\mu,\lambda+\mu}$ the canonical map $M_\lambda\to\bar V_\mu\otimes V_{\lambda+\mu}$. The module M_λ is considered as a topological $U_q\mathfrak{g}$-module with a base of neighborhoods of zero formed by the kernels of the maps $\operatorname{tr}_{\mu,\lambda+\mu}$, while all modules in our category $\mathcal{C}_q(\mathfrak{g})$ are considered with discrete topology. Then $\operatorname{Hom}_{U_q\mathfrak{g}}(M_\lambda,V)$ is the inductive limit of the spaces $\operatorname{Hom}_{U_q\mathfrak{g}}(\bar V_\mu\otimes V_{\lambda+\mu},V)$. The vectors $\bar\xi_\mu\otimes\xi_{\lambda+\mu}$ define a topologically cyclic vector $\Omega_\lambda\in M_\lambda$. For any finite dimensional admissible $U_q\mathfrak{g}$-module V the map

$$\eta_V\colon \operatorname{Hom}_{U_q\mathfrak{g}}(\oplus_\lambda M_\lambda,V)\to V,\quad \eta_V(f)=\sum_\lambda f(\Omega_\lambda),$$

is an isomorphism.

The results of [5, Section 4] up to Lemma 4.3 can be summarized by saying that for every invariant cocycle $\mathcal{E}$ satisfying (2) there exist a character χ of P/Q, an invertible morphism $\mathcal{E}_0$ of $\oplus_\lambda M_\lambda$ onto itself preserving the direct sum decomposition, and an invertible element c in the center of $\mathcal{U}(G_q)$ such that

$$\operatorname{tr}_{\mu,\lambda+\mu}\mathcal{E}_0=\chi(\mu)^{-1}\mathcal{E}\operatorname{tr}_{\mu,\lambda+\mu}\quad\text{and}\quad \eta_V(f\mathcal{E}_0)=c\,\eta_V(f)\tag{3}$$

for all $\mu \in P_+$, $\lambda \in P$ such that $\lambda + \mu \in P_+$, all finite dimensional admissible $U_q\mathfrak{g}$-modules V and $f \in \mathrm{Hom}_{U_q\mathfrak{g}}(M_\lambda, V)$.

Proof of Proposition 3. As we have already remarked, by [5, Corollary 4.4] it suffices to show that $\mathcal{R}_\hbar\mathcal{E} = \mathcal{E}_{21}\mathcal{R}_\hbar$ for some $\hbar$ such that $q = e^{\pi i \hbar}$. We will prove a stronger statement: $\sigma\mathcal{E} = \mathcal{E}\sigma$ for any braiding σ on $\mathcal{C}_q(\mathfrak{g})$.

By (3), since $\mathrm{tr}_{\mu,\lambda+\mu}(\Omega_\lambda) = \bar\xi_\mu \otimes \xi_{\lambda+\mu}$, for any $\mu, \eta, \nu \in P_+$ and $f \in \mathrm{Hom}_{U_q\mathfrak{g}}(\bar V_\mu \otimes V_\eta, V_\nu)$ we have

$$\chi(\mu)^{-1} f\mathcal{E}(\bar\xi_\mu \otimes \xi_\eta) = c(\nu) f(\bar\xi_\mu \otimes \xi_\eta).$$

As the vector $\bar\xi_\mu \otimes \xi_\eta$ is cyclic, this means that $f\mathcal{E} = \chi(\mu)c(\nu)f$. Since this is true for all f, we conclude that $\mathcal{E}$ acts on the isotypic component of $\bar V_\mu \otimes V_\eta$ with highest weight ν as multiplication by $\chi(\mu)c(\nu)$. In other words, $\mathcal{E}$ acts on the isotypic component of $V_\mu \otimes V_\eta$ with highest weight ν as multiplication by $\chi(\bar\mu)c(\nu)$. It follows that

$$\sigma\mathcal{E} = \chi(\bar\mu - \bar\eta)\mathcal{E}\sigma \quad \text{on} \quad V_\mu \otimes V_\eta.$$

But by assumption (2) the element $\mathcal{E}$ is the identity on the isotypic component of $V_\mu \otimes V_\eta$ with highest weight $\mu + \eta$, so by considering the above identity on this isotypic component we conclude that $\chi(\bar\mu - \bar\eta) = 1$. Thus χ is the trivial character and $\sigma\mathcal{E} = \mathcal{E}\sigma$. This finishes the proof of Proposition 3 and hence of Theorem 1. ☐

By a result of McMullen [3] any automorphism of the fusion ring of $\mathcal{C}_q(\mathfrak{g})$, mapping irreducibles into irreducibles, is implemented by an automorphism of the based root datum of $\mathfrak{g}$, hence by an automorphism of the Hopf algebra $U_q\mathfrak{g}$. Hence, similarly to [6, Theorem 2.5], we get the following consequence of Theorem 1.

Theorem 4. *The group of $\mathbb{C}$-linear monoidal autoequivalences of the tensor category $\mathcal{C}_q(\mathfrak{g})$ is canonically isomorphic to $H^2(P/Q;\mathbb{T}) \rtimes \mathrm{Aut}(\Psi)$, where Ψ is the based root datum of $\mathfrak{g}$.*

The group P/Q is canonically identified with the dual of the center $Z(G)$ of the group G, so for $q = 1$ Theorem 1 can be formulated as $H^2_G(\hat G;\mathbb{C}^*) \cong H^2(\widehat{Z(G)};\mathbb{C}^*)$. In this form it can be extended to a larger class of groups.

Theorem 5. *For any compact connected separable group G we have a canonical isomorphism*

$$H^2_G(\hat G;\mathbb{C}^*) \cong H^2(\widehat{Z(G)};\mathbb{C}^*).$$

Proof. For Lie groups the proof is essentially the same as above, with P replaced by the weight lattice of a maximal torus of G. In the general case we have a homomorphism $H^2(\widehat{Z(G)};\mathbb{C}^*) \to H^2_G(\hat G;\mathbb{C}^*)$ obtained by considering $\mathcal{U}(Z(G))$ as a subring of $\mathcal{U}(G)$. To construct the inverse homomorphism, for every quotient H of G which is a Lie group consider the composition

$$H^2_G(\hat G;\mathbb{C}^*) \to H^2_H(\hat H;\mathbb{C}^*) \to H^2(\widehat{Z(H)};\mathbb{C}^*),$$

where the first homomorphism is defined using the quotient map $\mathcal{U}(G) \to \mathcal{U}(H)$. The map $Z(G) \to Z(H)$ is surjective (since this is true for Lie groups), so $Z(G)$ is the inverse limit of the groups $Z(H)$. Then $H^2(\widehat{Z(G)};\mathbb{C}^*)$ is the inverse limit of the groups $H^2(\widehat{Z(H)};\mathbb{C}^*)$. Therefore the above maps $H^2_G(\hat G;\mathbb{C}^*) \to H^2(\widehat{Z(H)};\mathbb{C}^*)$ define a homomorphism $H^2_G(\hat G;\mathbb{C}^*) \to H^2(\widehat{Z(G)};\mathbb{C}^*)$. It is clearly a left inverse of the map $H^2(\widehat{Z(G)};\mathbb{C}^*) \to H^2_G(\hat G;\mathbb{C}^*)$, so it remains to show that it is injective.

In other words, we have to check that if $\mathcal{E}$ is an invariant cocycle on $\hat G$ such that its image in $\mathcal{U}(H \times H)$ is a coboundary for every Lie group quotient H of G, then $\mathcal{E}$ itself is a coboundary. If $\mathcal{E}$ were unitary, this could be easily shown by taking a weak operator limit point of cochains, see the proof of [6, Theorem 2.2], and would not require separability of G. In the non-unitary case we can argue as follows.

Since G is separable, there exists a decreasing sequence of closed normal subgroups N_n of G such that $\cap_{n\ge1} N_n = \{e\}$ and the quotients $H_n = G/N_n$ are Lie groups. Let $\mathcal{E}_n$ be the image of $\mathcal{E}$ in $\mathcal{U}(H_n \times H_n)$. By assumption there exist invertible central elements $c_n \in \mathcal{U}(H_n)$ such that $\mathcal{E}_n = (c_n \otimes c_n)\hat{\Delta}(c_n)^{-1}$. For a fixed n consider the image a of c_{n+1} in $\mathcal{U}(H_n)$. Then $c_n a^{-1}$ is a central group-like element in $\mathcal{U}(H_n)$. By [5, Theorem A.1] it is therefore defined by an element of the center of the complexification $(H_n)_{\mathbb{C}}$ of H_n. Since the homomorphism $(H_{n+1})_{\mathbb{C}} \to (H_n)_{\mathbb{C}}$ is surjective, we conclude that there exists a central group-like element b in $\mathcal{U}(H_{n+1})$ such that its image in $\mathcal{U}(H_n)$ is $c_n a^{-1}$. Replacing c_{n+1} by $c_{n+1}b$ we get an element such that $\mathcal{E}_{n+1} = (c_{n+1} \otimes c_{n+1})\hat{\Delta}(c_{n+1})^{-1}$ and the image of c_{n+1} in $\mathcal{U}(H_n)$ is c_n. Applying this procedure inductively we can therefore assume that the image of c_{n+1} in $\mathcal{U}(H_n)$ is c_n for all $n \geq 1$. Then the elements c_n define a central element $c \in \mathcal{U}(G)$ such that $\mathcal{E} = (c \otimes c)\hat{\Delta}(c)^{-1}$. □

In [6, Theorem 2.5] we computed the group of autoequivalences of the C*-tensor category of finite dimensional unitary representations of G. The above theorem allows us to get a similar result ignoring the C*-structure.

Theorem 6. *For any compact connected separable group G, the group of $\mathbb{C}$-linear monoidal autoequivalences of the category of finite dimensional representations of G is canonically isomorphic to $H^2(\widehat{Z(G)}; \mathbb{C}^*) \rtimes \mathrm{Out}(G)$.*

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