COMPACTNESS OF THE $\overline{\partial}$ - NEUMANN OPERATOR ON WEIGHTED $(0,q)$- FORMS.

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Abstract.
As an application of a new characterization of compactness of the $\overline{\partial}$-Neumann operator we derive a sufficient condition for compactness of the $\overline{\partial}$- Neumann operator on $(0,q)$-forms in weighted L^2-spaces on $\mathbb{C}^n$.

1. Introduction.

In this paper we continue the investigations of [12] ans [11] concerning existence and compactness of the canonical solution operator to $\overline{\partial}$ on weighted L^2-spaces over $\mathbb{C}^n$. Let $\varphi : \mathbb{C}^n \longrightarrow \mathbb{R}^+$ be a plurisubharmonic $\mathcal{C}^2$-weight function and define the space

$$L^2(\mathbb{C}^n, \varphi) = \{ f : \mathbb{C}^n \longrightarrow \mathbb{C} \ : \ \int_{\mathbb{C}^n} |f|^2 \, e^{-\varphi} \, d\lambda < \infty \},$$

where λ denotes the Lebesgue measure, the space $L^2_{(0,q)}(\mathbb{C}^n, \varphi)$ of $(0,q)$-forms with coefficients in $L^2(\mathbb{C}^n, \varphi)$, for $1 \leq q \leq n$. Let

$$(f,g)_\varphi = \int_{\mathbb{C}^n} f \, \overline{g} e^{-\varphi} \, d\lambda$$

denote the inner product and

$$\|f\|^2_\varphi = \int_{\mathbb{C}^n} |f|^2 e^{-\varphi} \, d\lambda$$

the norm in $L^2(\mathbb{C}^n, \varphi)$.
We consider the weighted $\overline{\partial}$-complex

$$L^2_{(0,q-1)}(\mathbb{C}^n, \varphi) \underset{\overline{\partial}^*_\varphi}{\overset{\overline{\partial}}{\underset{\longleftarrow}{\longrightarrow}}} L^2_{(0,q)}(\mathbb{C}^n, \varphi) \underset{\overline{\partial}^*_\varphi}{\overset{\overline{\partial}}{\underset{\longleftarrow}{\longrightarrow}}} L^2_{(0,q+1)}(\mathbb{C}^n, \varphi),$$

where for $(0,q)$-forms $u = \sum'_{|J|=q} u_J \, d\overline{z}_J$ with coefficients in $\mathcal{C}^\infty_0(\mathbb{C}^n)$ we have

$$\overline{\partial} u = \sum_{|J|=q}{}' \sum_{j=1}^n \frac{\partial u_J}{\partial \overline{z}_j} \, d\overline{z}_j \wedge d\overline{z}_J,$$

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and

$$\overline{\partial}^*_\varphi u = - \sum_{|K|=q-1}{}' \sum_{k=1}^n \delta_k u_{kK}\, d\overline{z}_K,$$

where $\delta_k = \frac{\partial}{\partial z_k} - \frac{\partial \varphi}{\partial z_k}$.
There is an interesting connection between $\overline{\partial}$ and the theory of Schrödinger operators with magnetic fields, see for example [5], [2], [8] and [6] for recent contributions exploiting this point of view.
The complex Laplacian on $(0,q)$-forms is defined as

$$\Box_\varphi := \overline{\partial}\,\overline{\partial}^*_\varphi + \overline{\partial}^*_\varphi \overline{\partial},$$

where the symbol $\Box_\varphi$ is to be understood as the maximal closure of the operator initially defined on forms with coefficients in $\mathcal{C}_0^\infty$, i.e., the space of smooth functions with compact support.
$\Box_\varphi$ is a selfadjoint and positive operator, which means that

$$(\Box_\varphi f, f)_\varphi \geq 0 \ , \ \text{for } f \in dom(\Box_\varphi).$$

The associated Dirichlet form is denoted by

$$Q_\varphi(f,g) = (\overline{\partial} f, \overline{\partial} g)_\varphi + (\overline{\partial}^*_\varphi f, \overline{\partial}^*_\varphi g)_\varphi, \tag{1.1}$$

for $f,g \in dom(\overline{\partial}) \cap dom(\overline{\partial}^*_\varphi)$. The weighted $\overline{\partial}$-Neumann operator $N_{\varphi,q}$ is - if it exists - the bounded inverse of $\Box_\varphi$.

We indicate that a $(0,1)$-form $f = \sum_{j=1}^n f_j \, d\overline{z}_j$ belongs to $dom(\overline{\partial}^*_\varphi)$ if and only if

$$\sum_{j=1}^n \left(\frac{\partial f_j}{\partial z_j} - \frac{\partial \varphi}{\partial z_j}\, f_j \right) \in L^2(\mathbb{C}^n, \varphi)$$

and that forms with coefficients in $\mathcal{C}_0^\infty(\mathbb{C}^n)$ are dense in $dom(\overline{\partial}) \cap dom(\overline{\partial}^*_\varphi)$ in the graph norm $f \mapsto (\|\overline{\partial} f\|^2_\varphi + \|\overline{\partial}^*_\varphi f\|^2_\varphi)^{\frac{1}{2}}$ (see [10]).

We consider the Levi - matrix

$$M_\varphi = \left(\frac{\partial^2 \varphi}{\partial z_j \partial \overline{z}_k} \right)_{jk}$$

of φ and suppose that the sum s_q of any q (equivalently: the smallest q) eigenvalues of M_φ satisfies

$$\liminf_{|z| \to \infty} s_q(z) > 0. \tag{1.2}$$

We show that (2.2) implies that there exists a continuous linear operator

$$N_{\varphi,q} : L^2_{(0,q)}(\mathbb{C}^n, \varphi) \longrightarrow L^2_{(0,q)}(\mathbb{C}^n, \varphi),$$

such that $\Box_\varphi \circ N_{\varphi,q} u = u$, for any $u \in L^2_{(0,q)}(\mathbb{C}^n, \varphi)$.

If we suppose that that the sum s_q of any q (equivalently: the smallest q) eigenvalues of M_φ satisfies

$$\lim_{|z|\to\infty} s_q(z) = \infty. \tag{1.3}$$

Then the $\overline{\partial}$-Neumann operator $N_{\varphi,q} : L^2_{(0,q)}(\mathbb{C}^n, \varphi) \longrightarrow L^2_{(0,q)}(\mathbb{C}^n, \varphi)$ is compact. This generalizes results from [12] and [11], where the case of $q = 1$ was handled. Finally we discuss some examples in $\mathbb{C}^2$.

2. The weighted Kohn-Morrey formula

First we compute

$$(\Box_\varphi u, u)_\varphi = \|\overline{\partial} u\|^2_\varphi + \|\overline{\partial}^*_\varphi u\|^2_\varphi$$

for $u \in dom(\Box_\varphi)$.
We obtain

$$\|\overline{\partial} u\|^2_\varphi + \|\overline{\partial}^*_\varphi u\|^2_\varphi = \sum_{|J|=|M|=q}{}' \sum_{j,k=1}^n \epsilon^{kM}_{jJ} \int_{\mathbb{C}^n} \frac{\partial u_J}{\partial \overline{z}_j} \overline{\frac{\partial u_M}{\partial \overline{z}_k}} \, e^{-\varphi} \, d\lambda$$
$$+ \sum_{|K|=q-1}{}' \sum_{j,k=1}^n \int_{\mathbb{C}^n} \delta_j u_{jK} \overline{\delta_k u_{kK}} \, e^{-\varphi} \, d\lambda,$$

where $\epsilon^{kM}_{jJ} = 0$ if $j \in J$ or $k \in M$ or if $k \cup M \neq j \cup J$, and equals the sign of the permutation $\binom{kM}{jJ}$ otherwise. The right-hand side of the last formula can be rewritten as

$$\sum_{|J|=q}{}' \sum_{j=1}^n \left\| \frac{\partial u_J}{\partial \overline{z}_j} \right\|^2_\varphi + \sum_{|K|=q-1}{}' \sum_{j,k=1}^n \int_{\mathbb{C}^n} \left(\delta_j u_{jK} \overline{\delta_k u_{kK}} - \frac{\partial u_{jK}}{\partial \overline{z}_k} \overline{\frac{\partial u_{kK}}{\partial \overline{z}_j}} \right) e^{-\varphi} \, d\lambda,$$

see [18] Proposition 2.4 for the details. Now we mention that for $f, g \in \mathcal{C}^\infty_0(\mathbb{C}^n)$ we have

$$\left(\frac{\partial f}{\partial \overline{z}_k}, g \right)_\varphi = -(f, \delta_k g)_\varphi$$

and hence

$$\left(\left[\delta_j, \frac{\partial}{\partial \overline{z}_k} \right] u_{jK}, u_{kK} \right)_\varphi = -\left(\frac{\partial u_{jK}}{\partial \overline{z}_k}, \frac{\partial u_{kK}}{\partial \overline{z}_j} \right)_\varphi + (\delta_j u_{jK}, \delta_k u_{kK})_\varphi.$$

Since

$$\left[\delta_j, \frac{\partial}{\partial \overline{z}_k} \right] = \frac{\partial^2 \varphi}{\partial z_j \partial \overline{z}_k},$$

we get

$$\|\overline{\partial} u\|^2_\varphi + \|\overline{\partial}^*_\varphi u\|^2_\varphi = \sum_{|J|=q}{}' \sum_{j=1}^n \left\| \frac{\partial u_J}{\partial \overline{z}_j} \right\|^2_\varphi + \sum_{|K|=q-1}{}' \sum_{j,k=1}^n \int_{\mathbb{C}^n} \frac{\partial^2 \varphi}{\partial z_j \partial \overline{z}_k} u_{jK} \overline{u}_{kK} \, e^{-\varphi} \, d\lambda. \tag{2.1}$$

Formula (2.1) is a version of the Kohn -Morrey formula, compare [18] or [16].

Proposition 2.1. *Let $1 \leq q \leq n$ and suppose that the sum s_q of any q (equivalently: the smallest q) eigenvalues of M_φ satisfies*

$$\liminf_{|z|\to\infty} s_q(z) > 0. \tag{2.2}$$

Then there exists a uniquely determined bounded linear operator

$$N_{\varphi,q} : L^2_{(0,q)}(\mathbb{C}^n, \varphi) \longrightarrow L^2_{(0,q)}(\mathbb{C}^n, \varphi),$$

such that $\Box_\varphi \circ N_{\varphi,q} u = u$, for any $u \in L^2_{(0,q)}(\mathbb{C}^n, \varphi)$.

Proof. Let $\mu_{\varphi,1} \leq \mu_{\varphi,2} \leq \cdots \leq \mu_{\varphi,n}$ denote the eigenvalues of M_φ and suppose that M_φ is diagonalized. Then, in a suitable basis,

$$\begin{aligned}\sum_{|K|=q-1}{}' \sum_{j,k=1}^{n} \frac{\partial^2 \varphi}{\partial z_j \partial \overline{z}_k} u_{jK} \overline{u}_{kK} &= \sum_{|K|=q-1}{}' \sum_{j=1}^{n} \mu_{\varphi,j} |u_{jK}|^2 \\ &= \sum_{J=(j_1,\ldots,j_q)}{}' (\mu_{\varphi,j_1} + \cdots + \mu_{\varphi,j_q}) |u_J|^2 \\ &\geq s_q |u|^2\end{aligned}$$

It follows from (2.1) that there exists a constant $C > 0$ such that

$$\|u\|^2_\varphi \leq C(\|\overline{\partial} u\|^2_\varphi + \|\overline{\partial}^*_\varphi u\|^2_\varphi) \tag{2.3}$$

for each $(0,q)$-form $u \in$dom $(\overline{\partial}) \cap$ dom $(\overline{\partial}^*_\varphi)$. For a given $v \in L^2_{(0,q)}(\mathbb{C}^n, \varphi)$ consider the linear functional L on dom $(\overline{\partial}) \cap$ dom $(\overline{\partial}^*_\varphi)$ given by $L(u) = (u, v)_\varphi$. Notice that dom $(\overline{\partial}) \cap$ dom $(\overline{\partial}^*_\varphi)$ is a Hilbertspace in the inner product Q_φ. Since we have by 2.3

$$|L(u)| = |(u, v)_\varphi| \leq \|u\|_\varphi \, \|v\|_\varphi \leq C Q_\varphi(u, u)^{1/2} \, \|v\|_\varphi.$$

Hence by the Riesz reprentation theorem there exists a uniquely determined $(0,q)$-form $N_{\varphi,q} v$ such that

$$(u, v)_\varphi = Q_\varphi(u, N_{\varphi,q} v) = (\overline{\partial} u, \overline{\partial} N_{\varphi,q} v)_\varphi + (\overline{\partial}^*_\varphi u, \overline{\partial}^*_\varphi N_{\varphi,q} v)_\varphi,$$

from which we immediately get that $\Box_\varphi \circ N_{\varphi,q} v = v$, for any $v \in L^2_{(0,q)}(\mathbb{C}^n, \varphi)$. If we set $u = N_{\varphi,q} v$ we get again from 2.3

$$\begin{aligned}\|\overline{\partial} N_{\varphi,q} v\|^2_\varphi + \|\overline{\partial}^*_\varphi N_{\varphi,q} v\|^2_\varphi &= Q_\varphi(N_{\varphi,q} v, N_{\varphi,q} v) = (N_{\varphi,q} v, v)_\varphi \leq \|N_{\varphi,q} v\|_\varphi \, \|v\|_\varphi \\ &\leq C_1 (\|\overline{\partial} N_{\varphi,q} v\|^2_\varphi + \|\overline{\partial}^*_\varphi N_{\varphi,q} v\|^2_\varphi)^{1/2} \, \|v\|_\varphi,\end{aligned}$$

hence

$$(\|\overline{\partial} N_{\varphi,q} v\|^2_\varphi + \|\overline{\partial}^*_\varphi N_{\varphi,q} v\|^2_\varphi)^{1/2} \leq C_2 \|v\|_\varphi$$

and finally again by 2.3

$$\|N_{\varphi,q} v\|_\varphi \leq C_3 (\|\overline{\partial} N_{\varphi,q} v\|^2_\varphi + \|\overline{\partial}^*_\varphi N_{\varphi,q} v\|^2_\varphi)^{1/2} \leq C_4 \|v\|_\varphi,$$

where $C_1, C_2, C_3, C_4 > 0$ are constants. Hence we get that $N_{\varphi,q}$ is a continuous linear operator from $L^2_{(0,q)}(\mathbb{C}^n, \varphi)$ into itself (see also [13] or [4]). $\square$

3. Compactness of $N_{\varphi,q}$

We use a characterization of precompact subsets of L^2-spaces, see [1]:
A bounded subset $\mathcal{A}$ of $L^2(\Omega)$ is precompact in $L^2(\Omega)$ if and only if for every $\epsilon > 0$ there exists a number $\delta > 0$ and a subset $\omega \subset\subset \Omega$ such that for every $u \in \mathcal{A}$ and $h \in \mathbb{R}^n$ with $|h| < \delta$ both of the following inequalities hold:

$$(3.1) \qquad (i) \int_\Omega |\tilde{u}(x+h) - \tilde{u}(x)|^2 \, dx < \epsilon^2 \quad , \quad (ii) \int_{\Omega \setminus \overline{\omega}} |u(x)|^2 \, dx < \epsilon^2.$$

In addition we define an appropriate Sobolev space and prove compactness of the corresponding embedding, for related settings see [3], [14], [15] .

Definition 3.1. *Let*

$$\mathcal{W}_q^{Q_\varphi} = \{u \in L^2_{(0,q)}(\mathbb{C}^n, \varphi) \ : \ \|\overline{\partial} u\|_\varphi^2 + \|\overline{\partial}^*_\varphi u\|_\varphi^2 < \infty\}$$

with norm

$$\|u\|_{Q_\varphi} = (\|\overline{\partial} u\|_\varphi^2 + \|\overline{\partial}^*_\varphi u\|_\varphi^2)^{1/2}.$$

Remark: $\mathcal{W}_q^{Q_\varphi}$ coincides with the form domain $dom(\overline{\partial}) \cap dom(\overline{\partial}^*_\varphi)$ of Q_φ (see [9], [10]).

Proposition 3.2. *Let φ be a plurisubharmonic $\mathcal{C}^2$- weight function. Let $1 \leq q \leq n$ and suppose that the sum s_q of any q (equivalently: the smallest q) eigenvalues of M_φ satisfies*

$$(3.2) \qquad \lim_{|z| \to \infty} s_q(z) = \infty.$$

Then $N_{\varphi,q} : L^2_{(0,q)}(\mathbb{C}^n, \varphi) \longrightarrow L^2_{(0,q)}(\mathbb{C}^n, \varphi)$ is compact.

Proof. For $(0,q)$ forms one has by (2.1) and Proposition 2.1 that

$$(3.3) \qquad \|\overline{\partial} u\|_\varphi^2 + \|\overline{\partial}^*_\varphi u\|_\varphi^2 \geq \int_{\mathbb{C}^n} s_q(z) \, |u(z)|^2 \, e^{-\varphi(z)} \, d\lambda(z).$$

We indicate that the embedding

$$j_{\varphi,q} : \mathcal{W}_q^{Q_\varphi} \hookrightarrow L^2_{(0,q)}(\mathbb{C}^n, \varphi)$$

is compact by showing that the unit ball of $\mathcal{W}_q^{Q_\varphi}$ is a precompact subset of $L^2_{(0,q)}(\mathbb{C}^n, \varphi)$, which follows by the above mentioned characterization of precompact subsets in L^2-spaces with the help of Gårding's inequality to verify (3.1) (i)(see for instance [7] or [4]) and to verify (3.1) (ii) : we have

$$\int_{\mathbb{C}^n \setminus \mathbb{B}_R} |u(z)|^2 e^{-\varphi(z)} \, d\lambda(z) \leq \int_{\mathbb{C}^n \setminus \mathbb{B}_R} \frac{s_q(z) |u(z)|^2}{\inf\{s_q(z) : |z| \geq R\}} e^{-\varphi(z)} d\lambda(z),$$

which implies by (3.3) that

$$\int_{\mathbb{C}^n \setminus \mathbb{B}_R} |u(z)|^2 e^{-\varphi(z)} \, d\lambda(z) \leq \frac{\|u\|_{Q_\varphi}^2}{\inf\{s_q(z) \ : \ |z| \geq R\}} < \epsilon,$$

if R is big enough, see [11] for the details.
This together with the fact that $N_{\varphi,q} = j_{\varphi,q} \circ j^*_{\varphi,q}$, (see [18]) gives the desired result. $\square$

Remark 3.3. *If $q = 1$ condition (3.2) means that the lowest eigenvalue $\mu_{\varphi,1}$ of M_φ satisfies*

$$\lim_{|z|\to\infty} \mu_{\varphi,1}(z) = \infty. \tag{3.4}$$

This implies compactness of $N_{\varphi,1}$ (see [11]).

Examples: a) We consider the plurisubharmonic weight function $\varphi(z,w) = |z|^2|w|^2 + |w|^4$ on $\mathbb{C}^2$. The Levi matrix of φ has the form

$$\begin{pmatrix} |w|^2 & \overline{z}w \\ \overline{w}z & |z|^2 + 4|w|^2 \end{pmatrix}$$

and the eigenvalues are

$$\mu_{\varphi,1}(z,w) = \frac{1}{2}\left(5|w|^2 + |z|^2 - \sqrt{9|w|^4 + 10|z|^2|w|^2 + |z|^4}\right)$$

$$= \frac{16|w|^4}{2\left(5|w|^2 + |z|^2 + \sqrt{9|w|^4 + 10|z|^2|w|^2 + |z|^4}\right)},$$

and

$$\mu_{\varphi,2}(z,w) = \frac{1}{2}\left(5|w|^2 + |z|^2 + \sqrt{9|w|^4 + 10|z|^2|w|^2 + |z|^4}\right).$$

It follows that (3.4) fails, since even

$$\lim_{|z|\to\infty} |z|^2 \mu_{\varphi,1}(z,0) = 0,$$

but

$$s_2(z,w) = \frac{1}{4}\,\Delta\varphi(z,w) = |z|^2 + 5|w|^2,$$

hence (3.2) is satisfied for $q = 2$.

b)In the next example we consider decoupled weights. Let $n \geq 2$ and

$$\varphi(z_1, z_2, \dots, z_n) = \varphi(z_1) + \varphi(z_2) + \cdots + \varphi(z_n)$$

be a plurisubharmonic decoupled weight function and suppose that $|z_\ell|^2 \Delta\varphi_\ell(z_\ell) \to +\infty$, as $|z_\ell| \to \infty$ for some $\ell \in \{1, \dots, n\}$. Then the $\overline{\partial}$-Neumann operator$N_{\varphi,1}$ acting on $L^2_{(0,1)}(\mathbb{C}^n, \varphi)$ fails to be compact (see [12], [9], [17]).
Finally we discuss two examples in $\mathbb{C}^2$: for $\varphi(z_1,z_2) = |z_1|^2 + |z_2|^2$ all eigenvalues of the Levi matrix are 1 and $N_{\varphi,1}$ fails to be compact by the above result on decoupled weights, for the weightfunction $\varphi(z_1,z_2) = |z_1|^4 + |z_2|^4$ the eigenvalues are $4|z_1|^2$ and $4|z_2|^2$ and $N_{\varphi,1}$ fails to be compact again by the above result, whereas $N_{\varphi,2}$ is compact by 3.2.

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