Sp(2) Renormalization

Peter M. Lavrov[1]

Department of Mathematical Analysis,
Tomsk State Pedagogical University,
634061, Kievskaya St. 60, Tomsk, Russia

Abstract. The renormalization of general gauge theories on flat and curved space-time backgrounds is considered within the Sp(2)-covariant quantization method. We assume the existence of a gauge-invariant and diffeomorphism invariant regularization. Using the Sp(2)-covariant formalism one can show that the theory possesses gauge invariant and diffeomorphism invariant renormalizability to all orders in the loop expansion and the extended BRST symmetry after renormalization is preserved. The advantage of the Sp(2)-method compared to the standard Batalin-Vilkovisky approach is that, in reducible theories, the structure of ghosts and ghosts for ghosts and auxiliary fields is described in terms of irreducible representations of the Sp(2) group. This makes the presentation of solutions to the master equations in more simple and systematic way because they are Sp(2)- scalars.

Keywords: Gauge theories, curved space, renormalization, extended BRST symmetry, antibrackets.

PACS: 11.10.Gh, 04.60.Gw, 04.62.+v, 11.30.Pb.

MSC-AMS: 81T15, 81T20.

1 Introduction

It is well known that Green's functions in Quantum Field Theory (QFT) contain divergences [1, 2]. Renormalization should be considered as one of important issue of QFT especially in gauge theories which form a basis for formulating modern theories of fundamental interactions (electromagnetic, weak, strong and gravitational). In the first papers by 't Hooft and Veltman [3, 4] devoted to solving the problem of renormalization in the Yang-Mills theories within Faddeev-Popov quantization [5], this achievement required a great effort. In particular, it was necessary the invention of construction of special gauges and also special technics to prove the gauge invariant renormalizability. Later on, after deriving the Slavnov-Taylor identity [6], discovering BRST symmetry [7] and presenting this symmetry in the form of non-linear unique equation (Zinn-

[1]E-mail address: lavrov@tspu.edu.ru

Justin equation) [8], the proof of the gauge invariant renormalizability of Yang-Mills theories became more simple [9, 10].

After discovering supergravity theories [11, 12, 13] it was realized that direct application of the Faddeev-Popov answers leads in the case of these theories to an incorrect result; namely, the violation of the physical S-matrix unitarity. The reason lies in the structure of gauge transformations for these theories. In this case, the invariance transformations for the initial action do not form a gauge group. The arising structure coefficients may depend on the fields of the initial theory, and the gauge algebra of these transformations may be opened by terms proportional to the equations of motion. Moreover, attempts of covariant quantization of gauge theories with linearly-dependent generators of gauge transformations result in the understanding of the fact that it is impossible to use the Faddeev-Popov rules to construct a suitable quantum theory [14, 15, 16]. Therefore, the quantization of gauge theories requires taking into account many new aspects (in comparison with QED) such as open algebras, reducible generators and so on. It was realized how to quantize them using different types of ghosts, antighosts, ghosts for ghosts (Nielsen, Kallosh ghosts etc) [17, 18, 19, 20, 21, 22, 23, 24].

A unique closed approach to the problem of covariant quantization summarized all these attempts was proposed by Batalin and Vilkovisky [25, 26]. The Batalin-Vilkovisky (BV) formalism gives the rules for the quantization of general gauge theories. Now it is known that using new concept of renormalizability (beyond the Dyson criterion [27, 28]) proposed in [29] this formalism enables one to prove the gauge-invariant renormalizability of general gauge theories when all fields under consideration are quantum ones. Later this point of view was supported by Gomis and Weinberg [30] (see also [31, 32] for an extensive review and further references).

Renormalizations in curved space-time within the Dyson criterion are under intense investigations beginning with paper by Utiyama and DeWitt [33] (see [34, 35, 36, 37, 38] and references therein). In the present work we continue our recent investigation of gauge invariant renormalizability in curved space-time with the help of new concept of renormalizability [29]. In [39] it was done in the framework of BV formalism [25, 26]. We have extended these considerations to the case when the QFT is defined in the presence of external conditions, in particular in curved space-time and proved that in this case the gauge invariant renormalizability is compatible with preserving general covariance.

Except the BV formalism, there is an alternative approach for quantization of general gauge theories, which is based on the principle of invariance under extended BRST symmetry including BRST and anti-BRST transformations on an equal footing [40, 41, 42] (compare with alternative approach [43]). We are going to consider the problem of gauge invariant renormalizability of general gauge theories in the framework of Sp(2)-method, since it was not done before, and also prove general covariance of renormalization in the presence of a gravitational background field.

The paper is organized as follows. In Section 2 a short exposition of Sp(2) quantization approach in Lagrangian formalism for general gauge theories is given. Section 3 is devoted to the proof of gauge invariant renormalizability of general gauge theories preserving extended BRST symmetry. In Section 4 within Sp(2) formalism the general gauge theories in the presence of an external gravitational field are considered. In Section 5 general covariance of renormalization in the Sp(2) method is proved. In Section 6 concluding remarks are given.

We use the condensed notations as given by DeWitt [44]. Derivatives with respect to sources and antifields are taken from the left, and those with respect to fields, from the right. Left derivatives with respect to fields are labeled by the subscript *"l"*. The Grassmann parity of any quantity A is denoted by $\epsilon(A)$.

2 Gauge theories in Sp(2)-covariant method

In this section we present a very brief review of the Sp(2)-covariant formalism [40, 41, 42], which will be used in the rest of the paper to prove the gauge invariant and general covariant renormalizability of the quantum field theory on curved background.

2.1 Configuration space

To construct the $Sp(2)$-quantization for general gauge theory one needs in introduction of configuration space. To this end we consider the initial classical action $S_0(A)$ of fields A^i. This action $S_0(A)$ is assumed to have at least one stationary point $A_0 = \{A_0^i\}$

$$S_{0,i}(A)|_{A_0} = 0, \tag{1}$$

and to be regular in the neighborhood of A_0. Equation (1) defines a surface Σ in space of functions A^i. Invariance of the action $S_0(A)$ under the gauge transformations $\delta A^i = R^i_\alpha(A)\xi^\alpha$ in the neighborhood of the stationary point is assumed:

$$S_{0,i}(A)R^i_\alpha(A) = 0, \quad \alpha = 1, 2, ..., m, \quad 0 < m < n, \quad \varepsilon(\xi^\alpha) = \varepsilon_\alpha. \tag{2}$$

Here ξ^α are arbitrary functions of space-time coordinates , and $R^i_\alpha(A)$ ($\varepsilon(R^i_\alpha) = \varepsilon_i + \varepsilon_\alpha$) are generators of gauge transformations. We have also used DeWitt's condensed notations [44], when any index includes all particular ones (space - time, index of internal group, Lorentz index and so on). Summation over repeated indices implies integration over continuous ones and usual summation over discrete ones.

Then it is necessary to introduce the total configuration space Φ^A, which coincides, in fact, with the total configuration space in the BV formalism [25, 26], but there is difference in arrangement of the ghost and antighost fields:

$$\Phi^A = (A^i,\ B^{\alpha|a_1\cdots a_s},\ C^{\alpha|a_0\cdots a_s},\ s = 0, ..., L;\ a_i = 1, 2), \quad \varepsilon(\Phi^A) = \varepsilon_A, \tag{3}$$

where L denotes the stage of initial action reducibility. Auxiliary fields $B^{\alpha|a_1\cdots a_s}$ and ghost fields $C^{\alpha|a_0\cdots a_s}$ are symmetric Sp(2) tensors of corresponding ranks. The following values of the Grassmann parity are ascribed to these fields:

$$\begin{aligned} \varepsilon(B^{\alpha|a_1\cdots a_s}) &= \varepsilon_{\alpha_s} + s \text{ (mod2)}, \\ \varepsilon(C^{\alpha|a_0\cdots a_s}) &= \varepsilon_{\alpha_s} + s + 1 \text{ (mod2)}, \quad s = 0, ..., L \end{aligned}$$

together with the following values of the ghost number:

$$gh(B^{\alpha_0}) = 0, \quad gh(B^{\alpha|a_1\cdots a_s}) = \sum_{s'=1}^{s}(3 - 2a_{s'}), \quad gh(C^{\alpha|a_0\cdots a_s}) = \sum_{s'=0}^{s}(3 - 2a_{s'}).$$

To each field Φ^A of the total configuration space one introduces three sets of antifields Φ^*_{Aa}, $\varepsilon(\Phi^*_{Aa}) = \varepsilon_A + 1$ and $\bar{\Phi}_A, \varepsilon(\bar{\Phi}_A) = \varepsilon_A$. We know the meaning of antifields in the BV-approach. They are sources of BRST transformations. In the extended BRST algebra, there are three kinds of transformations; namely, BRST-transformations, antiBRST-transformations and mixed transformations. The antifields Φ^*_{Aa} form $Sp(2)$ doublets with respect to the index a and can be treated as sources of BRST- and antiBRST-transformations, while $\bar{\Phi}_A$ are sources of combined transformation.

2.2 Extended antibrackets

On the space of fields Φ^A and antifields Φ^*_{Aa} one defines odd symplectic structures $(\ ,\ )^a$, called the extended antibrackets

$$(F,G)^a \equiv \frac{\delta F}{\delta \Phi^A}\,\frac{\delta G}{\delta \Phi^*_{Aa}} - (F \leftrightarrow G)\,(-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)}. \tag{4}$$

As usually the derivatives with respect to fields are understood as acting from the right and those with respect to antifields, as acting from the left.

The extended antibrackets (4) have the following properties:

$$\begin{aligned}
&\varepsilon((F,G)^a) = \varepsilon(F) + \varepsilon(G) + 1,\\
&(F,G)^a = -(G,F)^a(-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)},\\
&(F,GH)^a = (F,G)^a H + (F,H)^a G(-1)^{\varepsilon(G)\varepsilon(H)},\\
&((F,G)^{\{a},H)^{b\}}(-1)^{(\varepsilon(F)+1)(\varepsilon(H)+1)} + \text{cycl.perm.}(F,G,H) \equiv 0,
\end{aligned} \tag{5}$$

where curly brackets denote symmetrization with respect to the indices a, b of the $Sp(2)$ group:

$$A^{\{a}B^{b\}} \equiv A^a B^b + B^b A^a.$$

The last relations in (5) are the graded Jacobi identities for the extended antibrackets. In particular, for any bosonic functional S, $\varepsilon(S) = 0$, one can establish that

$$((S,S)^{\{a},S)^{b\}} \equiv 0. \tag{6}$$

2.3 Extended quantum master equations

In addition the operators V^a, Δ^a are introduced

$$V^a = \varepsilon^{ab}\,\Phi^*_{Ab}\,\frac{\delta}{\delta \bar{\Phi}_A}, \quad \Delta^a = (-1)^{\varepsilon_A}\frac{\delta_l}{\delta \Phi^A}\frac{\delta}{\delta \Phi^*_{Aa}}, \tag{7}$$

where ε^{ab} is the antisymmetric tensor for raising and lowering $Sp(2)$-indices

$$\varepsilon^{ab} = -\varepsilon^{ba}, \quad \varepsilon^{12} = 1 \quad \varepsilon_{ab} = -\varepsilon^{ab}.$$

It can be readily established that the algebra of the operators (7) has the form

$$\Delta^{\{a}\Delta^{b\}} = 0, \quad \Delta^{\{a}V^{b\}} + V^{\{a}\Delta^{b\}} = 0, \quad V^{\{a}V^{b\}} = 0. \tag{8}$$

The action of the operators (7) on a product of functionals F and G gives

$$\Delta^a(F \cdot G) = (\Delta^a F) \cdot G + F \cdot (\Delta^a G)(-1)^{\varepsilon(F)} + (F,\ G)^a(-1)^{\varepsilon(F)}, \tag{9}$$

$$\begin{aligned} V^a(F,G)^b &= (V^a F,\ G)^b - (-1)^{\varepsilon(F)}(F, V^a G)^b - \\ &\quad -\varepsilon^{ab}\left(\frac{\delta F}{\delta \phi^A}\frac{\delta G}{\delta \bar{\phi}_A} - \frac{\delta G}{\delta \phi^A}\frac{\delta F}{\delta \bar{\phi}_A}(-1)^{\varepsilon(F)(\varepsilon(G)+1)}\right). \end{aligned}$$

Therefore only the symmetrized form of V^a acting on the extended antibrackets observes the Leibniz rule

$$V^{\{a}(F,G)^{b\}} = (V^{\{a}F,G)^{b\}} - (-1)^{\varepsilon(F)}(F,V^{\{a}G)^{b\}}. \tag{10}$$

For any bosonic functional S we have

$$\frac{1}{2}V^{\{a}(S,S)^{b\}} = (V^{\{a}S,S)^{b\}}. \tag{11}$$

It is advantageous to introduce an operator $\bar{\Delta}^a$

$$\bar{\Delta}^a = \Delta^a + \frac{i}{\hbar}V^a$$

with the properties

$$\bar{\Delta}^{\{a}\bar{\Delta}^{b\}} = 0. \tag{12}$$

For a boson functional $S = S(\Phi, \Phi^*, \bar{\Phi})$, we introduce extended quantum master equations

$$\frac{1}{2}(S,S)^a + V^a S = i\hbar\Delta^a S \tag{13}$$

with the boundary condition

$$S\bigg|_{\Phi^*=\bar{\Phi}=\hbar=0} = S_0(A), \tag{14}$$

where $S_0(A)$ is the initial classical action.

The generating equation for the bosonic functional S is a set of two equations. It should be verified that these equations are compatible. The simplest way to establish this fact is to rewrite the extended master equations in an equivalent form of linear differential equations

$$\bar{\Delta}^a \exp\left\{\frac{i}{\hbar}S\right\} = 0. \tag{15}$$

Due to the properties of the operators $\bar{\Delta}^a$ (12), we immediately establish the compatibility of the equations.

2.4 Gauge fixing

The action S is gauge-degenerate. To lift the degeneracy, we should introduce a gauge. We denote the action modified by gauge as $S_{ext} = S_{ext}(\Phi, \Phi^*, \bar{\Phi})$. The gauge should be introduced so as, first, to lift the degeneracy in ϕ and, second, to retain the extended master equation, which provides the invariance properties of the theory for S_{ext}. To meet these conditions, the gauge is introduced as

$$\exp\left\{\frac{i}{\hbar}S_{ext}\right\} = \exp\left\{-i\hbar\hat{T}(F)\right\}\exp\left\{\frac{i}{\hbar}S\right\} \tag{16}$$

where $F = F(\Phi)$ is a bosonic functional fixing a gauge in the theory. The explicit form of the operator $\hat{T}(F)$ is

$$\hat{T}(F) = \frac{\delta F}{\delta\Phi^A}\frac{\delta}{\delta\bar{\Phi}_A} + \frac{i\hbar}{2}\varepsilon_{ab}\frac{\delta}{\delta\Phi^*_{Aa}}\frac{\delta^2 F}{\delta\Phi^A\delta\phi^B}\frac{\delta}{\delta\Phi^*_{Bb}}. \tag{17}$$

Due to the properties of the operators $\bar{\Delta}^a$, it is not difficult to check the equality

$$\bar{\Delta}^a\exp\left\{-i\hbar\hat{T}(F)\right\} = \exp\left\{-i\hbar\hat{T}(F)\right\}\bar{\Delta}^a. \tag{18}$$

Therefore, the action S_{ext} satisfies the extended master equations

$$\bar{\Delta}^a\exp\left\{\frac{i}{\hbar}S_{ext}\right\} = 0. \tag{19}$$

2.5 Generating functional of Green's functions

We next define the generating functional $Z(J)$ of Green's functions by the rule

$$Z(J) = \int d\Phi\,\exp\left\{\frac{i}{\hbar}[S_{eff}(\Phi) + J_A\Phi^A]\right\}, \tag{20}$$

where

$$S_{eff} = S_{ext}(\Phi, \Phi^*, \bar{\Phi})|_{\Phi^*=\bar{\Phi}=0}. \tag{21}$$

It can be represented in the form

$$\begin{aligned} Z(J) \;\; = & \int d\Phi\; d\Phi^*\; d\bar{\Phi}\; d\lambda\; d\pi^a\;\; \exp\left\{\frac{i}{\hbar}\left(S(\Phi, \Phi^*, \bar{\Phi}) + \Phi^*_{Aa}\pi^{Aa} + \right.\right. \\ & \left.\left. +\left(\bar{\Phi}_A - \frac{\delta F}{\delta\Phi^A}\right)\lambda^A - \frac{1}{2}\varepsilon_{ab}\pi^{Aa}\frac{\delta^2 F}{\delta\Phi^A\delta\Phi^B}\pi^{Bb} + J_A\Phi^A\right)\right\}, \end{aligned} \tag{22}$$

where we have introduced a set of auxiliary fields π^{Aa}, λ^A

$$\varepsilon(\pi^{Aa}) = \varepsilon_A + 1, \quad \varepsilon(\lambda^A) = \varepsilon_A.$$

2.6 Extended BRST symmetry

An important property of the integrand for $J_A = 0$ is its invariance under the following global transformations (which, for its part , is a consequence of the extended master equation for S_{ext})

$$\begin{aligned}
&\delta\Phi^A = \pi^{Aa}\mu_a, \quad \delta\Phi^*_{Aa} = \mu_a \frac{\delta S}{\delta\Phi^A}, \quad \delta\bar{\Phi}_A = \varepsilon^{ab}\mu_a\Phi^*_{Ab}, \\
&\delta\pi^{Aa} = -\varepsilon^{ab}\lambda^A\mu_b, \quad \delta\lambda^A = 0,
\end{aligned} \tag{23}$$

where μ_a is an Sp(2) doublet of constant anticommuting Grassmann parameters. These transformations realize the extended BRST transformations in the space of the variables Φ, Φ^*, $\bar{\Phi}$, π and λ.

2.7 Gauge independence of vacuum functional

The existence of these transformations enables one to establish the independence of the vacuum functional from the choice of gauge. Indeed, suppose $Z_F \equiv Z(0)$. We shall change the gauge $F \to F + \Delta F$. In the functional integral for $Z_{F+\Delta F}$ we make the above-mentioned change of variables with the parameters chosen as

$$\mu_a = \frac{i}{2\hbar}\varepsilon_{ab}\frac{\delta\Delta F}{\delta\Phi^A}\pi^{Ab}. \tag{24}$$

Then we find

$$Z_F = Z_{F+\Delta F} \tag{25}$$

and therefore the S-matrix is gauge-independent.

2.8 Ward identities

Let us now derive the Ward identities, which follow from the fact that the boson functional $S(\phi, \phi^*, \bar{\phi})$ satisfies the extended master equations. To do this, we introduce the extended generating functional of Green's functions

$$\mathcal{Z}(J, \Phi^*, \bar{\Phi}) = \int d\Phi \exp\left\{\frac{i}{\hbar}[S_{ext}(\Phi, \Phi^*_a, \bar{\Phi}) + J_A\Phi^A]\right\}. \tag{26}$$

From this definition it follows that

$$\mathcal{Z}(J, \Phi^*, \bar{\Phi})|_{\Phi^*=\bar{\Phi}=0} = Z(J) \tag{27}$$

where $Z(J)$ has been introduced above.

We have,

$$\int d\Phi \ \exp\left\{\frac{i}{\hbar}J_A\Phi^A\right\}\bar{\Delta}^a \exp\left\{\frac{i}{\hbar}S_{ext}(\Phi, \Phi^*, \bar{\Phi})\right\} = 0.$$

Integrating by parts, under the assumption that the integrated expression vanishes, we can write this equality as

$$\widehat{\omega}^a\mathcal{Z}(J, \Phi^*, \bar{\Phi}) = 0, \tag{28}$$

where

$$\widehat{\omega}^a = \left(J_A \frac{\delta}{\delta \Phi^*_{Aa}} - \varepsilon^{ab} \Phi^*_{Ab} \frac{\delta}{\delta \bar{\Phi}_A} \right), \quad \widehat{\omega}^{\{a} \widehat{\omega}^{b\}} = 0. \tag{29}$$

Eqs. (28) are the Ward identities for the generating functional of Green's functions. For the generating functional $\mathcal{W}(J, \Phi^*, \bar{\Phi})$ of connected Green's functions we have

$$\widehat{\omega}^a \mathcal{W}(J, \Phi^*, \bar{\Phi}) = 0, \tag{30}$$

Finally, for the generating functional of vertex functions

$$\Gamma(\Phi, \Phi^*, \bar{\Phi}) = \mathcal{W}(J, \Phi^*, \bar{\Phi}) - J_A \Phi^A, \quad \Phi^A = \frac{\delta \mathcal{W}}{\delta J_A}$$

we obtain the Ward identities

$$\frac{1}{2}(\Gamma, \Gamma)^a + V^a \Gamma = 0 \tag{31}$$

in the form of the classical part of the extended quantum master equations.

3 Extended BRST invariant renormalizability

Here we can prove the preservation of the extended BRST-symmetry under renormalization within the usual assumptions on perturbation theory as well as on a regularization repeating main arguments used in [29] to state the gauge invariant renormalizability in the BV formalism. It will be shown that if [2]

$$\frac{1}{2}(S, S)^a + V^a S = i\hbar \Delta^a S, \tag{32}$$

$$\frac{1}{2}(\Gamma, \Gamma)^a + V^a \Gamma = 0 \tag{33}$$

then the renormalized action S_R and the effective action Γ_R satisfy the same equations

$$\frac{1}{2}(S_R, S_R)^a + V^a S_R = i\hbar \Delta^a S_R, \tag{34}$$

$$\frac{1}{2}(\Gamma_R, \Gamma_R)^a + V^a \Gamma_R = 0. \tag{35}$$

(here and elsewhere we drop the index *ext*).

Let us represent S in the form

$$S = \sum_{n=0}^{\infty} \hbar^n S_{(n)} = S_{(0)} + \hbar S_{(1)} + \hbar^2 S_{(2)} + \cdots.$$

Then we have the following recurrent equations to define $S_{(n)}$ step by step beginning with $S_{(0)}$

$$\frac{1}{2}(S_{(0)},\ S_{(0)})^a + V^a S_{(0)} = 0, \tag{36}$$

[2] The action of Δ^a-operators on local functionals is proportional to $\delta(0)$. Usually they say that a regularization (likes dimensional one) is used when $\delta(0) = 0$. Here we give a formal proof without using this assumption.

The $S_{(1)}$ and $S_{(2)}$ satisfy the following linear equations:

$$(S_{(0)},\ S_{(1)})^a + V^a S_{(1)} = i\Delta^a S_{(0)}.$$

$$(S_{(0)},\ S_{(2)})^a + V^a S_{(2)} = i\Delta^a S_{(0)} - \frac{1}{2}(S_{(1)},\ S_{(1)})^a.$$

In general

$$(S_{(0)},\ S_{(n)})^a + V^a S_{(n)} = i\Delta^a S_{(n-1)} - \frac{1}{2}\sum_{k=1}^{n-1}(S_{(k)}, S_{(n-k)})^a, \quad n = 1, 2, 3, \ldots \tag{37}$$

In papers [40, 41] the existence theorem for the equations (36) has been proved in the form of Taylor series in the antifields $\Phi^*_{Aa}, \bar{\Phi}_A$. For the gauge theories discussed above the solution to (36) in the lower order in antifields can be presented as

$$\begin{aligned} S_{(0)} = {} & S_0(A) + A^*_{ia} R^i_\alpha C^{\alpha|a} + \bar{A}_i R^i_\alpha B^\alpha - \varepsilon^{ab} C^*_{\alpha a|b} B^\alpha + \\ & + \sum_{s=1}^{L}\Big(C^*_{\alpha_s a|a_0\ldots a_{s-1}} C^{\alpha_s|aa_0\ldots a_{s-1}} + \bar{C}_{\alpha_s|a_1\ldots a_s} B^{\alpha_s|a_1\ldots a_s} - \varepsilon^{ab} C^*_{\alpha_s a|ba_1\ldots a_s} B^{\alpha_s|a_1\ldots a_s} - \\ & \quad - \frac{s}{s+1} B^*_{\alpha_s a_0|a_1\ldots a_{s-1}} B^{\alpha_s|a_0 a_1\ldots a_{s-1}} - \varepsilon^{ab} C^*_{\alpha_s a|ba_1\ldots a_{s-1}} B^{\alpha_s|a_1\ldots a_{s-1}}\Big) + \cdots. \end{aligned} \tag{38}$$

It is important to note that the functional $S_{(0)}$ (38) is by construction a local functional if one operates with the gauge algebra underlying a given gauge theory described in terms of gauge generators being local functions.

Equations (37) can be presented in the form

$$W^a S_{(n)} = F^a_n, \tag{39}$$

where

$$W^a = \frac{\delta S_{(0)}}{\delta \Phi^A}\frac{\delta}{\delta \Phi^*_{Aa}} + (-1)^{\epsilon_A}\frac{\delta S_{(0)}}{\delta \Phi^*_{Aa}}\frac{\delta_l}{\delta \Phi^A} + V^a, \tag{40}$$

$$F^a_n = i\Delta^a S_{(n-1)} - \frac{1}{2}\sum_{k=1}^{n-1}(S_{(k)}, S_{(n-k)})^a. \tag{41}$$

The structure and properties of equations (39) formally coincide with ones used in [40, 41] to prove the existence theorem. Indeed, operators W^a obey the relations

$$W^{\{a}W^{b\}} = 0 \tag{42}$$

as consequences of equations (36) and the properties of V^a (8). It follows from (39) and (42) that F^a_n should satisfy the equations

$$W^{\{a}F^{b\}}_n = 0. \tag{43}$$

To prove these equations let us consider the identity (6) and rewrite it in the form

$$(\tfrac{1}{2}(S,S)^{\{a} + V^{\{a}S - i\hbar\Delta^{\{a}S, S)^{b\}} - (V^{\{a}S - i\hbar\Delta^{\{a}S, S)^{b\}} = 0, \tag{44}$$

or

$$(S, \tfrac{1}{2}(S,S)^{\{a} + V^{\{a}S - i\hbar\Delta^{\{a}S)^{b\}} + (V^{\{a}S - i\hbar\Delta^{\{a}S, S)^{b\}} = 0, \tag{45}$$

Using properties of operators Δ^a and V^a (8), (11) from (45) one derives

$$\begin{aligned}(S, \tfrac{1}{2}(S,S)^{\{a} + V^{\{a}S - i\hbar\Delta^{\{a}S)^{b\}} + V^{\{a}[\tfrac{1}{2}(S,S)^{b\}} + V^{b\}}S - i\hbar\Delta^{b\}}S] - \\ -i\hbar\Delta^{\{a}[\tfrac{1}{2}(S,S)^{b\}} + V^{b\}}S - i\hbar\Delta^{b\}}S] = 0. \end{aligned} \tag{46}$$

Note that

$$\frac{1}{2}(S,S)^a + V^aS - i\hbar\Delta^aS = \hbar^n(W^aS_{(n)} - F^a_n) + O(\hbar^{n+1}), \tag{47}$$

in the lower order in $\hbar$ we have

$$(S_{(0)}, W^{\{a}S_{(n)} - F^{\{a}_n)^{b\}} + V^{\{a}[W^{b\}}S_{(n)} - F^{b\}}_n] = 0, \tag{48}$$

or

$$W^{\{a}[W^{b\}}S_{(n)} - F^{b\}}_n] = 0 \tag{49}$$

that proves (43). Repeating arguments given in [40, 41] one can state the existence of solutions to the Eqs. (39) and therefore to (32). We suppose that the action S is a local functional.

Now let us represent Γ in the form

$$\Gamma = S + \hbar(\Gamma^{(1)}_{div} + \Gamma^{(1)}_{fin}) + O(\hbar^2) = S_{(0)} + \hbar(\Gamma^{(1)}_{div} + \bar{\Gamma}^{(1)}_{fin}) + O(\hbar^2),$$

where $\bar{\Gamma}^{(1)}_{fin} = \Gamma^{(1)}_{fin} + S_{(1)}$. Besides, $\Gamma^{(1)}_{div}$ and $\Gamma^{(1)}_{fin}$ denote the divergent and finite parts of the one-loop approximation for Γ.

The functional $\Gamma^{(1)}_{div}$ determines the counterterms of the one-loop renormalized action S_{1R} which is the local functional:

$$S_{1R} = S - \hbar\Gamma^{(1)}_{div}$$

and satisfies the equation

$$(S_{(0)},\ \Gamma^{(1)}_{div})^a + V^a\Gamma^{(1)}_{div} = 0.$$

Let us consider

$$\begin{aligned}&\frac{1}{2}(S_{1R},\ S_{1R})^a + V^aS_{1R} - i\hbar\Delta^aS_{1R} = \\ &= \frac{1}{2}(S,\ S)^a + V^aS - i\hbar\Delta^aS - \hbar(S,\ \Gamma^{(1)}_{div})^a + \frac{1}{2}\hbar^2(\Gamma^{(1)}_{div},\ \Gamma^{(1)}_{div})^a + i\hbar^2\Delta^a\Gamma^{(1)}_{div} = \\ &= \hbar^2\left(\frac{1}{2}(\Gamma^{(1)}_{div},\ \Gamma^{(1)}_{div})^a + i\Delta^a\Gamma^{(1)}_{div} - (S_{(1)},\ \Gamma^{(1)}_{div})^a\right) + O(\hbar^3).\end{aligned}$$

We find that S_{1R} satisfies the master equation

$$\frac{1}{2}(S_{1R},\ S_{1R})^a + V^aS_{1R} - i\hbar\Delta^aS_{1R} = \hbar^2E^a_2 + O(\hbar^3)$$

up to certain terms E_2^a

$$E_2^a = \frac{1}{2}(\Gamma^{(1)}_{div},\ \Gamma^{(1)}_{div})^a + i\Delta^a\Gamma^{(1)}_{div} - (S_{(1)},\ \Gamma^{(1)}_{div})^a$$

of the second order in $\hbar$.

Let us construct the effective action Γ_{1R} with the help of the action S_{1R}. This functional is finite in the one-loop approximation and satisfies the equation

$$\frac{1}{2}(\Gamma_{1R},\ \Gamma_{1R})^a + V^a\Gamma_{1R} = \hbar^2 E_2^a + O(\hbar^3).$$

Represent Γ_{1R} in the form

$$\begin{aligned}\Gamma_{1R} &= S + \hbar\Gamma^{(1)}_{fin} + +\hbar^2(\Gamma^{(2)}_{1,div} + \Gamma^{(2)}_{1,fin}) + O(\hbar^3) = \\ &= S_{(0)} + \hbar\bar{\Gamma}^{(1)}_{fin} + +\hbar^2(\Gamma^{(2)}_{1,div} + \bar{\Gamma}^{(2)}_{1,fin}) + O(\hbar^3),\end{aligned}$$

where $\bar{\Gamma}^{(2)}_{1,fin} = \Gamma^{(2)}_{1,fin} + S_{(2)}$. The divergent part $\Gamma^{(2)}_{1,div}$ of the two - loop approximation for Γ_{1R} determines the two - loop renormalization for S_{2R}

$$S_{2R} = S_{1R} - \hbar^2\Gamma^{(2)}_{1,div}$$

and satisfies the equations

$$(S_{(0)},\ \Gamma^{(2)}_{1,div})^a + V^a\Gamma^{(2)}_{1,div} = E_2^a.$$

Let us now consider

$$\begin{aligned}&\frac{1}{2}(S_{2R},\ S_{2R})^a + V^aS_{2R} - i\hbar\Delta^aS_{2R} = \\ &= \frac{1}{2}(S_{1R},\ S_{1R})^a - i\hbar\Delta^aS_{1R} - \hbar^2(S_{1R},\ \Gamma^{(2)}_{1,div})^a + i\hbar^3\Delta^a\Gamma^{(2)}_{1,div} = \\ &= \hbar^3\left((\Gamma^{(1)}_{div},\ \Gamma^{(2)}_{1,div})^a + i\Delta^a\Gamma^{(2)}_{1,div} - (S_{(2)},\Gamma^{(1)}_{div})^a - (S_{(1)},\Gamma^{(2)}_{1,div})^a\right) + O(\hbar^4) = \\ &= \hbar^3E_3^a + O(\hbar^4).\end{aligned}$$

We find that S_{2R} satisfies the master equations up to terms E_3^a

$$E_3^a = (\Gamma^{(1)}_{div},\ \Gamma^{(2)}_{1,div})^a + i\Delta^a\Gamma^{(2)}_{1,div} - (S_{(2)},\Gamma^{(1)}_{div})^a - (S_{(1)},\Gamma^{(2)}_{1,div})^a$$

of the third order in $\hbar$. Then the corresponding effective action Γ_{2R} generated by S_{2R} is finite in the two - loop approximation

$$\begin{aligned}\Gamma_{2R} &= S + \hbar\Gamma^{(1)}_{fin} + \hbar^2\Gamma^{(2)}_{1,fin} + \hbar^3(\Gamma^{(3)}_{2,div} + \Gamma^{(3)}_{2,fin}) + O(\hbar^4) = \\ &= S_{(0)} + \hbar\bar{\Gamma}^{(1)}_{fin} + \hbar^2\bar{\Gamma}^{(2)}_{1,fin} + \hbar^3(\Gamma^{(3)}_{2,div} + \bar{\Gamma}^{(3)}_{2,fin}) + O(\hbar^4)\end{aligned}$$

and satisfies the equations

$$\frac{1}{2}(\Gamma_{2R},\ \Gamma_{2R})^a + V^a\Gamma_{2R} = \hbar^3E_3^a + O(\hbar^4)$$

up to certain terms E_3 of the third order in $\hbar$.

Applying the induction method we establish that the totally renormalized action S_R

$$S_R = S - \sum_{n=1}^{\infty} \hbar^n \Gamma^{(n)}_{n-1,div} \tag{50}$$

satisfies the quantum master equations exactly:

$$\frac{1}{2}(S_R,\ S_R)^a + V^a = i\hbar\Delta^a S_R, \tag{51}$$

while the renormalized effective action Γ_R is finite in each order of $\hbar$ powers:

$$\Gamma_R = S + \sum_{n=1}^{\infty} \hbar^n \Gamma^{(n)}_{n-1,fin} = S_{(0)} + \sum_{n=1}^{\infty} \hbar^n \bar{\Gamma}^{(n)}_{n-1,fin}, \tag{52}$$

and satisfies the identities

$$\frac{1}{2}(\Gamma_R,\ \Gamma_R)^a + V^a\Gamma_R = 0. \tag{53}$$

Here, we have denoted by $\Gamma^{(n)}_{n-1,div}$ and $\Gamma^{(n)}_{n-1,fin}$ the divergent and finite parts, respectively, of the n - loop approximation for the effective action which is finite in (n-1)th approximation and is constructed from the action $S_{(n-1)R}$.

Thus, we have established the fact that the renormalized action S_R and the effective action Γ_R satisfy the quantum master equations and the Ward identities, respectively.

4 General gauge theories in curved space within Sp(2) formalism

Let us consider a theory of gauge fields A^i in an external gravitational field $g_{\mu\nu}$. The classical theory is described by the action which depends on both dynamical fields and external metric,

$$S_0 = S_0(A, g)\,. \tag{54}$$

Here and below we use the condensed notation $g \equiv g_{\mu\nu}$ for the metric, when it is an argument of some functional or function. The action (54) is assumed to be gauge invariant,

$$S_{0,i}R^i_a = 0, \quad \delta A^i = R^i_a(A, g)\lambda^a\,, \quad \lambda^a = \lambda^a(x) \quad (a = 1, 2, ..., n)\,, \tag{55}$$

as well as covariant,

$$\delta_g S_0 \;=\; \frac{\delta S_0}{\delta A^i}\delta_g A^i + \frac{\delta S_0}{\delta g_{\mu\nu}}\delta_g g_{\mu\nu} = 0\,, \tag{56}$$

where λ^a are independent parameters of the gauge transformation, corresponding to the symmetry group of the theory. The diffeomorphism transformation of the metric in Eq. (56) has the form

$$\begin{aligned} \delta_g g_{\mu\nu} &= -g_{\mu\alpha}\partial_\nu\xi^\alpha - g_{\nu\alpha}\partial_\mu\xi^\alpha - \partial_\alpha g_{\mu\nu}\xi^\alpha \\ &= -g_{\mu\alpha}\nabla_\nu\xi^\alpha - g_{\nu\alpha}\nabla_\mu\xi^\alpha \;=\; -\nabla_\mu\xi_\nu - \nabla_\nu\xi_\mu\,. \end{aligned} \tag{57}$$

Here ξ^α are the parameters of the coordinates transformation,

$$\xi^\alpha = \xi^\alpha(x) \quad (\alpha = 1, 2, ..., d)\,. \tag{58}$$

The generating functional $Z(J,\Phi^*,\bar{\Phi},g)$ of the Green functions can be constructed in the form of the functional integral

$$Z(J,\Phi^*,\bar{\Phi},g) = \int d\Phi \exp\Big\{\frac{i}{\hbar}\Big[S_{ext}(\Phi,\Phi^*,\bar{\Phi},g) + J_A\Phi^A\Big]\Big\}. \tag{59}$$

Here Φ^A represents the full set of fields of the complete configuration space of the theory under consideration and $\Phi^*_{Aa}, \bar{\Phi}_A$ are antifields. Finally, $S_{ext}(\Phi,\Phi^*,\bar{\Phi}_A,g)$ is the quantum action constructed with the help of the solution $S = S(\Phi,\Phi^*,\bar{\Phi}_A,g)$ to the master equations

$$\frac{1}{2}(S,S)^a + V^a S = i\hbar\Delta^a S\,, \qquad S(\Phi,\Phi^*,\bar{\Phi},g)|_{\Phi^*=\bar{\Phi}=\hbar=0} = S_0(A,g) \tag{60}$$

in the form given in Eqs. (16), (17). Note that S_{ext} satisfies the master equations

$$\frac{1}{2}(S_{ext},S_{ext})^a + V^a S_{ext} = i\hbar\Delta^a S_{ext}. \tag{61}$$

From gauge invariance of initial action (55) in usual manner one can derive the BRST symmetry and the Ward identities for generating functionals $Z = Z(\Phi,\Phi^*,\bar{\Phi},g), W = W(\Phi,\Phi^*,\bar{\Phi},g)$ and $,\Gamma = \Gamma(\Phi,\Phi^*,\bar{\Phi},g)$ in the form (28), (30) and (31) respectively.

In what follows we assume the general covariance of $S = S(\Phi,\Phi^*,\bar{\Phi},g)$,

$$\delta_g S(\Phi,\Phi^*,\bar{\Phi},g) = \frac{\delta S}{\delta\Phi^A}\delta_g\Phi^A + \delta_g\Phi^*_{Aa}\frac{\delta S}{\delta\Phi^*_{Aa}} + \frac{\delta S}{\delta g_{\mu\nu}}\delta_g g_{\mu\nu} = 0. \tag{62}$$

Let us choose the gauge fixing functional $F = F(\Phi,g)$ in a covariant form

$$\delta_g F = 0\,, \tag{63}$$

then the quantum action $S_{ext} = S_{ext}(\Phi,\Phi^*,\bar{\Phi},g)$ obeys the general covariance too

$$\delta_g S_{ext} = 0\,. \tag{64}$$

From the Eq. (64) and the assumption that the term with the sources J_A in (59) is covariant

$$\delta_g(J_A\Phi^A) = (\delta_g J_A)\Phi^A + J_A(\delta_g\Phi^A) = 0\,, \tag{65}$$

it follows the general covariance of $Z = Z(J,\Phi^*,\bar{\Phi},g)$. Indeed,

$$\begin{aligned}\delta_g Z(J,\Phi^*,\bar{\Phi},g) &= \frac{i}{\hbar}\int d\Phi\Big[\delta_g\Phi^*_{Aa}\frac{\delta S_{ext}(\Phi,\Phi^*,\bar{\Phi},g)}{\delta\Phi^*_{Aa}} + \frac{\delta S_{ext}(\Phi,\Phi^*,\bar{\Phi},g)}{\delta g_{\mu\nu}}\delta_g g_{\mu\nu} + \\ &+\delta_g\bar{\Phi}_A\frac{\delta S_{ext}(\Phi,\Phi^*,\bar{\Phi},g)}{\delta\bar{\Phi}_A} + (\delta_g J_A)\Phi^A\Big]\exp\Big\{\frac{i}{\hbar}\Big[S_{ext}(\Phi,\Phi^*,\bar{\Phi},g) + J_A\Phi^A\Big]\Big\}.\end{aligned} \tag{66}$$

Making change of integration variables in the functional integral, (66),

$$\Phi^A \quad\rightarrow\quad \Phi^A + \delta_g\Phi^A\,, \tag{67}$$

we arrive at the relation

$$\begin{aligned}\delta_g Z(J,\Phi^*,\bar{\Phi},g) &= \frac{i}{\hbar}\int d\Phi\Big[\frac{\delta S_{ext}}{\delta\Phi^A}\delta_g\Phi^A + \delta_g\Phi^*_A\frac{\delta S_{ext}}{\delta\Phi^*_A} + \delta_g\bar{\Phi}_A\frac{\delta S_{ext}}{\delta\bar{\Phi}_A} + \frac{\delta S_{ext}}{\delta g_{\mu\nu}}\delta_g g_{\mu\nu} + \\ &+(\delta_g J_A)\Phi^A + J_A(\delta_g\Phi^A)\Big]\exp\Big\{\frac{i}{\hbar}\Big[S_{ext}(\Phi,\Phi^*,\bar{\Phi},g) + J_A\Phi^A\Big]\Big\} \\ &= \frac{i}{\hbar}\int d\Phi\Big[\delta_g S_{ext} + \delta_g(J_A\Phi^A)\Big]\exp\Big\{\frac{i}{\hbar}\Big[S_{ext}(\Phi,\Phi^*,\bar{\Phi},g) + J_A\Phi^A\Big]\Big\} \;=\; 0\,.\end{aligned} \tag{68}$$

From (68) it follows that the generating functional of connected Green functions $W(J,\Phi^*,\bar{\Phi},g))$

$$W(J,\Phi^*,\bar{\Phi},g)=\frac{i}{\hbar}\ln Z(J,\Phi^*,\bar{\Phi},g) \tag{69}$$

obeys the pr$\frac{1}{2}$operty of the general covariance as well

$$\delta_g W(J,\Phi^*,\bar{\Phi},g)=0\,. \tag{70}$$

Consider now the generating functional of vertex functions $\Gamma=\Gamma(\Phi,\Phi^*,\bar{\Phi},g)$

$$\Gamma(\Phi,\Phi^*,\bar{\Phi},g)=W(J,\Phi^*,\bar{\Phi},g)-J_A\Phi^A\,, \tag{71}$$

where

$$\Phi^A=\frac{\delta W(J,\Phi^*,\bar{\Phi},g)}{\delta J_A},\quad J_A=-\frac{\delta\Gamma(\Phi,\Phi^*,\bar{\Phi},g)}{\delta\Phi^A}. \tag{72}$$

From definition of Φ^A (72) and the general covariance of $W(J,\Phi^*,\bar{\Phi},g)$ we can conclude the general covariance of $J_A\Phi^A$. Therefore,

$$\delta_g\Gamma(\Phi,\Phi^*,\bar{\Phi},g)=\delta_g W(J,\Phi^*,\bar{\Phi},g)=0. \tag{73}$$

5 Covariant renormalization in curved space-time

Up to now we consider non-renormalized generating functionals of Green functions. We are going to prove the general covariance for renormalized generating functionals. For this end, let us first consider the one-loop approximation for $\Gamma=\Gamma(\Phi,\Phi^*,\bar{\Phi},g)$,

$$\Gamma=S+\hbar\big[\Gamma^{(1)}_{div}+\Gamma^{(1)}_{fin}\big]+O(\hbar^2)\,, \tag{74}$$

where $\bar{\Gamma}^{(1)}_{div}$ and $\bar{\Gamma}^{(1)}_{fin}$ denote the divergent and finite parts of the one-loop approximation for Γ. The divergent local term $\Gamma^{(1)}_{div}$ gives the first counterpart in one-loop renormalized action S_{1R}

$$S\to S_{1R}=S-\hbar\Gamma^{(1)}_{div}. \tag{75}$$

From (64) and (73) it follows that in one-loop approximation we have

$$\delta_g\big[\Gamma^{(1)}_{div}+\Gamma^{(1)}_{fin}\big]=0 \tag{76}$$

and therefore $\Gamma^{(1)}_{div}$ and $\Gamma^{(1)}_{fin}$ obey the general covariance independently

$$\delta_g\Gamma^{(1)}_{div}=0\,,\qquad \delta_g\Gamma^{(1)}_{fin}=0\,. \tag{77}$$

In its turn the one-loop renormalized action S_{1R} is covariant

$$\delta_g S_{1R}=0\,. \tag{78}$$

Constructing the generating functional of one-loop renormalized Green functions $Z_1(J,\Phi^*,\bar{\Phi},g)$, with the action $S_{1R}=S_{1R}(\Phi,\Phi^*,\bar{\Phi},g)$, and repeating arguments given above, we arrive at the relation

$$\delta_g Z_1=0\,,\qquad \delta_g W_1=0\,,\qquad \delta_g\Gamma_1=0\,. \tag{79}$$

The generating functional of vertex functions $\Gamma_1 = \Gamma_1(\Phi, \Phi^*, \bar{\Phi}, g)$ which is finite in one-loop approximation

$$\Gamma_1 = S + \hbar\Gamma^{(1)}_{fin} + \hbar^2\big[\Gamma^{(2)}_{1,div} + \Gamma^{(2)}_{1,fin}\big] + O(\hbar^3)\,, \tag{80}$$

contains the divergent part $\Gamma^{(2)}_{1,div}$ and defines renormalization of the action S in the two-loop approximation

$$S \to S_{2R} = S_{1R} - \hbar^2\Gamma^{(2)}_{1,div}\,. \tag{81}$$

Starting from (77), (78) and (79) we derive

$$\delta_g\Gamma^{(2)}_{1,div} \;=\; 0\,, \qquad \delta_g\Gamma^{(2)}_{1,fin} = 0\,, \tag{82}$$

that means general covariance of the divergent and finite parts of Γ_1 in two-loop approximation. Therefore the two-loop renormalized action $S_{2R} = S_{2R}(\Phi, \Phi^*, \bar{\Phi}, g)$ is covariant

$$\delta_g S_{2R} = 0. \tag{83}$$

Applying the induction method we can repeat the procedure to an arbitrary order of the loop expansion. In this way we prove that the full renormalized action, $S_R = S_R(\Phi, \Phi^*, \bar{\Phi}, g)$,

$$S_R \;=\; S - \sum_{n=1}^{\infty}\hbar^n\Gamma^{(n)}_{n-1,div}\,, \tag{84}$$

which is local in each finite order in $\hbar$, obeys the general covariance

$$\delta_g S_R \;=\; 0\,; \tag{85}$$

and the renormalized generating functional of vertex functions, $\Gamma_R = \Gamma_R(\Phi, \Phi^*, \bar{\Phi}, g))$,

$$\Gamma_R \;=\; S + \sum_{n=1}^{\infty}\hbar^n\Gamma^{(n)}_{n-1,fin}\,, \tag{86}$$

which is finite in each finite order in $\hbar$, is covariant

$$\delta_g\Gamma_R \;=\; 0\,. \tag{87}$$

Therefore, taking into account results of Section 5 we can state that in presence of an external gravitational field the gauge invariant renormalizability can be arrived with preserving general covariance of functional Γ (87).

6 Conclusions

We have considered the general scheme of gauge-invariant and covariant renormalization of the quantum gauge theories of matter fields in flat and curved space-time. Using the Sp(2) formalism we have proved that in the theory which admits gauge invariant and diffeomorphism invariant regularization, these two symmetries hold in the counterterms to all orders of the loops expansion together with extended BRST symmetry. To arrive at these results we have shown the gauge invariant renormalizability of general gauge theories in the Sp(2) formalism without assuming the use of regularization for which acting by Δ^a on a local functionals gives zero. This result has required to find a new way to prove the gauge invariant renormalizability, which is different from the previous considerations. The key point of the new method is that first one has to perform the proof of the existence of solution of the master equation for the classical action. After that one can prove the gauge invariant renormalizability of the effective action which is given as a series of the the loop expansions. If one uses a regularization scheme when $\delta(0) = 0$ then from the begining we have a sulution $S_{(0)}$ to the classical master equations (36) (see [40]) and the proof of Sp(2) gauge invariant renormalizability is given in the way proposed in Section 3 when $S_{(n)} = 0, n = 1, 2, ...$

Acknowledgments

This paper was done during the visit of the author to Juiz de Fora, supported by the FAPEMIG grant, project No. CEX-BPV-00045-10. The author is grateful to I.L. Shapiro and I.V. Tyutin for stimulating discussions, to the Department of Physics of the Federal University of Juiz de Fora for warm hospitality and to FAPEMIG for the grant. The work was also supported by the grant for LRSS, project No. 3558.2010.2, the RFBR-Ukraine grant, project No. 10-02-90446, the DFG grant, project No. 436 RUS 113/669/0-4 and the RFBR grant, project No. 09-02-00078.

References

[1] N.N. Bogoliubov and D.V. Shirkov, *The Theory of Quantized Fields* (New York, Interscience, 1959).

[2] J.C. Collins, *Renormalization* (Cambridge University Press, Cambridge, UK, 1984).

[3] G. 't Hooft and M.J.G. Veltman, *Regularization and renormalization of gauge fields* Nucl. Phys. B44 (1972) 189.

[4] G. 't Hooft and M.J.G. Veltman, *Combinatorics in gauge fields.* Nucl. Phys. B50 (1972) 318.

[5] L.D. Faddeev and V.N. Popov, *Feynman diagrams for the Yang-Mills field*, Phys. Lett. B25B (1967) 29.

[6] A.A. Slavnov, *Ward identities in gauge theories*, Theor. Math. Phys. 10 (1972) 99; J.C. Taylor, *Ward identities and charge renormalization of the Yang-Mills field*, Nucl. Phys. B33 (1971) 436.

[7] C. Becchi, A. Rouet and R. Stora, *Renormalization of the abelian Higgs-Kibble model*, Commun. Math. Phys. 42 (1975) 127; I.V. Tyutin, *Gauge invariance in field theory and statistical physics in operator formalism*, Lebedev Inst. preprint N 39 (1975).

[8] J. Zinn-Justin, *Renormalization of gauge theories*, Lect, Notes in Phys. 37 (1975).

[9] H. Kluberg-Stern and J.B. Zuber, *Renormalization of non-Abelian gauge theories in a background-field gauge. I. Green's functions*, Phys. Rev. D12 (1975) 482.

[10] R. Kallosh, *Renormalization in non-Abelian gauge theories*, Nucl. Phys. B78 (1974) 293.

[11] D.Z. Freedman, P. van Nieuwenhuizen and S. Ferrara, *Progress toward a theory of supergravity*, Phys. Rev. D13 (1976) 3214.

[12] S. Deser and B. Zumino, *Consistent supergravity*, Phys. Lett. B62 (1976) 335.

[13] D.Z. Freedman and P. van Nieuwenhuizen, *Properties of supergravity theory*, Phys. Rev. D14 (1976) 912.

[14] P.K. Townsend, *Covariant quantization of antisymmetric gauge fields*, Phys. Lett. **88B** (1979) 97.

[15] W. Siegel, *Hidden ghosts*, Phys. Lett. **93B** (1980) 170.

[16] H. Hata, T. Kugo and N. Ohta, *Skew symmetric tensor gauge field theory dynamically realized in QCD U(1) channel*, Nucl. Phys. **B178** (1981) 527.

[17] B. de Wit and J.W. van Holten, *Covariant quantization of gauge theories with open algebra*, Phys. Lett. **79B** (1978) 389.

[18] D.Z. Freedman and P.K. Townsend, *Antisymmetric tensor gauge theories and non-linear σ-models*, Nucl. Phys. **B177** (1981) 282.

[19] N.K. Nielsen, *Ghost counting in supergravity*, Nucl. Phys. **B140** (1978) 494.

[20] R.E. Kallosh, *Modified rules in supergravity*, Nucl. Phys. **B141** (1978) 141.

[21] N.K. Nielsen, *BRS invariance of supergravity in a gauge involving an extra ghost*, Phys. Lett. **103B** (1981) 197.

[22] S.P. de Alwis, M.T. Grisaru and L. Mezincescu, *Quantization and unitarity in antisymmetric tensor gauge theories*, Nucl. Phys. **B303** (1988) 57.

[23] S.A. Frolov and A.A. Slavnov, *Quantization of non-abelian antisymmetric tensor field*, Theor. Math. Phys. **75** (1988) 470.

[24] P.M. Lavrov and I.V. Tyutin, *Lagrange quantization of gauge theories and unitarity of the physical S-matrix*, Sov. J. Nucl. Phys. **50** (1989) 912.

[25] I.A. Batalin and G.A. Vilkovisky, *Gauge algebra and quantization*, Phys. Lett. 102B (1981) 27.

[26] I.A. Batalin and G.A. Vilkovisky, *Quantization of gauge theories with linearly dependent generators*, Phys. Rev. D28 (1983) 2567.

[27] F.J. Dyson, *The S matrix in quantum electrodynamics*, Phys. Rev. 75 (1949) 1736.

[28] P.T. Matthews and A. Salam, *Renormalization*, Phys. Rev. 94 (1954) 185.

[29] B.L. Voronov, P.M. Lavrov and I.V. Tyutin, *Canonical transformations and the gauge dependence in general gauge theories*, Sov. J. Nucl. Phys. 36 (1982) 292.

[30] J. Gomis and S. Weinberg, *Are nonrenormalizable gauge theories renormalizable?*, Nucl. Phys. B469 (1996) 473.

[31] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems*, (Princeton University Press, Princeton, New Jersey, 1992).

[32] J. Gomis, J. Paris, S. Samuel, *Antibracket, Antifields and Gauge-Theory Quantization* Phys. Rept. 259 (1995) 1.

[33] R. Utiyama and B.S. DeWitt, *Renormalization of a classical gravitational field interacting with quantized matter fields*, J. Math. Phys. 3 (1962) 608.

[34] I.L. Buchbinder, *Renormalization Group Equations In Curved Space-Time.* Theor. Math. Phys. 61 (1984) 1215.

[35] D.J. Toms, *Renormalization of Interacting Scalar Field Theories in Curved Space-Time*, Phys. Rev. D26 (1982) 2713; *The Effective Action and the Renormalization Group Equation in Curved Space-Time* Phys. Lett. B126 (1983) 37; *The Background - Field Method and the Renormalization of Nonabelian Gauge Theories in Curved Space-Time*, Phys. Rev. D27 (1983) 1803..

[36] P. Panangaden, *One Loop Renormalization of Quantum Electrodynamics in Curved Space-Time*, Phys. Rev. D23 (1981) 1735; B.L. Nelson, P. Panangaden, *Scaling Behavior of Interacting Quantum Fields in Curved Space-Time* Phys. Rev. D25 (1982) 1019.

[37] I.L. Buchbinder, S.D. Odintsov and I.L. Shapiro, *Effective Action in Quantum Gravity* (IOP Publishing, Bristol, 1992).

[38] I.L. Shapiro, *Effective Action of Vacuum: Semiclassical Approach*, Class. Quant. Grav. 25 (2008) 103001.

[39] P.M. Lavrov and I.L. Shapiro, *On the renormalization of gauge theories in curved space-time*, Phys. Rev. D81 (2010) 044026.

[40] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, *Covariant quantization of gauge theories in the framework of extended BRST symmetry*, J. Math. Phys. **31** (1990) 1487.

[41] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, *An* $Sp(2)$*-covariant quantization of gauge theories with linearly dependent generators*, J. Math. Phys. **32** (1991) 532.

[42] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, *Remarks on the* $Sp(2)$*-covariant quantization of gauge theories* J. Math. Phys. **32** (1991) 2513.

[43] C.M. Hull, *The BRST and anti-BRST Invariant Quantization of General Gauge Theories*, Mod.Phys.Lett. A5 (1990) 1871.

[44] B.S. DeWitt, *Dynamical Theory of Groups and Fields*, Gordon and Breach, New York, 1965.