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Asynchronous signal processing for brain-computer interfaces

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Abstract

Brain-computer interfaces (BCIs) provide a way to monitor and treat neurological diseases. An important application of BCIs is the monitoring and treatment of epilepsy, a neurological disorder characterized by recurrent unprovoked seizures, symptomatic of abnormal, excessive or synchronous neuronal activity in the brain. BCIs contain an array of sensors that gather and transmit data under the constrains of low-power and minimal data transmission. Asynchronous sigma delta modulators (ASDMs) are considered an alternative to synchronous analog to digital conversion. ASDMs are non-linear feedback systems that enable time-encoding of analog signals, from which the signal can be reconstructed. An efficient reconstruction of time-encoded signals can be achieved using a prolate spheroidal waveform (PSW) projection. PSWs have finite time support and maximum energy concentration within a given bandwidth. The ASDM time-encoding is related to the duty-cycle modulation and demodulation, which shows that sampling is non-uniform. For transmission of data from BCI, we propose a modified orthogonal frequency division multiplexing (OFDM) technique using chirp modulation. Our method generalizes the chirp modulation of binary streams with non-uniform symbol duration. Our theoretical results relate to recent continuous-time digital signal processing and compressive sampling theories.

Key Words: *Asynchronous sigma delta modulators (ASDM), brain computer interface (BCI), wireless OFDM communications, prolate spheroidal waveform (PSW), chirp modulation.*

1. Introduction

Acquisition and transmission of data from the brain, for monitoring or treatment, can be done using an array of sensors supported by analog circuitry. Two issues of special interest in the design and implementation of these brain-computer interfaces (BCI) [1] are energy management and use of clocks. The power dissipation due to analog to digital conversion and to wireless transmission is significant. In radio-frequency applications, it has been shown that the energy used to transmit one bit wirelessly is equivalent to that for executing a thousand 32-bit computations [2]. Furthermore, the presence of clocks in BCIs is problematic. In conventional sigma delta modulators, for instance, the required high frequency clocks may possibly affect the brain and cause electromagnetic interference corrupting the analog signal to be sampled [3].

Given the lack of clocks and the low power consumption required in bio-monitoring systems, asynchronous data acquisition is a viable alternative to analog to digital conversion [1, 3]. Processing signals without analog to digital converters, multiplexing and transmitting data from several channels under restrictive power conditions becomes a challenging problem. Furthermore, the intended small dimensions of the BCIs impose additional storage and computational constrains. The prototype that we propose is shown in Figure 1, where neural signals are acquired in the BCI and then transmitted to a personal digital assistant device (PDA) which is capable of processing and transferring data to a server or group of servers. We are interested in the acquisition and transmission of data in the BCI to the PDA under the mentioned constraints.

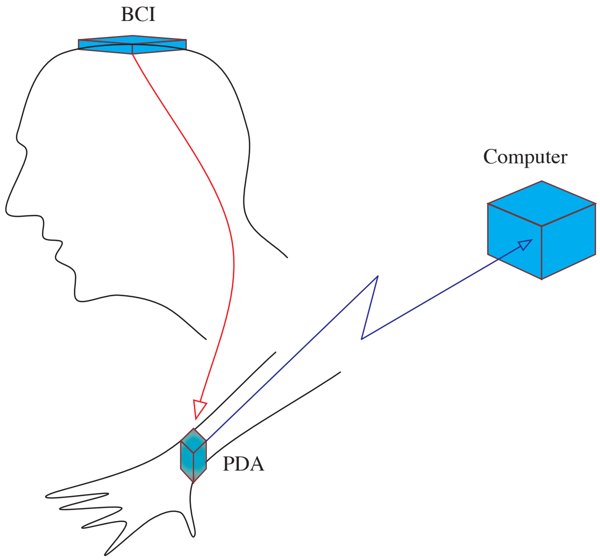

Figure 1. Brain-computer interface.

To satisfy the signal processing requirements of the BCI, we consider an asynchronous approach just like the continuous-time digital signal processing recently proposed [4]. Asynchronous sigma delta modulators (ASDMs) [5] are non-linear feedback systems, without a clock, that transform amplitude information into time information to represent analog signals in a binary form. Their simple circuitry allows them to operate at low power levels. Theoretically the ASDMs are related to the duty-cycle modulation. A band-limited signal can be reconstructed from the zero crossings of the ASDM binary signal [3]. In this paper, we present a reconstruction of the signal by means of the prolate spheroidal waveform (PSW) projection presented in [6]. This projection is based in the approximation of the sinc function in terms of the PSWs giving a lower order representation than the complex exponential-bases used in [3, 7].

As shown in Figure 2, the neural signals from different sensors are processed by ASDMs, multiplexed and transmitted via the skin to a PDA. The power consumption in the transmission can be reduced by using the skin as a short-range communication channel [8]. However, the non-uniformity of the zero-crossings of the time-encoded signals makes otherwise very efficient methods such as Orthogonal Frequency Division Multiplexing (OFDM) not applicable. We propose a combination of chirp and localized modulation of the ASDM time-

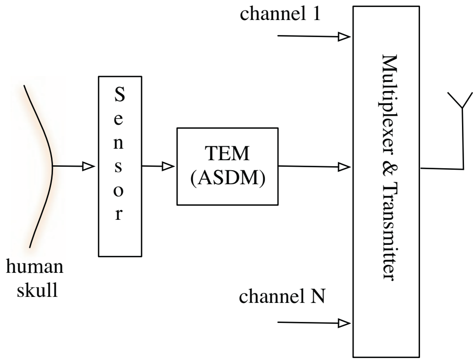

Figure 2. ASDM-based brain-computer interface.

encoded signals to achieve an efficient transmission with a modified OFDM system. OFDM is a multi-carrier communication technique that divides the bit stream into sub-streams that are more efficiently transmitted. Given that the communication channel is modeled as a linear time-varying system, chirp modulation and time-frequency processing of the signals in such a system is more appropriate than the conventional linear time-invariant modeling and Fourier domain processing [9].

A sequence of ortho-normal chirps can be used to transmit multichannel data in an efficient way and with robustness to additive noise. In [9, 10] it is shown that the transmission of a sequence of binary symbols $\{b_u(t)\}$, $u = 1, \cdots, U$, with uniform duration of T seconds and corresponding to U users, can be efficiently done by modulating each of the binary signals with a set of ortho-normal chirps. The orthonormality of these chirps can be obtained using the kernel of the fractional Fourier transform (FrFT) [11]. If the symbol duration is not constant, the ortho-normality of the chirps is not sufficient to recover the transmitted signal from a multiplexed version of it. As we will show, it is necessary to create a localized set of chirps capable of representing each of the non-uniform pulses.

2. Asynchronous data acquisition

In this section, we will show how the data collection in the BCI can be accomplished without a clock using ASDMs, and how the data can be used to reconstruct the neural signal.

2.1. Asynchronous sigma delta modulators

An ASDM is a nonlinear feedback system that operates at low power. It can be used to time encode a band-limited analog signal into a continuous-time signal with binary amplitude. The zero-crossing times of this signal permit recovery of the original signal. An ASDM is similar to a synchronous sigma-delta modulator but it differs in that no sampling is done in the ASDM and as such no quantization noise is input into the modulator. Recently, the ASDM shown in Figure 3, consisting of an integrator and a non-inverting Schmitt trigger, has been proposed for bio-monitoring [3]. This type of ASDM transforms amplitude information into time information by the limit cycles of the non-linear component.

The operation of the ASDM in Figure 3 can be related to the non-uniform sampling of a band-limited signal $x(t)$. To reconstruct $x(t)$ from non-uniform samples requires knowledge not only of the samples of the

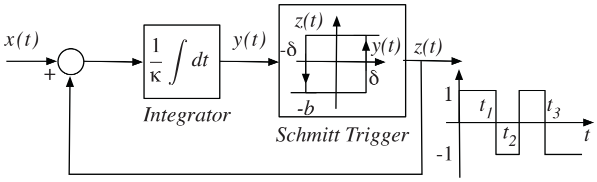

Figure 3. Example of ASDM.

signal but also of the times at which they occur. Although reconstruction from non-uniform samples can be posed as a generalization of the sinc interpolation of the Nyquist-Shannon sampling theorem, the problem is not well defined due to the infinite dimension of the matrices and vectors involved, and to the ill-conditioning of the matrix with sinc entries [6, 7].

Perfect reconstruction of $x(t)$ from non-uniform samples can be achieved provided that the zero-crossing time sequence $\{t_k\}$ satisfies the condition [3]:

$$\max_k (t_{k+1} - t_k) \leq T_N, \tag{1}$$

where $T_N = \pi/\Omega_{\max}$ is the Nyquist sampling period.

2.2. Time-encoding via ASDM

The zero-crossing time sequence can be obtained as follows from the feedback system shown in Figure 3. For a bounded signal $x(t)$

$$|x(t)| \leq c < b \tag{2}$$

and a certain value of κ, the output of the integrator $y(t)$ is also bounded, i.e., $|y(t)| < \delta$ for all t, and the output of the feedback system is binary, $z(t) = b(-1)^{k+1}$, $t_k \leq t \leq t_{k+1}$. If at a time $t_{k+1} > t_k$ the difference of outputs of the integrator is $y(t_{k+1}) - y(t_k) = \pm 2\delta$ and $z(t_k) = b(-1)^{k+1}$, then we have

$$y(t_{k+1}) - y(t_k) = \quad \frac{1}{\kappa}\int_{t_k}^{t_{k+1}} x(\tau)d\tau - \frac{b}{\kappa}(-1)^{k+1}(t_{k+1} - t_k).$$

After replacing the left hand-side term by $\pm 2\delta$, it becomes

$$\int_{t_k}^{t_{k+1}} x(\tau)d\tau = (-1)^k \left[-b(t_{k+1} - t_k) + 2\kappa\delta\right]. \tag{3}$$

Furthermore, from the property $|x(t)| \leq c$ and condition (1), we have

$$\frac{2\kappa\delta}{b+c} \leq t_{k+1} - t_k \leq \frac{2\kappa\delta}{b-c} \leq T_N \tag{4}$$

which gives us the way to choose the parameters δ, and κ in terms of the Nyquist sampling rate.

2.3. Duty-cycle modulation and time-encoding

A sequence of binary rectangular pulses (Figure 4) is characterized by the duty-cycle defined for two consecutive pulses of duration $T_k = \alpha_k + \beta_k$, where α_k in this case is the duration of the pulse of amplitude 1 and β_k the duration of the other pulse of amplitude -1. For $x(t)$, $t_k \le t \le t_{k+2}$, the duty-cycle is defined as

$$0 < \frac{\alpha_k}{T_k} = \frac{1 + x(t)}{2} < 1 \qquad t_k \le t \le t_{k+2}.$$

Thus, if the signal is zero for all times, $x(t) = 0,\ -\infty < t < \infty$, it is modulated into a train of square pulses with $\alpha_k = \beta_k$ or $\alpha_k/T_k = 0.5$. If $x(t) = A,\ |A| < 1,\ t_k \le t \le t_{k+2}$, then the two pulses for that time are rectangular, where

$$\alpha_k = \frac{(1 + A)T_k}{2}$$

and $\beta_k = T_k - \alpha_k$. If the signal is not constant in a time segment, the duty-cycle is not clearly defined with respect to the amplitude in each segment. Suppose $\bar{x}_k$ is the local average of the signal in $t_k \le t \le t_{k+2}$. From

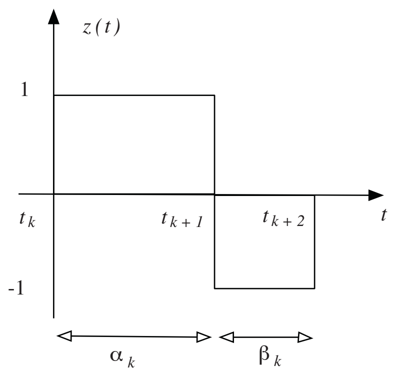

Figure 4. Duty-cycle modulation.

the above we can see that it can be obtained from the α_k and β_k obtained in the duty cycle modulation, i.e.,

$$\bar{x}_k = \frac{\alpha_k - \beta_k}{\alpha_k - \beta_k}$$

$$\alpha_k = t_{k+1} - t_k, \quad \beta_k = t_{k+2} - t_{k-1}. \tag{5}$$

Thus from the duty-cycle modulation, or the output $z(t)$ of the ASDM, giving the time-sequence $\{t_k\}$ we are only able to recover local averages in each segment. If the signal $x(t)$ is continuous in $t_k \le t \le t_{k+2}$, the local average coincides with one of the values $x(\zeta)$ for $t_k \le \zeta \le t_{k+2}$ and one could think then of a non-uniformly sampled signal for which we would like to interpolate the rest of the signal values in that segment. Equation (3) should provide the way to obtain that interpolation, as we will see in the next section.

3. Slepian reconstruction

According to the Whittaker–Kotel'nikov–Shannon-Nyquist sampling theory [12] a band-limited signal can be reconstructed from uniformly taken samples by a sinc interpolation. The problem with this is that not only the band-limited condition is idealized, but the use of sinc function of infinite support in time is not the appropriate functions to represent finite support signals. Thus if $x(t)$ is a finite-energy signal that is band-limited (i.e., with a maximum frequency $\Omega_{\max}$) according to the Shannon sampling theory, it can be reconstructed from uniform samples $\{x(kT_s)\}$, where $T_s \leq \pi/\Omega_{\max}$ is the Nyquist sampling period by the sinc interpolation

$$x(t) = \sum_{k=-\infty}^{\infty} x(kT_s)S(t - kT_s), \qquad S(t) = \frac{\sin(\Omega_{\max} t)}{\Omega_{\max} t}, \tag{6}$$

where $S(t)$ is the sinc function. In the non-uniform sampling, the reconstruction problem can be presented as a projection using $\{S(t-t_k)\}$ as a frame for $x(t)$ so that we need to find the coefficients $\mathbf{c}$ of the matrix equation obtained from (6):

$$\mathbf{Sc} = \mathbf{b}, \tag{7}$$

where $\mathbf{S}$ has $S(t_m - t_n)$ as (m, n) entries and $\mathbf{b}$ has as entries the samples $\{x(kT_s)\}$. Given the infinite dimensions of $\mathbf{S}$ and $\mathbf{b}$ it cannot be solved, and when we reduce the dimension the problems become ill-posed [7]. The problem is with the sinc functions which clearly not appropriate to represent finite support signals.

In [6] we have shown that the Prolate Spheroidal Wave Functions (PSWF), or Slepian functions, are more appropriate for sampling signals of finite time support and essentially band-limited, while reducing the number of samples for reconstruction. The Slepian functions $\{s_k(t)\}$ have finite time support, and their energy is optimally concentrated in a frequency band. Figure 5 display some of these functions and their Fourier transforms.

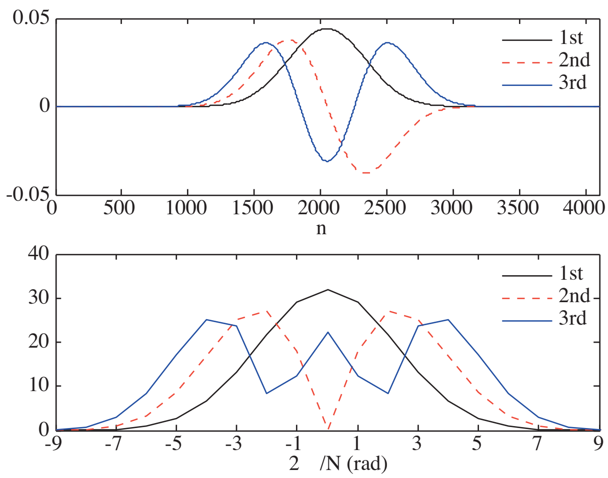

Figure 5. Slepian functions and their Fourier transforms.

Using the connection of the Slepian functions with the sinc function, the sinc interpolation can be converted into a finite Slepian projection of finite dimension L, in turn related to the time-frequency product of the signal. In general, $L < N_n$, where N_N is the number of samples required by the Nyquist criteria [6]. The projection of a signal $x(t)$ is given as

$$\mathbf{x(t_k)} = \mathbf{\Phi(t_k)}\gamma_{\mathbf{L}}, \tag{8}$$

where $\mathbf{\Phi}(\mathbf{t_k})$ is a matrix with entries Slepian functions computed at uniform times $\{t_k\}$, and $\gamma_{\mathbf{L}}$ are the projection coefficients. If the a non-uniform sampling, e.g., jitter sampling, occurs so that $\{\hat{t}_k\}$ is a subset of $\{t_k\}$, then the measurements are given as

$$\mathbf{x}(\hat{\mathbf{t}}_{\mathbf{k}}) = \mathbf{\Phi}(\hat{\mathbf{t}}_{\mathbf{k}})\gamma_{\mathbf{L}}, \tag{9}$$

where $\mathbf{\Phi}(\hat{\mathbf{t}}_{\mathbf{k}})$ is random because of the nature of the sampling. Due to this, we find the coefficients by means of the pseudoinverse

$$\gamma_{\mathbf{L}} = \left[\mathbf{\Phi}(\hat{\mathbf{t}}_{\mathbf{k}})\right]^{\dagger} \mathbf{x}(\hat{\mathbf{t}}_{\mathbf{k}}), \tag{10}$$

which are then used to reconstruct the signal. As an example, consider the reconstruction of a sub-dural EEG signal shown in Figure 6.

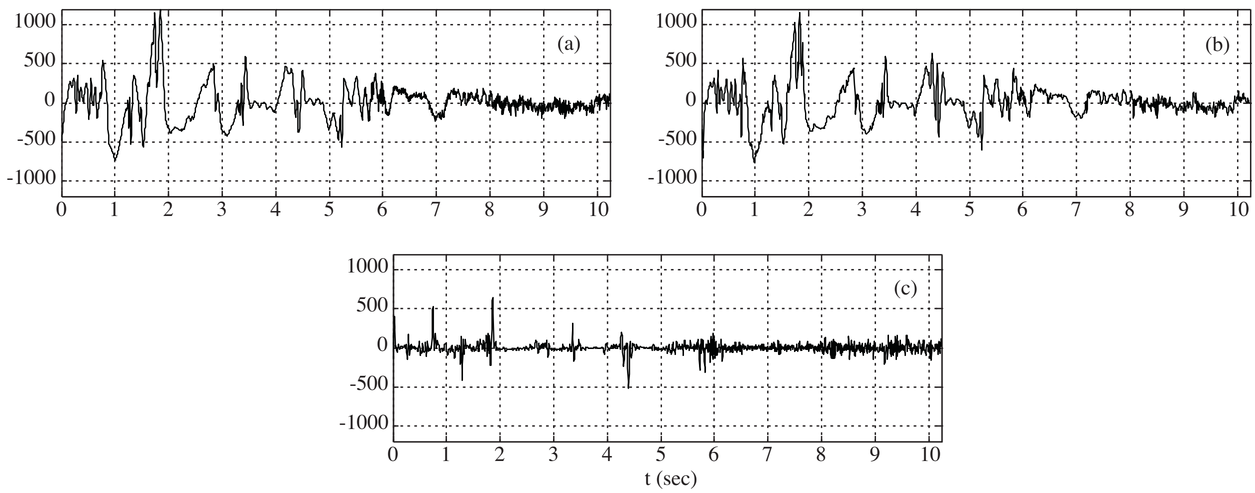

Figure 6. Reconstruction from non-uniform samples: original signal (top), reconstructed signal and error.

3.1. Reconstruction from ASDM output

The train of rectangular pulses $z(t)$ displays non-uniform zero-crossing times that depend on the input signal amplitude. The reconstruction of the neural signal $x(t)$ can be done by approximating the integral by the trapezoidal rule using $\Delta = (t_{k+1} - t_k)/D$ for an integer $D > 1$ (the larger this value the better the approximation), we have that

$$\int_{t_k}^{t_{k+1}} x(\tau)d\tau \approx \Delta \left[\frac{x(t_k)}{2} + \sum_{\ell=1}^{D-1} x(t_k + \ell\Delta) + \frac{x(t_{k+1})}{2} \right].$$

We then obtain the following reconstruction algorithm:

$$\begin{aligned} &(i) \qquad \mathbf{v} = \mathbf{Q}\mathbf{x} = \mathbf{Q}\mathbf{P}\gamma \\ &(ii) \qquad \gamma = [\mathbf{Q}\mathbf{P}]^{\dagger}\mathbf{v} \\ &(iii) \qquad \mathbf{x} = \mathbf{P}\gamma, \end{aligned}$$

where $\mathbf{v}$ is the right term in (3), $\mathbf{Q}$ is the matrix for the trapezoidal approximation, $\mathbf{x} = \mathbf{P}\gamma$ is the PSW projection, and † indicates pseudo-inverse. Thus the signal $x(t)$ can be reconstructed from the zero crossings $\{t_k\}$ of the output of the ASDM $z(t)$.

4. Chirp OFDM for ASDM signals

Consider then the transmission of binary signals $\{z_n(t)\}$, $n = 1, \cdots, N$ from an array of N ASDMs conforming a BCI. These signals need to be transmitted in the most efficient way from the BCI to an intermediate personal digital assistant (PDA) capable of transmitting the signal to a server where the signal analysis is performed. Each of the signals to transmit is a train of pulses with non-uniform zero-crossings. We explore the application of OFDM using orthonormal chirp basis for the modulation of the N time-encoded signals.

4.1. Uniform symbol period

Chirp modulation has been applied successfully in OFDM [9, 10], a multi-carrier technique that transmits data by dividing the bit stream into several parallel streams. This chirp modulation has been shown to mitigate the effects of the channel Doppler frequency shifts (due to a moving receiver or transmitter) and to be robust to the presence of noise in the transmitted signal. In the transmission of source symbols $+1$ or -1 with a uniform period T, if we have orthonormal chirps $c_k(t)$ for users $k = 1, \cdots, U$ the baseband transmitted signal for user k is given by

$$s_k(t) = b_k(t)c_k(t), \tag{11}$$

where $b_k(t)$ is either 1 or -1 for $t_0 \leq t \leq t_0 + T$. Assuming perfect synchronization between transmitter and receiver, and that the only channel effect is addition of Gaussian noise $\eta(t)$, the baseband received signal is

$$r(t) = \sum_{k=1}^{U} s_k(t) + \eta(t). \tag{12}$$

To recover the source symbols, multiplying the received signals by the conjugate of the chirps, $c_k^*(t)$, we obtain a decision variable for user k, y_k, by integrating over a period and using the orthogonality of the chirp signals:

$$\begin{aligned} y_k &= \int_{t_0}^{t_0+T} r(t)c_k^*(t)dt \\ &= \sum_{n=1}^{U} b_n(t) \int_{t_0}^{t_0+T} [c_n(t)c_k^*(t)dt + \eta(t)c_k^*(t)]dt \\ &= b_k(t) + \int_{t_0}^{t_0+T} \eta(t)c_k^*(t)dt \end{aligned}$$

in $t_0 \leq t \leq t_0 + T$. The value $b_k(t)$, which is either 1 or -1, is estimated by a thresholder. The orthonormality of the chirps mitigates the multiple-access interference caused by users different from the user in which we are interested.

Consider a set of frequency-modulated linear chirps $\{c_k(t)\}$ with instantaneous frequencies

$$\phi_k(t) = \theta t + 2f_k, \qquad k = 1, \cdots, U, \tag{13}$$

where θ is the chirp rate, common to all the chirps, and $f_k = k/T$ is a multiple of the frequency corresponding to the symbol period T. The chirps are given by

$$c_k(t) = e^{j\pi t\phi_k(t)} = e^{j\pi\theta t^2} e^{j2\pi f_k t}.$$

The orthonormality of the chirps $\{c_k(t)\}$ depends on the orthonormality of the $\{e^{j2\pi f_k t}\}$ terms. Indeed, the common chirp rate makes it so that

$$\begin{aligned} \frac{1}{T}\int_{t_0}^{t_0+T} c_k(t)c_n^*(t)dt &= \frac{1}{T}\int_{t_0}^{t_0+T} e^{j2\pi(f_k-f_n)t}dt \\ &= \begin{cases} 1 & k=n \\ 0 & k \neq n. \end{cases} \end{aligned} \tag{14}$$

In [9, 10] the orthonormal chirps are obtained from the properties of the kernel of the fractional Fourier transform (FrFT), but such relation is unnecessary as shown above, but it could be used to connect the whole procedure with the FrFT later. Given a signal $x(t)$ its FrFT is

$$\begin{aligned} F_\alpha[x(t)](u) &= \int_{-\infty}^{\infty} x(t)K_\alpha(t,u)dt \\ K_\alpha(t,u) &= A_\alpha e^{j(\pi t^2+u^2} e^{-j2\pi\csc(\alpha)tu}, \end{aligned}$$

where $K_\alpha(t,u)$ is the FrFT kernel. In [9, 10] the authors showed that for the special case the kernel can be shown to be orthonormal, i.e.,

$$\int_0^T K_\alpha(t,u_m)K_\alpha(t,u_n)dt = |A_\alpha|^2 T\delta[u_n,u_m],$$

where

$$\delta[u_n,u_m] = \begin{cases} 1 & u_n = u_m \\ 0 & u_n - u_m = \sin(\alpha/T), \end{cases}$$

so that the chirps

$$c_n(t) = \frac{1}{|A_\alpha|\sqrt{T}} K_\alpha^*(t, \sin(\alpha/T))$$

are orthonormal and can be used as carriers.

4.2. Non-uniform symbol period

Applying the chirp-modulated OFDM for the transmission of the time-encoded signals obtained from N ASDMs is complicated by the fact that the pulses, corresponding to the symbols, do not have a uniform period as before. Figure 7 illustrates the ASDM output corresponding to an arbitrary signal.

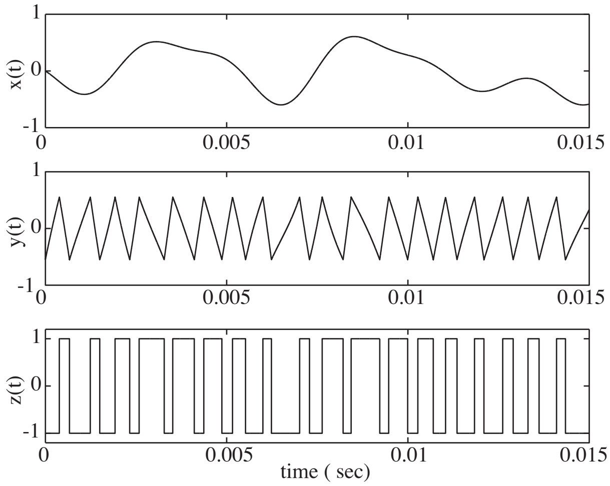

Figure 7. Output of ASDM for arbitrary signal. Notice the non-uniform duration of the pulses.

In this case we will again consider chirps with a common chirp rate θ, but with frequencies $f_n = 1/\hat{T}$, where

$$\hat{T} = \min\{T_n(k)\}$$

and $T_n(k) = t_n(k+1) - t_n(k)$ are the time intervals from the signals $\{z_n(t),\ n = 1, \cdots, N\}$. The bandwidth allocated to the n^{th}-ASDM, $F_n = f_{n+1} - f_n$, is divided into M sub-bands with frequencies

$$f_n(m) = f_n + \frac{F_n}{M}m, \qquad m = 0, \cdots, M-1. \tag{15}$$

Using these frequencies and the zero crossings $\{t_n(k)\}$ from $z_n(t)$, we create an array of chirps with instantaneous frequencies

$$\phi_{n,m}(t) = \theta t + 2f_n(m) \tag{16}$$

where $t \in [t_n(m), t_n(m+1)]$ and $-\infty$ otherwise, (so that the chirp is zero outside $[t_n(k), t_n(k+1)]$). Thus the chirp

$$c_{nm}(t) = e^{j\pi t\phi_{nm}(t)} = e^{j\pi\theta t^2} e^{j2\pi f_n(m)t} \tag{17}$$

for $t_n(m) \le t \le t_n(m+1)$ and zero otherwise.

Considering an analysis time segment $t_0 \le t \le t_0 + T_f$, where $T_f = \beta\hat{T}$ for a small integer β, the orthonormality of the chirps $c_{nm}(t)$ is kept by the common chirp rate and by the orthogonality of the complex exponentials with frequencies $\{f_n(m)\}$. Each consecutive pulse in $z_n(t)$ is multiplied by a chirp with an increasing frequency $f_n(m)$.

Assuming again that the effect of the channel is only the addition of Gaussian noise, the received signal is now

$$\begin{aligned} r(t) &= \sum_{n=1}^{N}\sum_{m=0}^{M-1} s_{nm}(t) + \eta(t) \\ &= \sum_{n=1}^{N}\sum_{m=0}^{M-1} z_n(t)c_{nm}(t) + \eta(t). \end{aligned} \tag{18}$$

Multiplying this signal by $e^{-j\pi\theta t^2}$ gives

$$\begin{aligned} y(t) &= r(t)e^{-j\pi\theta t^2} = \sum_{n=1}^{N}\sum_{m=0}^{M-1} z_n(t)e^{j2\pi f_n(m)t} \\ &+ \eta(t)e^{-j\pi\theta t^2}, \end{aligned} \tag{19}$$

and when we pass this signal through a band-pass filter of bandwidth F_n gives

$$\tilde{y}_n(t) = \sum_{m=0}^{M-1} z_n(t)e^{j2\pi f_n(m)t} + \tilde{\eta}(t), \tag{20}$$

which is a combination of sinusoids in the bandwidth assigned to channel n, and $\tilde{\eta}(t)$ is the noise within that band-width.

If we express $z_n(t)$ for $t_0 \le t \le t_0 + T_f$ as a concatenation of rectangular pulses using the unit-step signal $u(t)$ and let $d_\ell = \pm 1$ for the subchannels being occupied and zero for those that are not, we get

$$z_n(t) = \sum_{\ell=0}^{M-1} d_\ell[u(t - t_n(\ell+1)) - u(t - t_n(\ell))].$$

The Fourier transform of $z_n(t)$ is

$$Z_n(\omega) = \sum_{\ell=0}^{M-1} d_\ell \int_{t_n(\ell)}^{t_n(\ell+1)} e^{-j\omega t}dt, \tag{21}$$

and then the Fourier transform of $\tilde{y}_n(t)$ is given by

$$\tilde{Y}_n(\omega) = \sum_{m=0}^{M-1} Z_n(\omega - 2\pi f_n(m)) + \tilde{\eta}(\omega).$$

If we filter $\tilde{Y}_n(\omega)$ with a band-pass filter of center frequency $f_n(m)$ and determine the value of this function at the frequencies $f_n(m)$, for $m \in [0, \cdots, M-1]$ we obtain

$$\begin{aligned} \hat{Y}_n(f_n(m)) &= Z_n(0) + \tilde{\eta}(f_n(m)) \\ &= d_m\,[t_n(m+1) - t_n(m)] \\ &\quad + \tilde{\eta}(f_n(m)) \end{aligned} \tag{22}$$

so that $|\hat{Y}_n(f_n(m))| \approx t_n(m+1) - t_n(m)$. We thus have that for the m-subchannel in the n^{th}-ASDM outputl with high signal to noise ratio the corresponding period is

$$T_n(m) = t_n(m+1) - t_n(m),$$

and the magnitude of $\hat{Y}_n(f_n(m))$ is d_m. A bank of filters like the one shown in Figure 8 is used to reconstruct the neural signals.

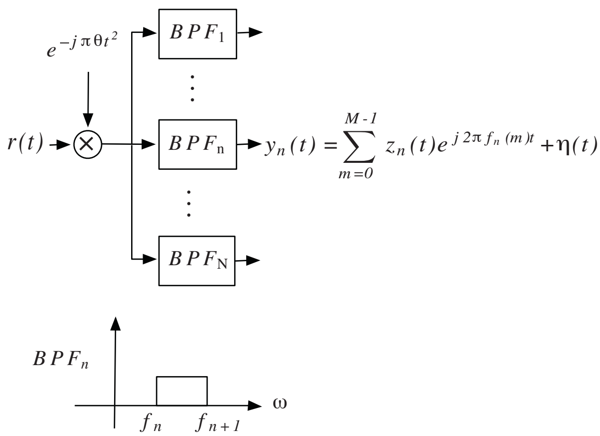

Figure 8. Filter reconstruction bank.

4.3. Simulations

The transmission of four outputs $\{z_n(t),\ n = 1,2,3,4\}$, assumed to come from arbitrary signals, is illustrated in Figure 9. To illustrate the performance of our procedure a Monte Carlo simulation with 500 trials for each signal to noise ratio (SNR) between -10 and 10 dBs (with increments of 5 dBs) was implemented. Gaussian noise is added to the chirp-modulated signal to obtain the different SNR's. The binary signals $\{z_n(t),\ n = 1,2,3,4\}$ in a window of 4 msec are shown in the top plot of Figure 3 displaying different widths for the two pulses in each $z_n(t)$. The magnitudes $|\hat{Y}_n(f_n(m))|$ corresponding to different frequencies in the middle plot are estimates of the width of the pulses in each of the $\{z_n(t),\ n = 1,2,3,4\}$. The axis showing this information is labeled symbol duration. The horizontal axis displays the frequency at which the chirp originates. The effect of the noise (this corresponds to an SNR of 10 dBs) is shown. Thus our algorithm provides the duration of each of the symbols in seconds from which we compute the zero-crossing times needed to reconstruct the original signals in each of the channels. The plot at the bottom of Figure 9 displays the error probability when estimating the width of each of the pulses in the binary signals for each of the SNR used in the Monte-Carlo simulation.

5. OFDM implementation

The above procedure requires a time-frequency implementation, as we need to know frequency information from each of the subchannels as it changes with time. An easier implementation can be obtained by generating the signals (see Figure 10)

$$y_i(t) = |z_i(t) - z_i(t - \Delta)| \qquad i = 1, \cdots, N,$$

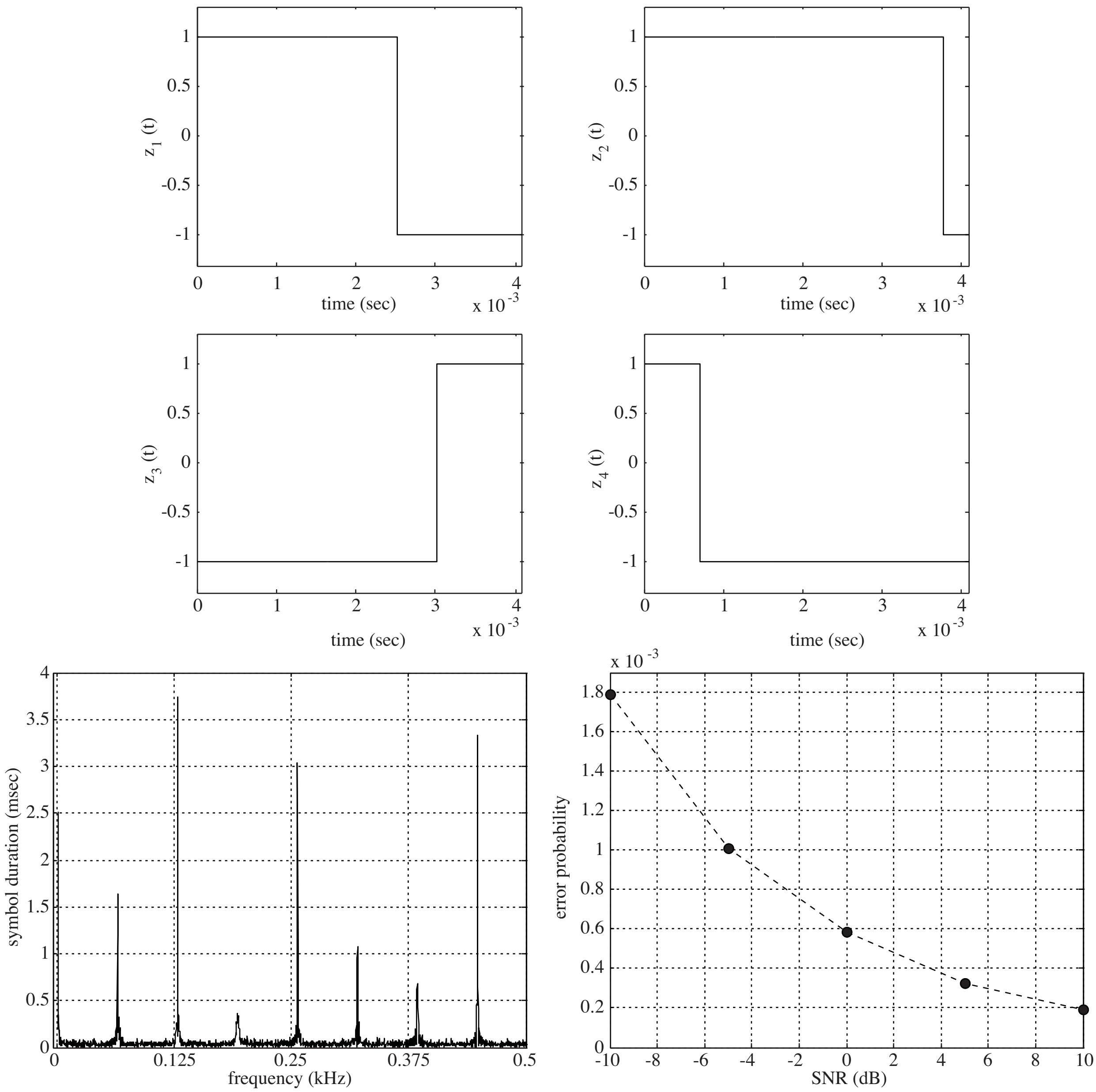

Figure 9. Monte-Carlo simulation for the transmission of four channel ASDM binary signals $\{z_n(t),\ n = 1, 2, 3, 4\}$. Top: non-uniform widths of the binary signals $\{z_n(t)\}$. Middle: estimated widths for each of the pulses in $\{z_n(t),\ n = 1, 2, 3, 4\}$ when noise is added (SNR=10 dB). Bottom: error probability of the estimation of the widths for different SNRs.

which are approximately the absolute value of the derivative of the binary signals $z_i(t)$, or narrow pulses that occur at the zero-crossing times. Putting the vector $\{y_i(t)\},\ i = 1, \cdots, N$ as the input of an FFT, and demodulating with an inverse FFT in the receiver, ideally we should be able to obtain an approximation of the vector

$$\{y_i(t)\},\ i = 1, \cdots, N,$$

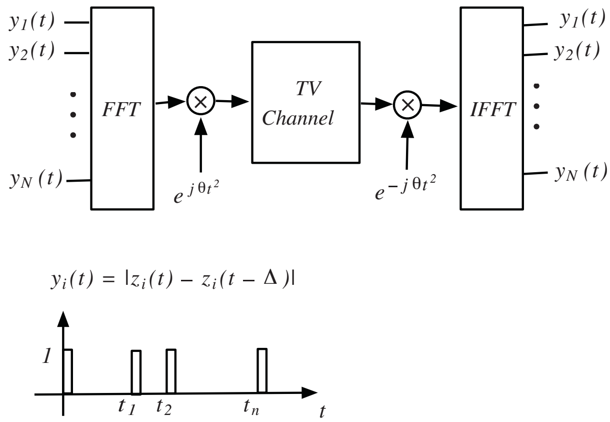

Figure 10. OFDM implementation. Chirps are used for channel characterization.

for which we can measure the time at which the pulses appear giving us the desired time sequence to recover the original signals $x_i(t)$. If we wish to obtain a model for the channel we could use the chirps to detect the channel parameters. As shown in [13], the time-varying model of a channel with P paths, and attenuation factors α_k, Doppler frequency-shift d_k and time shifts τ_k for each of the $k = 1, \cdots, P$ paths is

$$r(t) = \sum_{k=1}^{P} \alpha_k s(t - \tau_k) e^{j d_k t^2},$$

where the input of the channel is $s(t)$ and $r(t)$ the output. If we let the input be $s(t) = e^{j\theta t^2}$, i.e., a linear chirp with chirp rate θ, we then get that the output of the channel is

$$r(t) = \sum_{k=1}^{P} \hat{\alpha}_k s(t - \tau_{k,eq}), \tag{23}$$

which resembles a linear time-invariant channel with an equivalent time shift

$$\tau_{k,eq} = d_k - 2\tau_k \tag{24}$$

that depends on the Doppler frequency and the time shift.

6. Conclusions

In this paper we consider asynchronous data acquisition using ASDMs, multiplexing and transmission of outputs of several channels with ASDMs and their reconstruction. The advantages of using ASDMs are the low-power consumed and the lack of clocks. For the transmission of the outputs of a number of ASDMs we propose using chirp modulation OFDM, which is robust to Doppler and time-shifting caused by time-varying channels. Since the conventional approach cannot be implemented given the non-uniformity of the pulses, we propose a novel approach that uses a sequence of localized linear chirps that are orthonormal. The results are encouraging, especially its robustness to noise. The neural signals can be recovered by means of a Slepian interpolation. Connecting our procedure to either Fractional Fourier Transform or to the evolutionary spectral theory will permits us to investigate the performance of the proposed chirp OFDM under the constrains of the channel. We will also like to explore a different approach where an ASDM and a level crossing system can be used for the data acquisition and transmission, possibly reducing the overall complexity which is desirable given the computational constrains of BCIs.

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