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Determination of the continuous casting cross-section with prescribed average temperature

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Abstract

This paper presents the method of determination of the continuous casting cross-section, in which average temperature was equal to a prescribed value. The method proposed here does not require evaluation of temperature distribution. On the basis of input data, a linear or non-linear equation is created (depending on the heat flux form on the region boundaries), which solution enabled determination of the cross-section.

Keywords: Solidification, Stefan problem, Heat equation, Continuous casting

1. Introduction

Let us consider a task consisting of determination of such place in a solidified part of a flat continuous casting, where the average temperature in the cross-section would equal a prescribed value, specified by technological requirements. The discussion below refers to a vertical device for continuous casting operating in an undisturbed cycle with an assumption that the variable cooling conditions, depending on the casting drawing direction, are identical throughout the casting circumference and its section dimensions fulfil the condition: $a << b$, where a means the casting thickness and b, its width. Let us also assume that the heat flux takes place only in a direction perpendicular to the casting axis. This assumption results from the fact that the amount of heat conducted in the casting motion direction, compared to the amount of heat conducted in a direction perpendicular to the casting axis, is negligible [1,2].

There is a possibility of finding an analytical solution of the proposed problem only in specific cases of one-dimensional problem and most often, for one-phase problems [3,4]. In simple cases, we can also use the Adomian decomposition method or a variational iteration method to solve the Stefan problem [5-9]. Then a solution in the form of a continuous function defined as a linear combination of the prescribed base functions is obtained. The coefficients of this combination are so determined numerically, to minimize the functional description of solution deviation from boundary conditions.

For other cases only approximated methods can be used (see for example [10-17]), which however, require a tremendous amount of effort and time for calculations. The temperature field must be determined in such cases for the entire region under consideration. It is only then possible to determine the cross-section of a casting with a prescribed average temperature. The method proposed does not require determination of temperature distribution. Based on the input data a linear or non-linear equation is created (depending on the heat flux form on the boundary Γ_a), which solution enables determination of the cross-section being the object of our interest.

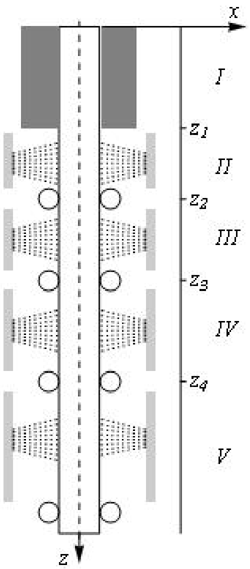

Fig. 1. Modelled object

2. Mathematical model

With the assumptions made, as well as due to thermal symmetry, the D casting region can be treated as a two-dimensional domain composed of two subdomains: D_1 - liquid phase, and D_2 - solid phase, where, with the space orientation as in fig. 2, the heat exchange process, including the quasi-steady thermal field, is described by a two-phase Stefan problem determined from the following system of equations and conditions:

$$c_k\,\rho_k\,v\frac{\partial T_k}{\partial z}(x,z)=\lambda_k\frac{\partial^2 T_k}{\partial x^2}(x,z), \qquad (x,z)\in D_k,\ k=1,2, \tag{1}$$

$$T_1(x,0)=T_0, \qquad x\in[0,a], \tag{2}$$

$$-\lambda_1\frac{\partial T_1}{\partial x}(0,z)=0, \qquad z\in\left(0,z^*\right], \tag{3}$$

$$-\lambda_2\frac{\partial T_2}{\partial x}(0,z)=0, \qquad z\in\left(z^*,z_e\right], \tag{4}$$

$$-\lambda_2\frac{\partial T_2}{\partial x}(a,z)=q(z), \qquad z\in\left(0,z_e\right], \tag{5}$$

$$T_1(\xi(z),z)=T_2(\xi(z),z)=T^*, \qquad z\in\left[0,z^*\right], \tag{6}$$

$$L\,\rho_2\frac{d\xi(z)}{dz}=-\lambda_1\frac{\partial T_1(\xi(z),z)}{\partial x}+\lambda_2\frac{\partial T_2(\xi(z),z)}{\partial x}, \qquad z\in\left[0,z^*\right], \tag{7}$$

$$\xi(0)=a, \tag{8}$$

$$\frac{1}{a}\int_0^a T_2(x,z_e)\,dx=T_{av}, \tag{9}$$

where T_k is the temperature, x and z refer to spatial location, λ_k, c_k and ρ_k are, respectively: the thermal conductivity, the specific heat and the mass density in liquid phase ($k=1$) and in solid phase ($k=2$), v is the constant casting velocity, T^* is the temperature of the phase change, T_0 is the pouring temperature, L is the latent heat of fusion, $\xi(z)$ is the function describing the position of the phase change moving interface, z^* means the maximum depth of liquid metal deposition, T_{av} and z_e mean the average temperature of casting cross-section and the place where this value is reached (casting cutting place), respectively, whereas $q(z)$ means the heat flux emitted.

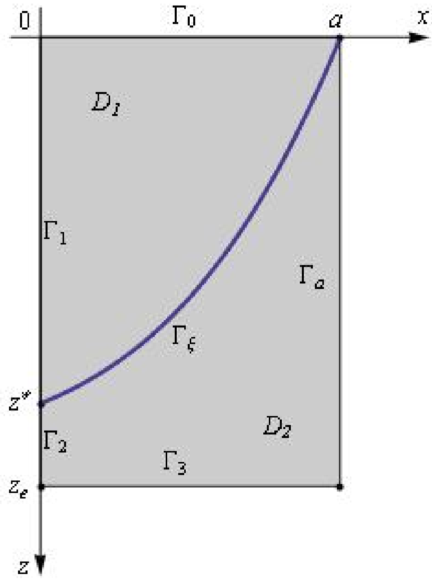

Fig. 2. Domain of the problem

3. Solution to the problem

By integrating equations (1) on appropriate domains D_k ($k=1,2$), we obtain:

$$\iint_{D_k}\left(\lambda_k\frac{\partial^2 T_k}{\partial x^2}(x,z)-c_k\,\rho_k\,v\frac{\partial T_k}{\partial z}(x,z)\right)dxdz=0. \tag{10}$$

Now, using the Green's formula for a double integral, the equations (10) can be presented in the form:

$$\oint_{\partial D_k}c_k\,\rho_k\,v\,T_k(x,z)dx+\lambda_k\frac{\partial T_k}{\partial x}(x,z)dz=0, \quad k=1,2, \tag{11}$$

where ∂D_k, $k=1,2$, denote the boundaries of domains D_k. Since boundaries ∂D_k of domains D_k can be presented in the following form (see Fig. 2):

$$\partial D_1 = \Gamma_0 \cup \Gamma_1 \cup \Gamma_\xi,$$
$$\partial D_2 = \Gamma_2 \cup \Gamma_3 \cup \Gamma_a \cup \Gamma_\xi,$$

where

$$\Gamma_0 = \{(x,0);\ x \in [0,a]\},$$
$$\Gamma_1 = \{(0,z);\ z \in [0,z^*]\},$$
$$\Gamma_2 = \{(0,z);\ z \in [z^*,z_e]\},$$
$$\Gamma_3 = \{(x,z_e);\ x \in [0,a]\},$$
$$\Gamma_a = \{(a,z);\ z \in [0,z_e]\},$$
$$\Gamma_\xi = \{(\xi(z),z);\ z \in [0,z^*]\},$$

it means that the curves in the integrals can be parameterized. By utilizing such parametrization and taking into account the conditions: (2)-(6), we can transform the right sides of equations (11), which will assume the form:

$$\oint_{\partial D_1} c_1\,\rho_1\,v\,T_1(x,z)dx + \lambda_1 \frac{\partial T_1}{\partial x}(x,z)dz = = -c_1\,\rho_1\,v\,a\,T^* + c_1\,\rho_1\,v\,a\,T_0 + \int_0^{z^*} \lambda_1 \frac{\partial T_1}{\partial x}(\xi(z),z)dz, \tag{12}$$

$$\oint_{\partial D_2} c_2\,\rho_2\,v\,T_2(x,z)dx + \lambda_2 \frac{\partial T_2}{\partial x}(x,z)dz = c_2\,\rho_2\,v\,a\,T^* - \int_0^{z_e} q(z)dz - c_2\,\rho_2\,v\int_0^a T_2(x,z_e)dx - \int_0^{z^*} \lambda_2 \frac{\partial T_2}{\partial x}(\xi(z),z)dz. \tag{13}$$

Having substituted dependencies (12) and (13), respectively, for formulas (11), and adding side-by-side the obtained dependencies, with concurrently taking into account condition (7), we obtain:

$$c_2\,\rho_2\,v\int_0^a T_2(x,z_e)dx = (c_2\,\rho_2 - c_1\,\rho_1)v\,a\,T^* + + L\,\rho_2\,v\,a + c_1\,\rho_1\,v\,a\,T_0 - \int_0^{z_e} q(z)dz. \tag{14}$$

By using dependence (9) in the latter equation, we will obtain:

$$\int_0^{z_e} q(z)dz = c_2\,\rho_2\,v\,a\left(T^* - T_{av}\right) + c_1\,\rho_1\,v\,a\left(T_0 - T^*\right) + L\,\rho_2\,v\,a. \tag{15}$$

The value sought, z_e, is present within the first integral limit of integration and, since function $q(z)$ as well as all other quantities present in the above equation are known, we obtain a confounded equation for z_e. Whether or not we will be able to analytically determine the value of z_e from this equation, depends on whether we will be able to analytically integrate function $q(z)$, and on what equation the integration will yield. As regards a numerical solution (even if function $q(z)$ is not analytically enterable) reduces to solving the equation:

$$\int_0^{z_e} q(z)dz = A, \tag{16}$$

where, to simplify the notation, the following designation was introduced:

$$A = c_2\,\rho_2\,v\,a\left(T^* - T_{av}\right) + c_1\,\rho_1\,v\,a\,T_{sh} + L\,\rho_2\,v\,a, \tag{17}$$

where $T_{sh} = T_0 - T^*$ (temperature of superheating). The method of a numerical solution of a non-linear equation (e.g. bisection method) can be applied directly to equation (16) or the integral can be first substituted with the sum (e.g. in accordance with Newton-Cotes quadrature formula) and next, an approximated solution method can be applied for the obtained non-linear equation.

4. Example

Proposed solution method can be illustrated with following example of determination the cross-section of a copper casting with the following parameters [13]: $a = 0.1$ [m], $v = 0.002$ [m/s], $\lambda_1 = 370$ [W/(m K)], $\lambda_2 = 370$ [W/(m K)], $c_1 = 400$ [J/(kg K)], $c_2 = 400$ [J/(kg K)], $\rho_1 = 8900$ [kg/m^3], $\rho_2 = 8900$ [kg/m^3], $L = 200000$ [J/kg], $T^* = 1356$ [K], $T_0 = 1373$ [K] and $T_{av} = 800$ [K].

It is assumed for the calculations that the casting cooling area consists of five zones (fig. 1) of the following lengths: 0.2 [m], 0.4 [m], 0.8 [m], 0.6 [m], $z_e - 2.0$ [m], respectively, for which a constant flux value is assumed:

$$q(z) = \begin{cases} 220\cdot 10^3 & z \in [0,\ 0.2), \\ 140\cdot 10^3 & z \in [0.2,\ 0.6), \\ 110\cdot 10^3 & z \in [0.6,\ 1.4), \\ 90\cdot 10^3 & z \in [1.4,\ 2.0), \\ 80\cdot 10^3 & z \ge 2.0. \end{cases}$$

For the problem so posed, the calculation results show that the place for which the average cross-section temperature with value of 800 [K], should be as follows:

$z_e = 8.5247\,[m]$.

To verify accuracy of the method presented, the same case of Stefan problem (1)-(8) was solved by means of an alternating phase truncation method [14,15] combined with the finite difference method. Calculations were carried out on a mesh with discretisation steps equal: $\Delta z = 0.1$ and $\Delta x = a/500$. The temperature distribution for $z = z_e$ determined by this method is presented in fig. 3. The average temperature value in the section $z = z_e$ is 800.013 [K]. This corroborates accuracy of the method presented in this paper.

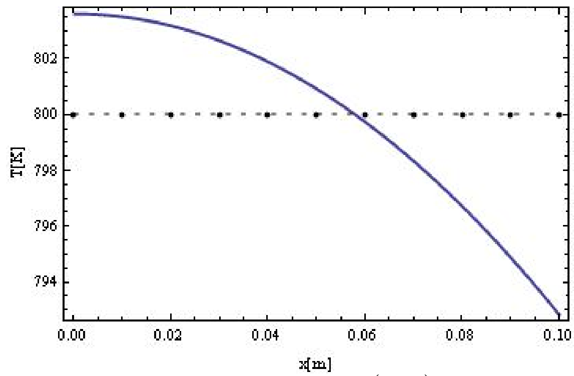

Fig. 3. The temperature distribution $T(x, z_e)$ obtained by the alternating phase truncation method (the broken line denotes the mean temperature obtained by the alternating phase truncation method and the dotted line shows the prescribed mean temperature of the section (800 [K]) used in proposed method)

5. Conclusion

The paper presents the method of determination of continuous casting cross-section position, in which the average temperature in the cross-section is equal to prescribed value, for example by technological requirements. It is worth emphasizing that the method applied herein does not require the determination of thermal fields for any of the phases and it is not necessary to determine the interface location. Other available methods, which are applied in practice to find the mean temperature for a chosen section, require the determination of temperature distribution throughout the entire area and the location of interface (since considering the Stefan problem), which however requires a considerable amount of effort and time for calculations. The method proposed does not require determination of the temperature distribution. Only with use of input data a linear or non-linear equation is created, which solution enables determination of the sought position of the cross-section. A comparison of the obtained solution with the thermal field from the Stefan problem solution obtained by the alternating phase truncation method corroborates accuracy of the method presented.

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