

# ON MANY BODY SYSTEM INTERACTIONS

NILS A. BAAS

ABSTRACT. We discuss possible relationships between geometric and topological interactions on one side and physical interactions on the other side. This paper is a follow up of [2].

We will here discuss possible interactions of families of particles. By a particle we mean a system or an (extended) object in some space. The families may be finite, countable or uncountably infinite.

$$\mathcal{P} = \{P_i\}_{i \in I}$$

What does it mean that the particles interact?

Often we have “state”-spaces associated to the particles

$$P_i \mapsto S_i.$$

An interaction is a rule telling us how the states influence each other or are related:

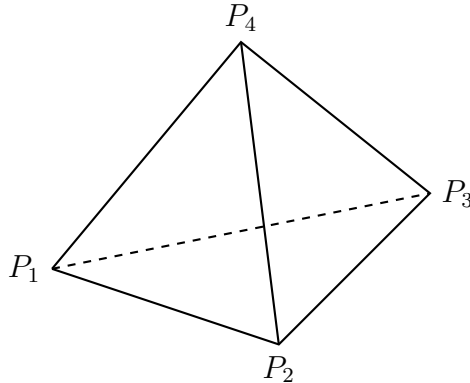
$$R \subset \prod S_i$$

If this is stable and time independent we may call it a *bond*.

## Simplicial model:

A *pair* interaction is geometrically represented as: 

An *n*-tuple interaction may be represented as an *n*-simplex, but it is reducible to pair interactions:



---

*Date:* November 1, 2010.

Here we represent the systems geometrically by *points* (in some Euclidean space):

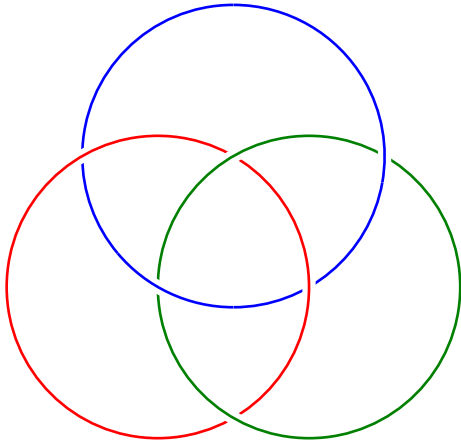
Particle (system)  $\mapsto$  point in f. ex.  $\mathbb{R}^3$

But we may have more sophisticated interactions like:

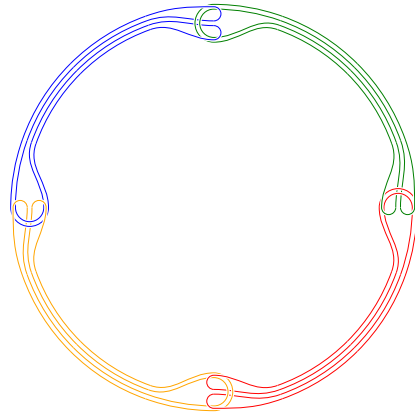
**Brunnian or Borromean model:**

$n$  particles interact in such a way that no subset of them interact. This suggest and is best understood by another representation:

Particle  $\mapsto$  ring (string) in  $\mathbb{R}^3$

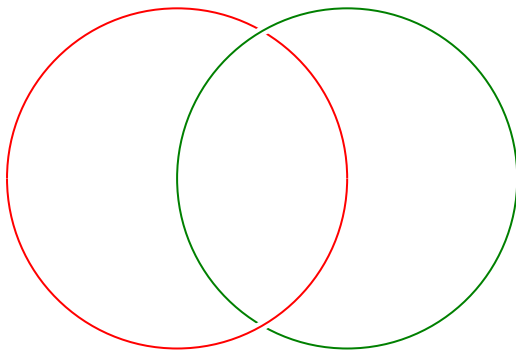


(a) Borromean

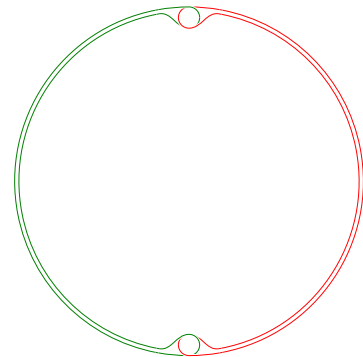


(b) Brunnian interactions of length  $k = 4$

In this representation a pair interaction is represented by Hopf links:



or



With this representation we have in [2] introduced a whole new hierarchy of (possible) higher order interactions represented by new higher order links of rings. This has been extensively developed in [2].

The interesting question is then:

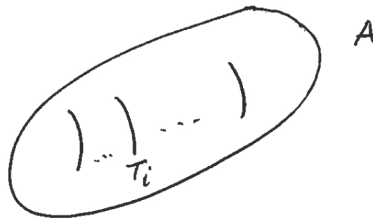
*What about other geometric (topological) representations, and what kind of new interactions do they suggest — both of first and higher order?*

We may proceed as follows.

Particle  $\mapsto$  Space (of some kind and f. ex. embedded in a fixed ambient space  $A$ )

$$P_i \longrightarrow T_i = \text{Space}_i \subseteq A$$

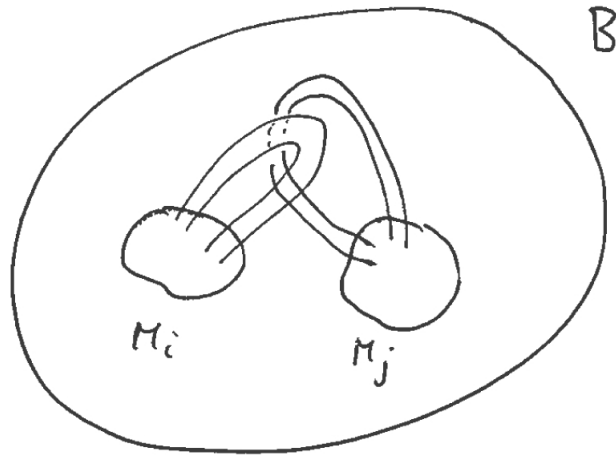
Pictorially this looks like



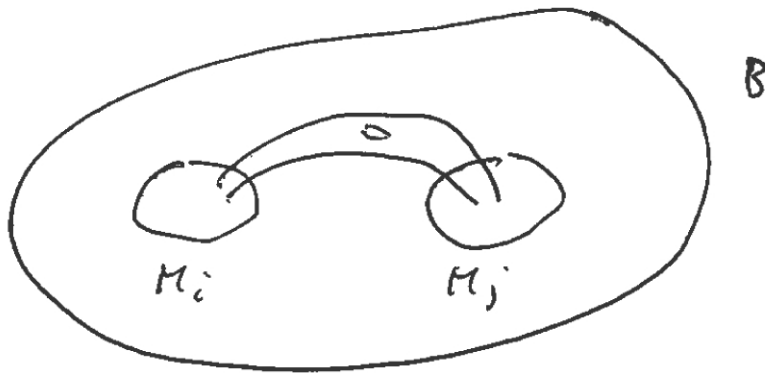
In the sense of [1]  $A$  is a bond of  $\{T_i\}$ . This can be iterated and one may form higher order bonds, ending up with a *hyperstructure*, defined and described in [1].

Let us be more specific and consider the situation where the representing spaces are manifolds embedded in a large ambient manifold or Euclidean space.

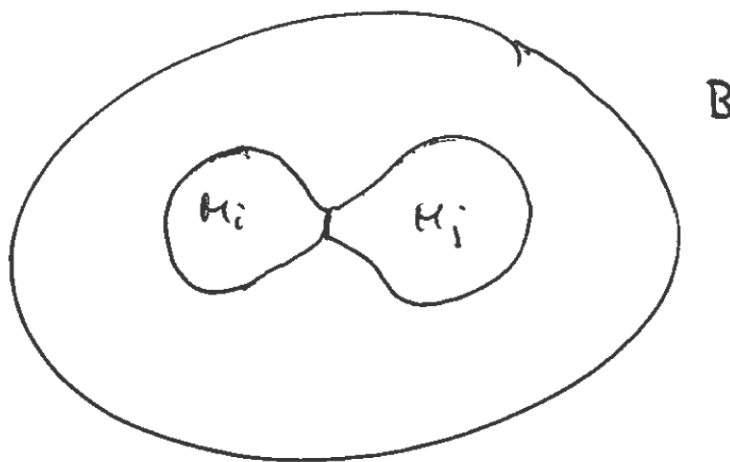
$$P_i \longrightarrow M_i \subseteq B(\subseteq \mathbb{R}^N)$$



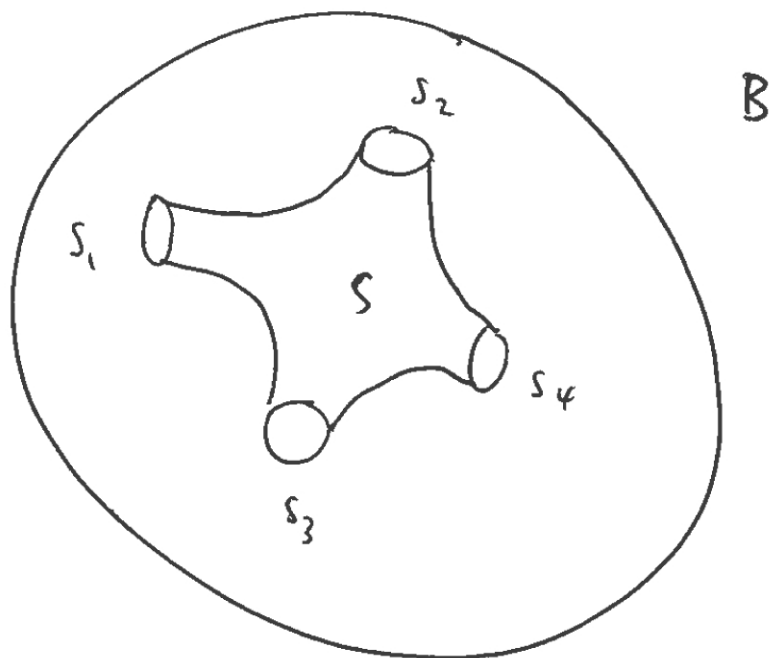
linked



Connected (by intermediate manifolds)



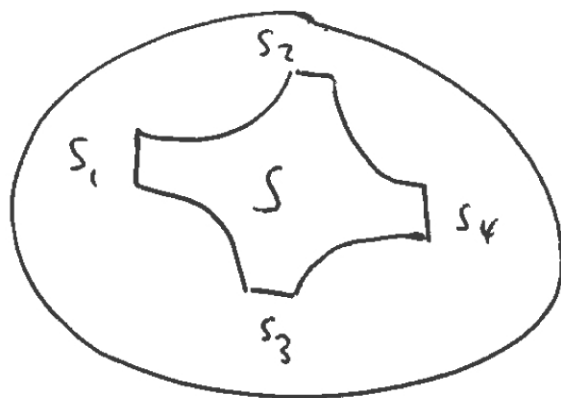
Glued



$$P_i \rightarrow S_i$$

$$\sqcup S_i = \partial S \subset B \quad \text{or} \quad S = B.$$

Cobordant,



$$\sqcup S_i \subset \partial S \subset B$$

Weakly cobordant

**Example.**  $B = \mathbb{R}^3$

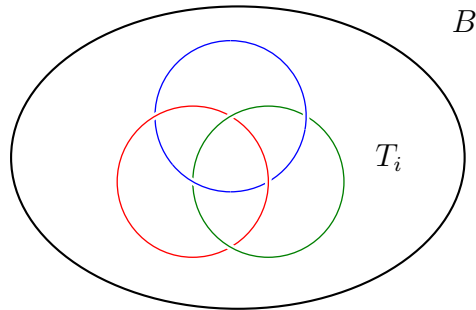


Figure 1: Borromean interactions

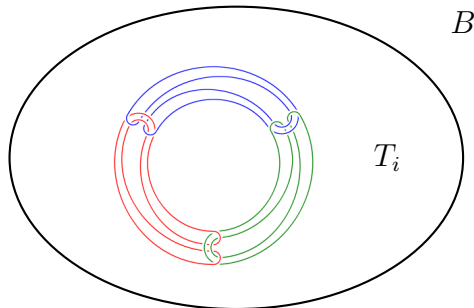


Figure 2: Brunnain interactions

See [2] for a whole hierarchy of extensions.

For higher dimensional manifolds there is a variety of ways to do this (spaces of embeddings).

One may manipulate or tune externally a system in such a way that it is represented by a desired (linked, ...) embedding of the  $M_i$ 's in  $B$ .

Hence it justifies calling  $B$  a bond, see [1].

Then one may — as already mentioned — iterate this process to higher order bonds which we have defined as hyperstructures. This gives higher order, hyperstructured interaction patterns of the particles.

*This means that for many body systems there is a whole new universe of new represented states.*

*The pertinent question is then: Which of these new types of states are realizable in real world systems (physical, chemical, biological, social, ...)?*

This discussion also shows that bonds of subspaces (like manifolds) and their associated hyperstructures may be a good laboratory for suggesting new states, designing and studying general many body systems and their interactions. But the geometric interactions in the geometric universe should then be interpreted back into interactions

in the real universe where the particles live. In cold gases Borromean or Brunnian states of first order correspond to Efimov states, see [2] for details.

In [2] we have studied and suggested connections between physical states and higher order links in the geometric universe of links.

#### REFERENCES

- [1] N.A. Baas, Hyperstructures as Abstract Matter, Adv. in Complex Systems
- [2] N.A. Baas, New States of Matter, Preprint, NTNU, 2010

DEPARTMENT OF MATHEMATICAL SCIENCES, NTNU, NO-7491 TRONDHEIM, NORWAY  
*E-mail address:* `baas@math.ntnu.no`