The dynamics of financial stability

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Abstract. - The social role of any company is to get the maximum profitability with the less risk. Due to Basel III, banks should now raise their minimum capital levels on an individual basis, with the aim of lowering the probability for a large crash to occur. Such implementation assumes that with higher minimum capital levels it becomes more probable that the value of the assets drop bellow the minimum level and consequently expects the number of bank defaults to drop also. We present evidence that in such new financial reality large crashes are avoid only if one assumes that banks will accept quietly the drop of business levels, which is counter-nature. Our perspective steams from statistical physics and gives hints for improving bank system resilience. Stock markets exhibit critical behavior and scaling features, showing a power-law for the amplitude of financial crisis. By modeling a financial network where critical behavior naturally emerges it is possible to show that bank system resilience is not favored by raising the levels of capital. Due to the complex nature of the financial network, only the probability of bank default is affected and not the magnitude of a money market crisis. Further, assuming that banks will try to restore business levels, raising diversification and lowering their individual risk, the dimension of the entire financial network will increase, which has the natural consequence of raising the probability of large crisis.

Introduction. – Since 1988 that bank system resilience is the main issue in financial regulation. It is so important that it is probably the only subject on which the all world agree on. Nations cannot reach a global agreement on saving the atmosphere or the seas, but they are quite successful on bank system protection treaties. The Basel Committee on Banking Supervision, located at the Bank for International Settlements, in Basel, is composed by bank regulators from all over the world and they issue what is known by the Basel Accords which act as the accepted law on banks of the developed countries. In very simple terms, these accords define the solvency level of the banks, i.e., the amount of the bank's own money capital - that is lend to the customers, with the remaining money coming from customers deposits.

As bank system resilience rules intend to protect the system resilience, the 2008 financial turmoil that lead to bank system freeze was not a very good sign, specially on the ability of regulators to make system protection rules. The first accord, dated from 1988 [1] become very important in the sense that it provided a way to prevent ad infinitum leverage and when the US mortgage crisis came about, the second version, known as Basel II [2], was already scheduled to start. Naturally, the social pressure over the Committee to tighten the rules become very strong and in 2010 they issue the third version, Basel III [3], to improve bank system resilience by raising the levels of capital. But, as we will show, the bank system resilience does not necessarily improve with such rising of the capital levels.

Since long ago, physics and in particular statistical physics have motivated the construction of models for explaining the evolution of economies and societies and for tackling major economic decisions in different contexts in general [7], and in particular in financial markets. The study of critical phenomena and multi-scale systems in physics lead to the development of tools that proved to be useful in non-physical contexts, in particular in financial systems [7, 8]. Two reasons for this. First, being subjected to well-defined rules, financial markets are described by indices with a dynamical behavior showing scaling character, typical of critical physical systems [6]. Second, showing the co-existence of several time-scales, determined by traders at different time-span terms, financial data exhibits an hierarchical structure which can be described within physical frameworks, such as the ones for self-organized criticality [4,6] and hydrodynamical turbulence [5].

With the aim of addressing the occurrence of large crisis in financial systems, we have focused [9] in the critical behavior aspects of financial systems, using the two basic types of economical interactions, namely the consumption connections and the production connections, to describe economic exchange of labor within a set of financial agents, represented by a proper agent-model. The exchange of labor is addressed through interactions among agents and are created or destroyed within the system, according to the basic economical principles of demand and supply. New connections are created due to the assumption that there is a natural propensity for agents to interact, with the aim of improving their profit. From the basic principles of demand and supply, agents should tend to prefer interactions with the most connected agents in the network. Additionally, the system remains open by imposing a leverage threshold to each agent bellow which the agent leaves the consumption network (it bankrupts), breaking its consumption connections with all other agents. These basic assumptions induce a stationary critical state in the financial network [9], where the abrupt crashes in the evolution of total leverage in the system occur with arbitrary amplitude. We have shown that [9] the amplitude of such crashes, that measures the size of crisis in the financial system, is closelly related to the underlying topology of agent relations and is well characterized by a power-law with the same exponent as the one observed in empirical data of financial indices, namely in the indices All Ords, CAC, CBOE, DAX, Dow Jones, FTSE, HSI and NIKKEI [9].

Results. – Here we show an important consequence of the relation between economic crisis and the topology of agent relations: by varying the leverage threshold in the system, the power-law for the size distribution of crisis implies non-trivial deviations of its exponent values. Since such exponent is directly related with the probability for large crisis to occur, as we explain below, such deviations have fundamental implications when addressing the problem of the definition of minimum levels of capital with the aim of improving the banking system resilience. In particular, the intentions of Basel III accords [3] may not be observed in the future after increasing minimum capital level.

To arrive to that conclusion, we start by noting that a well behaved and riskless economy, where the agents don't make consumption connections without making an equivalent production connection, has zero leverage, since we are not using any kind of credit. This situation is a situation of non-economy, representing a closed and static regime without growth, which obviously does not occur in our present society.

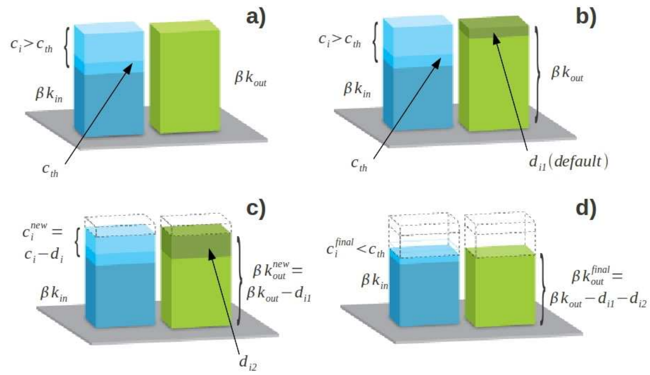

Fig. 1: Sketch of banks system resilience. **(a)** Bank balance with no defaults, having a capital $c_i = \beta(k_{out} - k_{in}) > c_{th}$ above the minimum capital level c_{th}. **(b)** After client default d_{i1}, the number of outgoing connections will be reduced, **(c)** reducing the number of shareholder connections to $\beta k_{out}^{new} = \beta k_{out} - d_{i1}$ and yielding a new capital investment $c_i^{new} = c_i - d_i$. If client default persists, at a new level d_{i2}, **(d)** the capital may drop bellow minimum level $c_i^{final} < c_{th}$, leading to bankruptcy (see text).

When, on the contrary, each agent can form both production or consumption connections without having an equivalent connection that cancel it in terms of leverage, it experiments the borders of the system and, obviously, the risk for it to move out of the system may occur whenever the other agents no longer believe that it can form production connections to net the consumption connections. Such agents can be regarded as persons or companies. There is also another kind of agents that have a mirrored relation with labor, and hence with money, because their production is credit and their consumption is inverse credit (deposits). These agents are called banks, and are the ones we are concerned with.

The natural propensity for agents to form consumption and production connections between each other lead to the formation of two co-existing complex networks, one of consumption connections and another of production. When an agent leaves the consumption network we call that event a bankruptcy and it occurs when the leverage of a particular agent goes bellow what the agent aggregate assumes to be the limit.

Figure 1 sketches the bank structure in the scope of our model. A bank provides production of credit (such as money) to other k_{out} agents, using a quantity the production from k_{in} other agents. As shown in Fig. 1a, the total credit provided by the bank is therefore βk_{out}, with β being a proper constant measuring the average amount received by one agent in exchange of its average labor [9]. Similarly, the total production amount used by the bank is βk_{in}. The difference between both is the capital c_i. Normalized to the total amount of income credit the capital reads

$$\bar{c}_i = \frac{k_{out}}{k_{in}} - 1 \tag{1}$$

which takes values above -1 and is well-defined for any agents with at least one incoming connections ($k_{in} > 0$) as it is the case of banks. When it takes a value of -1 that means that the bank has an infinite leverage (asymptotic level), $k_{in} \to \infty$.

The capital can be regarded as a sort of 'internal energy' that accounts for the amount of leverage a bank - or any other economic agent - has [9]. Since not all k_{out} agents will return their debt to the bank, an amount d_{i1} of defaults is expected (see Fig. 1b). As long as the capital c_i exceeds the defaults level ($c_i^{new} = c_i - d_{i1} > c_{th}$ in Fig. 1c) the bank survives. The reason why there is this kind of conservation law between consumption and production in a bank, even though we do not represent shareholder connections, is because a bank intermediates labor and we are disregarding the operational work needed to run the bank that, obviously, exists.

Bankruptcy occurs therefore when the number of destructed production connections the costumer defaults, d_{i2} in Fig. 1c - becomes such that the number of shareholder consumption connections drops bellow a regulatory minimum level, $c_i^{final} < \bar{c}_{th} k_{in}$ (see Fig. 1d). Such bankruptcy situation reflects the removal of all banks consumption connections from its clients ($k_{in} = 0$) and the bank therefore leaves the system. At the same time new banks enter the system by joining some of the remaining agents.

Two notes are due here. First, in practice, capital is a consumption of labor from agents called shareholders and could be represented by the amount βk_{in}. The reason why we don't do it is because banking regulation separate shareholders from depositors and to map the two approaches we need to make the same segregation. Second, the difference between production connections and consumption connections is not equal to the difference between assets and liabilities, it only represents it. That is, the break of the consumption connections on the agents that connect with the bank represents a destruction of a production connection, an asset. The main difference resides on the fact that we are not interested in fixed or non-performing assets. A default represents a destruction of a production connection in opposition to bank accounting where the loan becomes a depreciating stock.

Assuming what one knows from empirical data that financial systems exhibit self-organized criticality [4], the financial system remains trapped between two phases, one where trades increase in number and another where global crashes occur. Assuming that one given bank in the network experiences bankruptcy, if there is, on average, less than one customer to bankruptcy as a consequence of the first one, banks could raise capital with no extra cost in relation to deposits. Such situation corresponds to a non-economic state [9], called the infinite capital state, since it would imply an infinite amount of energy (capital) to each agent (bank). If, on the contrary, the average number of bankruptcies is bigger than one, this means that each bankruptcy leads to more than one new bankruptcy, in a chain reaction destroying the full financial network. This is of course also a non-economic state. Therefore, the average number of bankrupted agents that follow a bank bankruptcy in a chain reaction must be equal to one. And this makes all the difference in terms of banking system stability, especially when we study the magnitude of the system crisis.

Therefore, the system incorporating finance agents, some of them banks, is critical if one starting bankrupted agent leads to several other agents to bankrupt as the result of a cascade phenomenon. For that, the expected value for one neighbor to be also bankrupted must be one. This occurs when the event of a default leads the agent with $\hat{k}_{in}$ consumption connections to fulfill the following conditions $\frac{k_{out}}{\hat{k}_{in}} - 1 > \bar{c}_{th}$ and $\frac{k_{out}-1}{\hat{k}_{in}} - 1 < \bar{c}_{th}$, i.e.

$$\frac{k_{out} - 1}{1 + \bar{c}_{th}} < \hat{k}_{in} < \frac{k_{out}}{1 + \bar{c}_{th}} \tag{2}$$

and the probability for a bank to become insolvent is given by the probability $P_{ins} = P(\hat{k}_{in})$ for having $\hat{k}_{in}$ production connections.

It is reasonable to assume that agents select their production connections with agents with high production, i.e. large number of production connections. With such a preferential attachment [12] scheme, the probability of finding an agent with k_{in} connections follows a power-law $P(k_{in}) = k_{in}^{-\gamma}$, typically with $2 < \gamma < 3$ and therefore the probability of bank insolvency reads

$$P_{ins} = \left(\frac{1 + \bar{c}_{th}}{k_{out}} \right)^{\gamma}. \tag{3}$$

Equation (3) shows that a higher minimum capital value is more probable violated than a lower one. The aim of the regulators is not that banks become more insolvent but that they do not make outgoing connections without the proper level of capital. If we were talking about a system where the agents relate each other in a perfect randomly fashion, raising the minimum level would raise the total capitalization of the banking system and since banks would keep themselves above the threshold level, when a single bank violates that threshold, the amount of capital in the system would function as a pillow to hold the impact. This would be true if the system would not show self-organized criticality or if the probability of banks to negotiate loans with each other were independent from their size. Since it is not true and agents relate each other depending on their size and showing self-organized criticality, the phase transition state imposes an average of one child bankruptcy for each particular one, that is

$$\sum_{k_{out}=1}^{\infty} k_{out} P(k_{out}) P_{ins} = 1 \tag{4}$$

assuming that $P(k_{out}) \propto k_{out}^{-\gamma}$ also. These assumptions lead to

$$(1+\bar{c}_{th})^{\gamma} = M^{\gamma}\zeta(\gamma+1) \quad (5)$$

an expression which relates the capital minimum level $\bar{c}_{th}$ with economic growth M and the topology of the underlying network of financial connections γ by the imposition of the existence of an economy in a phase transition state, $\zeta(x)$ is the Riemann Zeta function of x. Equation (5) shows that raising or lowering levels of minimum capital has consequences both on the economic growth and on the topology of economic relations. The question now is how?.

Since the expected value of bankruptcies on banks that have deposits (consumption connections) in a bankrupted bank is one, the Otter theorem [10] tells us that the probability $P(r)$ of having r banks involved on the bankruptcy chain reaction is proportional to $P(r) \propto r^{-3/2}$. From Eq. (3) having a total number L of agents it follows

$$r = LP_{ins} \propto L\left(\frac{1+\bar{c}_{th}}{k_{out}}\right)^{\gamma}. \quad (6)$$

Consequently, the probability of having r banks involved on a bankruptcy chain reaction can be written as a function of the number of consumption connections involved, namely

$$P(k_{out}) \propto L^{-3/2}\left(\frac{1+\bar{c}_{th}}{k_{out}}\right)^{3\gamma/2}. \quad (7)$$

From Eq. (7) one easily concludes that raising capital levels will not have any influence on the magnitude of banking stability and, in fact, it makes the occurrence of a chain reaction more probable. If the minimum capital level $\bar{c}_{th}$ increases with everything else kept constant, the probability of a crisis of any size also increases.

Considering Eq. (4) one can write the probability for a crisis size of s or larger given by

$$P(k \geq s) \propto \int k^{-3\gamma/2}dk \propto s^{-3\gamma/2-1} \equiv s^{-m} \quad (8)$$

which shows that a larger exponent m reduces the probability for a large crisis to occur. But it is also possible that all quantities in Eq. (7) may also change when the minimum capital level increases, which makes the analytical treatment of this model more difficult. To overcome this we use an agent model of financial agents to simulate the occurrence of defaults in a banking like system. We consider a system with L agents, each one representing and economical entity capable of trading, and interactions between them represented by connections. The set of agents and interactions composes the financial subsystem where the agents establish their trading actions, which is, in general, a part of the full world-wide financial network. One connection is formed on each clock event between two of these agents, being one the consumption agent and the other the production agent. The agents are chosen using a standard preferential attachment algorithm [12] and the minimum level of capital $\bar{c}_{th}$ is provided to the model as a parameter.

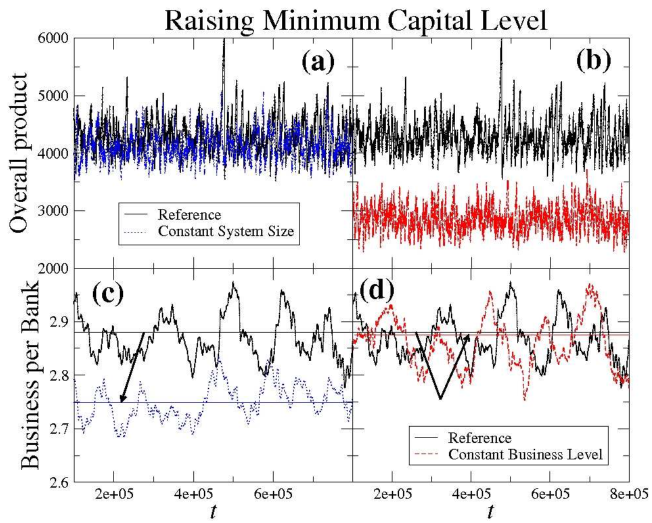

Fig. 2: Considering the overall product U_T of a reference minimum capital level (solid line), there are two different processes for achieving quasi-stationary states when minimum capital level is increased: **(a)** by keeping the number of agents constant (dotted line) or **(b)** by maintaining the business level constant (dotted line). **(c)** The average business level Ω per bank (see Eq. (10)) decreases when the number of agents is constant. **(d)** Assuming that banks do not want to see their business level dropping, it is natural to expect that they will choose a process with constant business level to face an increase of the minimum capital level. Under such natural assumption the change of the probability for new large crisis is not the one shown here. See Fig. 4.

The system of financial agents evolves with each agent summing up its own product which takes into account its production connections, those where its labor is delivered to neighboring agents which pay for it, and its consumption connections, those where the agent pays an amount for receiving one unit of labor. The amount paid in each case depends on the rules of supply and demand for the agent and its neighbor. An agent delivering labor to many neighbors (high supply) tends to impose a high price in return to its neighbors. At the same time that price depends on each particular neighbor: a neighbor receiving labor from many other agents will induce a reduction in the price impose by each agent. These principles can be incorporated in a simple *ansatz* between the labor W_{ij} delivered and the corresponding amount E_{ij} received as

$$\frac{W_{ij}}{E_{ij}} = \frac{2}{1+\mathrm{e}^{-(k_{out,i}-k_{in,j})}} \quad (9)$$

where $k_{out,i}$ is the number of production connections of agent i and $k_{in,j}$ is the number of consumption connections of neighbor j. Summing up the product due to the production connections in the network one gets the overall product U_T.

Figure 2 shows several examples of overall products for

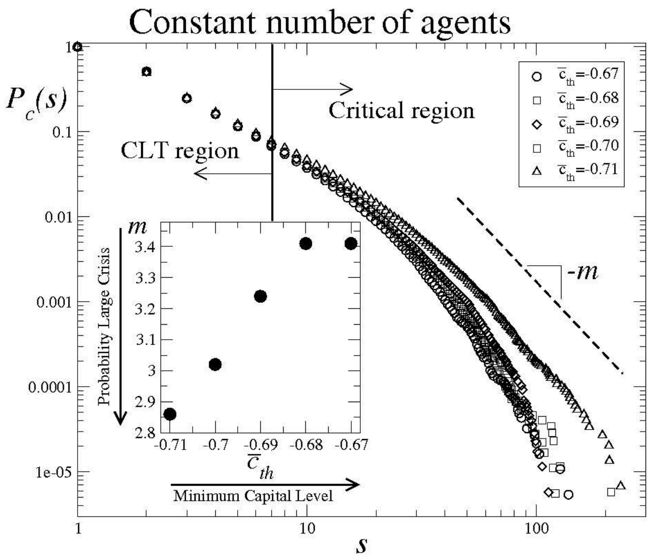

Fig. 3: Crisis size distributions for different scenarios of minimum capital level, keeping the same operating neighbourhood for each agent. The different curves match at small sizes, in the Central Limit Theorem (CLT) region, and deviate from each other for larger crises (critical region). For the critical region one observes (inset) that increasing minimum capital level decreases the probability for a large crisis to occur, which supports the intentions of Basel III accords. However, in this scenario one assumes that each bank will have a simultaneous decrease of their business level and accept it (see text). A more natural scenario would be one where each bank acts in order to keep its business level constant, which leads to a completely different crisis situation (see Fig. 4).

different minimum capital levels. In black one plot in Figs. 2a and 2b a reference situation of minimum capital level and with (blue) dotted and (red) dashed lines the overall product observed when this minimum capital level is increased in two different ways. The blue dotted line (Fig. 2a and 2c) shows what happens when the system size is kept constant and the red dashed line (Figs. 2b and 2d) the corresponding situation when the business level per bank is kept constant. The business level is define as

$$\Omega_{per\ bank} = \frac{1}{L}\frac{1}{T_S}\int U_T(t)dt \qquad (10)$$

where T_S is a sufficiently large period for taking time averages.

Figure 2 shows that keeping the size L of the operating neighborhood where banks operate, induces a decrease of the business level as a result of the minimum capital level raise. Differently, if the business level is kept constant, the size of the operating neighborhood shrinks.

Next we consider the first situation for minimum capital level rising, i.e. we consider L as a constant. Figure 3 shows the size distributions of crisis in our model for different minimum capital levels, remaining the number of agents constant ($L = 2000$). Having less frequent large crisis, the system shows a larger number of small crisis comprehending sets of one or two agents that collapse. Further, this overall scenario occurs under the imposition that the number of agents trading within the system is constant and each agent, in particular each bank, will maintain the same neighborhood as previously, before the raise of minimum capital level. As one sees in the inset of Fig. 3 the exponent increases in absolute value for larger minimum capital levels, which prevents large crisis to occur. Though, such scenario occurs only when the size (number of agents) of the financial subsystem where the agent makes its trades is kept constant. For a scenario where the size of the operating neighborhood is adapted to maintain the business level constant (Eq. (10) the situation is different as shown next, in Fig. 4.

Figure 4 shows the critical exponent m (Eq. (9)) and the business level per bank Ω (Eq. (10)) as a function of the minimum capital level $\bar{c}_{th}$ and the operating neighborhood size L. For easy comparison, both quantities are normalized in the unit interval of accessible values. Roughly, the critical exponent increases and the business level decreases with both the minimum capital level and the operating neighborhood size. Considering a reference state F_0 with $c_{th,0}$, L_0 and Ω_0 there is one isoline of constant minimum capital level, $\Gamma^0_{c_{th}}$, and another of constant system size, Γ^0_L, crossing at F_0. Assuming a transition of our system to a larger minimum capital level at isoline $\Gamma^f_{c_{th}}$ keeping the system size constant, i.e. along the isoline Γ^0_L, one arrives to a new state F_L with a larger critical exponent, which means a lower probability for large crisis to occur, as explained above. However in such situation the new business level $\Omega_f < \Omega_0$ is smaller than the previous one.

On the contrary, if we assume the transition to the higher minimum capital level occurring at constant business level, i.e. along the isoline Γ^0_Ω one arrives to a state F_Ω on the isoline $\Gamma^f_{c_{th}}$ for which the critical exponent is approximately the same, with the same probability for large crisis to occur. In fact the isolines of constant critical exponent match within good accuracy the isolines of constant business level. Since it is reasonable to expect a reaction from the agents facing an increase of minimum capital level to maintain their business level, this finding contradicts the expectations in Basel accords and raises the question if such regulation will indeed prevent a larger crisis to occur again in the future.

In Fig. 4 one finds for the reference state F_0, $m = 2.97 \pm 0.18$. An increase of minimum capital level at constant business level (state F_Ω), yields $m = 2.79 \pm 0.09$ which corresponds to a significant higher probability of large crisis than if one assumes minimum capital level increase at constant operating neighborhood size (state F_L), with a lower exponent ($m = 3.34 \pm 0.09$).

Discussion. – In summary, raising the minimum capital levels will in fact bring more capital to the system but it will not, on the opposite, improve bank system resilience. On the best hypothesis, it will keep the same if banks go after the same business levels, as one should

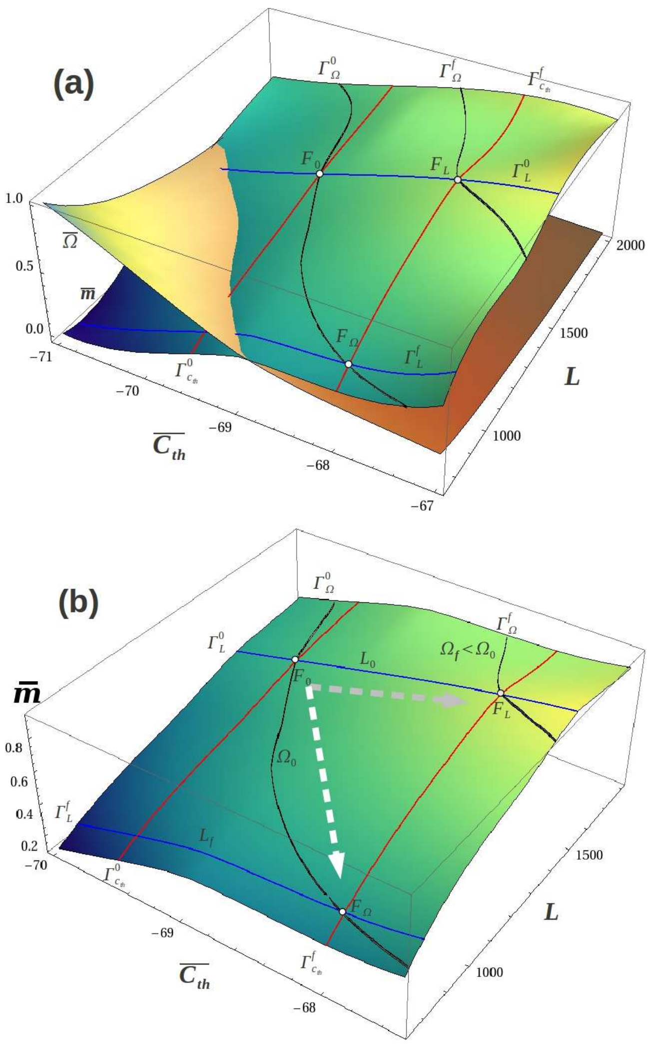

Fig. 4: **(a)** Normalized critical exponent $\bar{m}$ and business level Ω as a function of the minimum capital level $\bar{c}_{th}$ and system size L. For an initial financial state F_0 an increase of the minimum capital level means to follow one of the infinitely many paths from its (dashed) isoline $\Gamma^0_{c_{th}}$ at constant minimum capital level to a final one $\Gamma^f_{c_{th}}$. **(b)** If such path follows a (dot-dashed) isoline Γ^0_L at constant system size the critical exponent (probability for large crisis) increases (decreases), but its business level decreases ($\Omega_f < \Omega_0$), which is against to the natural intentions of financial agents. Contrarily, if it follows a (thick) isoline Γ^0_Ω at constant business level, as one natural expects the financial agents would do, the critical exponent does not change significantly, meaning that large financial crisis may still occur with the same probability as before (see text).

expect. Such findings can be helpful in the recent governmental measures for handling with the effects of 2008 financial crisis. In particular, governments have shown [3] the tendency for imposing a higher capital investment from private banks. If the threshold is increased, with the total trade amount remaining constant there will be less trades connections between the bank and clients which leads to smaller crisis in the evolution of the financial network. However, increasing their capital investment, banks will try to increase proportionally their level of business, to maintain the fraction of client loans to the banks capital constant. Such increase in the number of clients reflects into a network with a different number of agents and leads to a statistical distribution including larger crisis.

One other important effect is that, at constant capital, that is, if banks do not gather more capital for the business, the curve at constant business level will lead to fewer agents in the network. This means that the reorganization of the network, at constant capital level, will make agents disappear (bankruptcies or fusions) favoring the appearance of too big to fall financial companies.

Interestingly, while being controversial, our claims point in the direction of IMF reports in November 2010 [11], where it is argued that rapid growth in emerging economies periods can be followed by financial crises, and also to recent theoretical studies on risk of interbank markets [13, 14].

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