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Are "Nearly Exogenous Instruments" reliable?

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ABSTRACT

We show that when instruments are nearly exogenous, the two stage least squares *t*-statistic unpredictably over-rejects or under-rejects the null hypothesis that the endogenous regressor is insignificant and Anderson–Rubin test over-rejects the null. We prove that in the limit these tests are no longer nuisance parameter free.

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1. Introduction

Instrumental variable methods are used to identify causal relationships. Researchers pick relevant instruments that should be related to the endogenous explanatory variable both on the basis of an a priori argument and statistics.[1]

Instruments must also be exogenous; that is, they are not related to the outcome variable after controlling for relevant explanatory variables. Just whether or not the exclusion restriction is satisfied is controversial for many other seemingly exogenous instruments. For example, Angrist (1990) argues that draft lottery numbers are instruments for testing whether serving in Vietnam affects the earnings of men in the civilian sector because these numbers influence earnings purely through military service. However, Wooldridge (2002, p.88) argues that because civilian employers are more likely to invest in job training for employees who have high draft numbers, these numbers could also influence earnings through job training, which is unobservable.

We show that the standard *t*-test statistic is unreliable: even when the instrument is very close to being exogenous, the *t*-test grossly and unpredictably over-rejects or under-rejects the null that the endogenous regressor is insignificant, and the Anderson–Rubin test over-rejects the null. We prove these results in the limit and in small samples. And, to our knowledge, these are new theoretical results.

2. Inference using the standard test statistics

In this section we relax the assumption that instruments must be exogenous and introduce a definition of "near exogeneity." Suppose we want to check for whether not an institution, say property rights enforcement, influences long term growth in a sample of countries.[2] If we suspect that institutions are endogenous and we also believe that a linear specification is appropriate, we would estimate and compute test statistics for the following simple linear simultaneous equations model (Hausman, 1983; Phillips, 1983):

$$\mathbf{LRGr} = \beta_0 + \beta_1 \mathbf{INST} + \mathbf{u} \tag{1}$$

$$INST = \Pi_0 + Z\Pi_1 + V \tag{2}$$

Eq. (1) is the structural equation, where **LRGr** is an nx1 vector of long run growth, **INST** is an nx1 vector of institutions, and **u** is an nx1 vector of structural error terms that have zero mean and finite variance $\sigma_{\mathbf{u}}^2 < \infty$. Eq. (2) is the reduced form, $\boldsymbol{Z}$ is an nxk matrix of instruments and **V** is an nx1 vector of reduced form errors that have zero means and finite variance. $\sigma_{\mathbf{V}}^2 < \infty$. The error terms **u** and **V** may be correlated and *n* represents the number of countries. The parameters

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[1] Instruments that marginally satisfy this requirement are denoted weak and are the subject of a large and growing literature (see Staiger and Stock, 1997; Stock et al, 2002).

[2] We just consider one kind of institution and, hence, one endogenous variable for expositional simplicity. Our method also works for multiple endogenous variables. See Acemoglu and Johnson (2006) for an analysis of how instrumental variables can be used to identify how two endogenous institutions, property rights (measured by a survey of risk of expropriation) and efficiency of contracts (measured by an index of legal formalism), can affect long run growth.

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β_0, β_1, Π_0 *and* Π_1, are unknowns, and, for notational conventional, we denote $\beta=\{\beta_0, \beta_1\}$, $\Pi=\{\Pi_0, \Pi_1\}$. Other covariates, for example, population, latitude and education, can be added to the system in Eqs. (1) and (2) without loss of generality.[3]

In order to determine whether or not institutions matter, we estimate the unknown parameter β_1 and use test statistics to check whether $\beta_1=0$. To do this properly, we need valid instruments that are both relevant and exogenous. As previously discussed, relevant instruments are picked on the basis of a theoretical, institutional and/or historical argument, and are validated ex post by estimating the reduced form. The second criterion for validity is that instruments are exogenous, which implies they are orthogonal to the error term in the structural equation:

$$\text{Exogenous} \Rightarrow \ \text{Cov} \ \ \boldsymbol{Z'}_i\mathbf{u}_i = 0. \tag{3}$$

It is generally difficult, as we have previously argued, to find instruments that satisfy this strong condition. In particular, while these instruments influence long run growth in the structural equation primarily through institutions, they may also be weakly correlated with unobserved factors that can also influence long term growth. We model this potential small correlation as "nearly exogenous" which is a local to zero setup:

$$\text{Nearly Exogenous} \Rightarrow \ \text{Cov} \ \ \boldsymbol{Z'}_i\mathbf{u}_i = \mathbf{C}/\sqrt{n} \tag{4}$$

where **C** is an k * 1 vector of constants that is contained in compact set.

If we choose Cov $\boldsymbol{Z}_i'\mathbf{u}_i=\mathbf{C}$ to capture near exogeneity, then the test statistics always diverge in the limit. Thus, this assumption does not provide any guidance for finite sample behavior when there is some mild correlation between the instrument and error.

In what follows, small sample simulation methods are used to show that even a slight relaxation of the exogeneity assumption in Eq. (3) makes the standard test statistics unreliable. Suppose we employ the TSLS *t*-test to determine whether or not institutions matter. Denoting the H_0 and H_1 as the null and the alternative and $\beta_{1,\text{TSLS}}$ as the TSLS estimator of β_1, we use the *t*-statistic to test

$$\begin{aligned} &H_0 : \beta_1 = 0, \ \text{against} \\ &H_1 : \beta_1 \neq 0, \ \text{where the } t\text{−statistic is given by} \\ &t = \hat{\beta}_{1,\text{TSLS}}/\sqrt{a \ \ \text{var} \ \ \hat{\beta}_{1,\text{TSLS}}}. \end{aligned} \tag{5}$$

We generate i.i.d. data for the one instrument, the structural error term and reduced form, ($\boldsymbol{Z}$,**u**,**V**), from a joint normal distribution $N(0,\Lambda)$ and

$$\Lambda = \begin{pmatrix} 1 & \text{Cov} \ \ \boldsymbol{Z}_i\mathbf{u}_i & 0 \\ \text{Cov} \ \ \boldsymbol{Z}_i\mathbf{u}_i & 1 & \text{Cov} \ \ \mathbf{V}_i\mathbf{u}_i \\ 0 & \text{Cov} \ \ \mathbf{V}_i\mathbf{u}_i & 1 \end{pmatrix} \tag{6}$$

where Cov $\boldsymbol{Z}_i'\mathbf{u}_i$ measures the correlation between the instrument $\boldsymbol{Z}$ and the error term **u**, and Cov $\mathbf{V}_i\mathbf{u}_i$ measures the endogeneity of institutions, which is set to 0.25 in all simulations. When the i.i.d. data ($\boldsymbol{Z}$,**u**,**V**) are generated, we can derive the observations of **INST** and **LRGr** by using Eqs. (1) and (2) and specified true values of β_1 *and* Π_1. Based on the information of (**LRGr**, **INST**, $\boldsymbol{Z}$), we compute the *t*-statistic and then test whether the null of $\beta_1=0$ can be rejected at the 5% level by using the critical value 1.95. We replicate the simulation by 1000 times to derive the distribution of the *t*-statistic and calculate the actual rejection probability which is reported in Table 1.

Table 1 reports rates of right hand side and left hand false rejection when the instrument is more weakly correlated with the error term: Cov $\boldsymbol{Z}_i\mathbf{u}_i=0.06$ or -0.06 and illustrates that as the absolute value of the correlation decreases, the size problems of the two-sided *t*-test are mitigated. When the correlation is positive there is a 9.4% false rejection rate on the right hand side, a conservative 0.4% rate from the left hand side and an overall 9.8% false rejection rate. When, the correlation is negative, the rates of false rejection on the right hand and left hand sides are 0.6% and 7.2%, respectively, and the overall false rejection rate is 7.9%.

Table 1
Test statistics

Sample size = 100, and 1000 simulations
Truth is that institutions do not matter

Test statistic	Nominal 5% critical values	Cov $\boldsymbol{Z}_i'\mathbf{u}_i$	Actual rejection rate	Actual rejection rate (RHS)	Actual rejection rate (LHS)
t-statistic	±1.95	0.06	9.8%	9.4%	0.4%
t-statistic	±1.95	−0.06	7.9%	0.6%	7.2%
AR test	3.85	±0.06	9.4%	n.a.	n.a.
t-statistic	±1.95	0.10	19.4%	19.2%	0.2%
t-statistic	±1.95	−0.10	14.3%	0.3%	14.0%
AR test	3.85	±0.10	17.7%	n.a.	n.a.

Suppose we test the null against the alternative using Anderson–Rubin (Anderson and Rubin, 1949) test:[4]

$$\text{AR}(\beta_1 = 0) = \mathbf{LRGr}'\boldsymbol{P}_z\mathbf{LRGr}/(\mathbf{LRGr}'M_z\mathbf{LRGr})/(n-2) \tag{7}$$

Here, AR($\beta_1=0$) is the test statistic for the null, $\boldsymbol{P}_z=\boldsymbol{Z}(\boldsymbol{Z'Z})^{-1}\boldsymbol{Z}$ is the projection matrix and $M_z=1-\boldsymbol{P}_z$.

Table 1 illustrates that the small sample problems associated with the Anderson–Rubin test (for herein, denoted the AR test) are also diminished when the instrument is less endogenous. When the correlation decreases to 0.06, the AR test falsely rejects 9.4% of the time. Since it is not possible to calculate the absolute value of the correlation between the instruments and structural error, it is not possible to adjust for this small sample distortion and the AR test is also unreliable.

3. Large sample distributions

This section adds to the bad news: we show that the shifts in test-statistic distributions observed in the small sample simulations also hold in limit. For the rest of the paper, we generalize the simultaneous equation system Eqs. (1) and (2) to model a more general system with $m \geq 1$ endogenous explanatory variables, and $k \geq m$ instruments:

$$\mathbf{y} = \boldsymbol{Y}\beta + \mathbf{u} \tag{1*}$$

$$\boldsymbol{Y} = \boldsymbol{Z}\Pi + \boldsymbol{V} \tag{2*}$$

where **y** and $\boldsymbol{Y}$ are respectively nx1 vector and nxm matrix of endogenous explanatory variables, $\boldsymbol{Z}$ is an nxk matrix of instruments, **u** is an nx1 vector of structural errors, $\boldsymbol{V}$ is an nxm matrix of reduced form errors, and the errors have zero means and finite variance, and **u** and $\boldsymbol{V}$ are correlated with each other. As noted before, other exogenous covariates can be added to the system.

In the next theorem, we show that near exogeneity shifts the asymptotic distribution of the *t*-statistic to a normal distribution with non-zero mean.

Theorem 1. *Suppose that the instrument is nearly exogenous according to Eq.* (4)*, and the standard* Assumption 2 *in the* Appendix *holds. Then,*

$$t \xrightarrow{d} N\left[\sigma_{\mathbf{u}}^{-1}(\Pi'\boldsymbol{Q}_{zz}\Pi)^{-1/2}\Pi'\mathbf{C}, 1\right] \tag{8}$$

where $\sigma_{\mathbf{u}}$ *is the square root of* $\sigma^2_{\mathbf{u}}$, *and* $\mathbf{Q}_{zz}$ *is the second moment matrix of instruments.*

[3] By the Frisch–Waugh–Lovell Theorem, we can always project out these covariates and obtain the system in Eqs.(1) and (2) (see Davidson and McKinnon, 1993, p.19).

[4] We can generalize this test statistic to allow for multiple endogenous explanatory variables and at least as many instruments.

Proof. See the Appendix. □

According to Theorem 1, the mean of the distribution depends upon the parameter **C**, which, by Eq. (4), is related to the small correlation between structural error and instruments. When **C**=0 and the instruments are exogenous, the *t*-statistic converges to the standard normal distribution. When **C**>0 (given $\Pi>0$), the distribution shifts to the right. When **C**<0 (given $\Pi>0$), the distribution shifts to the left. Since we cannot consistently estimate **C** let alone know its sign, we cannot use this large sample theorem to improve inference.

The next theorem characterizes the impact of near exogeneity on the distribution of the AR test, which is now more generally defined from Eq. (7) for *k* instruments and *m* endogenous explanatory variables:

$$\mathrm{AR}(\beta_0)=(\mathbf{y}-\boldsymbol{Y}\beta_0)'\boldsymbol{P}_z(\mathbf{y}-\boldsymbol{Y}\beta_0)/(\mathbf{y}-\boldsymbol{Y}\beta_0)'M_z(\mathbf{y}-\boldsymbol{Y}\beta_0)/(n-k-m) \quad (7*)$$

We use this statistic to test H_0: $\beta=\beta_0$ against H_1: $\beta\neq\beta_0$ where β_0 is the true value.

Theorem 2. *Suppose that the instrument is nearly exogenous according to Eq.* (4)*, and the standard* Assumption 2 *in the* Appendix *holds. If the null hypothesis is* $\beta=\beta_0$*, then*

$$\mathrm{AR}(\beta_0)\xrightarrow{d}\chi^2_K(\varsigma) \quad (9)$$

where $\chi^2_K(\varsigma)$ *is a non-central chi-square distribution with k degrees of freedom and the non-centrality parameter* $\varsigma=\mathbf{C}'\Omega^{-1}\mathbf{C}/2$ *, where* $\Omega=\sigma^2_{\mathbf{u}}\mathbf{Q}_{zz}$.

Proof. See the Appendix. □

According to Theorem 2, the mean of the non-centrality parameter is quadratic in parameter **C**. Therefore, when **C**=0 the AR test converges to the centered chi-square distribution, and when **C**≠0 the distribution shifts to the right. Again, since we do not know **C**, we cannot use these theorems to obtain appropriate critical values. The convergence is uniform.

4. Conclusion

This article analyzes the limit theory when there are both weakly identified as well as nearly exogenous instruments. We show that Anderson–Rubin test is no longer asymptotically pivotal. In future research we consider how to remedy this problem by using a delete-d jackknife bootstrap procedure.

Appendix A

In the beginning of this Appendix, we first describe the near exogeneity assumption and some moment conditions that are required to obtain the theorems in the paper. Assumptions 1 and 2 are sufficient for Lemma 1, Theorem 1 and Theorem 2.

Assumption 1. Near exogeneity $E[\boldsymbol{Z}'_i\mathbf{u}_i]=\mathbf{C}/\sqrt{N}$, where **C** is a fixed $K\times 1$ vector.

Assumption 2. The following limits hold jointly when the sample size N converges to infinity:

(a) $(\mathbf{u}'\mathbf{u}/N,\boldsymbol{V}'\mathbf{u}/N,\boldsymbol{V}'\boldsymbol{V}/N)\xrightarrow{p}(\sigma^2_{\mathbf{u}},\Sigma_{V\mathbf{u}},\Sigma_{VV})$, where $\sigma^2_{\mathbf{u}}$, $\Sigma_{\mathbf{Vu}}$ and $\Sigma_{\mathbf{VV}}$ are respectively a 1×1 scalar, an $m\times 1$ vector and an $m\times m$ matrix.

(b) $\boldsymbol{Z}'\boldsymbol{Z}/N\xrightarrow{p}\boldsymbol{Q}_{ZZ}$ where $\boldsymbol{Q}_{ZZ}$ is a positive definite, finite $K\times K$ matrix.

(c) and

$$\left(\boldsymbol{Z}'\mathbf{u}/\sqrt{N},\boldsymbol{Z}'\boldsymbol{V}/\sqrt{N}\right)\xrightarrow{d}\left(\bar{\Psi}_{Z\mathbf{u}},\Psi_{ZV}\right)'$$

$$\begin{pmatrix}\bar{\Psi}_{Z\mathbf{u}}\\ \Psi_{ZV}\end{pmatrix}\equiv N\left[\begin{pmatrix}\mathbf{C}\\0\end{pmatrix},\Sigma\otimes\boldsymbol{Q}_{zz}\right]$$

where $\Sigma=\begin{pmatrix}\sigma^2_{\mathbf{u}} & \Sigma'_{V\mathbf{u}}\\ \Sigma_{V\mathbf{u}} & \Sigma_{VV}\end{pmatrix}$.

These convergences in Assumption 2 are not primitive assumptions but hold under weak primitive conditions. Parts (a) and (b) follow from the weak law of large numbers, and Part (c) follows from triangular arrays central limit theorem. Instead of a mean zero normal distribution in Staiger and Stock (1997), the $\bar{\Psi}_{\mathbf{Zu}}$ in (c) is a normal distribution with non-zero mean, which is a drift term **C** coming from the near exogeneity assumption. For any independent sequence $\boldsymbol{Z}'_i\mathbf{u}_i$, if $E[\boldsymbol{Z}'_i\mathbf{u}_i]^{2+\delta}<\Delta<\infty$ for some $\delta>0$ for all $i=1, 2, 3,\ldots, N$, then Liapunov's theorem leads to the limiting results in (c); see James Davidson (1994).

Lemma 1. *If Assumptions* 1 *and* 2 *hold for the model defined by Eqs.* (1*) and (2*)*, then the TSLS estimator* β_{TSLS} *is consistent and*

$$\sqrt{N}\left(\hat{\beta}_{\mathrm{TSLS}}-\beta_0\right)\xrightarrow{d}N\left((\Pi'\boldsymbol{Q}_{ZZ}\Pi)^{-1}\Pi'\mathbf{C},\sigma^2_{\mathbf{u}}(\Pi'\boldsymbol{Q}_{ZZ}\Pi)^{-1}\right)$$

where $\mathbf{u}'\mathbf{u}/N\xrightarrow{p}E(\mathbf{u}^2_i)=\sigma^2_{\mathbf{u}},\ \boldsymbol{Z}'\boldsymbol{Z}/N\xrightarrow{p}E(\boldsymbol{Z}'_i\boldsymbol{Z}_i)=\boldsymbol{Q}_{ZZ}$.

Proof of Lemma 1. We know that

$$\hat{\beta}_{\mathrm{TSLS}}=(\boldsymbol{Y}'\boldsymbol{P}_Z\boldsymbol{Y})^{-1}(\boldsymbol{Y}'\boldsymbol{P}_Z\mathbf{y}).$$

So we have

$$\sqrt{N}\left(\hat{\beta}_{\mathrm{TSLS}}-\beta_0\right)=\left[\left(\frac{\boldsymbol{Y}'\boldsymbol{Z}}{N}\right)\left(\frac{\boldsymbol{Z}'\boldsymbol{Z}}{N}\right)^{-1}\left(\frac{\boldsymbol{Z}'\boldsymbol{Y}}{N}\right)\right]^{-1}\left[\left(\frac{\boldsymbol{Y}'\boldsymbol{Z}}{N}\right)\left(\frac{\boldsymbol{Z}'\boldsymbol{Z}}{N}\right)^{-1}\left(\frac{\boldsymbol{Z}'\mathbf{u}}{\sqrt{N}}\right)\right].$$

By Assumption 2 and Eq. (2*), we can obtain that

$$\left[\left(\frac{\boldsymbol{Y}'\boldsymbol{Z}}{N}\right)\left(\frac{\boldsymbol{Z}'\boldsymbol{Z}}{N}\right)^{-1}\left(\frac{\boldsymbol{Z}'\boldsymbol{Y}}{N}\right)\right]^{-1}\xrightarrow{p}(\Pi'\boldsymbol{Q}_{ZZ}\Pi)^{-1}.$$

Now, we consider

$$\frac{\boldsymbol{Z}'\mathbf{u}}{\sqrt{N}}=\frac{1}{\sqrt{N}}\sum_{N}^{i=1}[\boldsymbol{Z}'_i\mathbf{u}_i-E(\boldsymbol{Z}'_i\mathbf{u}_i)]+\frac{1}{\sqrt{N}}\sum_{N}^{i=1}E(\boldsymbol{Z}'_i\mathbf{u}_i).$$

Combining Assumptions (1) and (2), we obtain

$$\frac{\boldsymbol{Z}'\mathbf{u}}{\sqrt{N}}\xrightarrow{d}N\left[\mathbf{C},\sigma^2_{\mathbf{u}}\boldsymbol{Q}_{ZZ}\right].$$

Then the result in the lemma follows directly. Q.E.D.

Lemma 1 summarizes the limiting results of the TSLS estimator under near exogeneity. The reason why we can obtain a consistent estimator under near exogeneity is because the correlation between instruments and structural errors shrinks toward zero asymptotically. When **C**=0, we can obtain the regular results of the TSLS estimator under the orthogonality condition. Instead of a normal distribution with a zero mean, near exogeneity can shift the distribution away from mean zero. The non-zero mean depends on an unknown local to zero parameter **C** which is impossible to be estimated consistently (Donald W.K. Andrews, 2000).

Proof of Theorem 1. The result in the theorem directly follows from Lemma 1. Q.E.D.

Proof of Theorem 2. The Anderson–Rubin test is given by

$$\mathrm{AR}(\beta_0)=(\mathbf{y}-\boldsymbol{Y}\beta_0)'\boldsymbol{P}_Z(\mathbf{y}-\boldsymbol{Y}\beta_0)/\frac{1}{N-K}(\mathbf{y}-\boldsymbol{Y}\beta_0)'M_Z(\mathbf{y}-\boldsymbol{Y}\beta_0).$$

We first observe that

$$(\mathbf{y}-\boldsymbol{Y}\beta_0)'\boldsymbol{P}_Z(\mathbf{y}-\boldsymbol{Y}\beta_0)=\mathbf{u}'\boldsymbol{P}_Z\mathbf{u}.$$

Define $\xi=\boldsymbol{P}_Z^{1/2}\mathbf{u}$. Parts (b) and (c) in Assumption 2 implies:

$$\xi\xrightarrow{d}\boldsymbol{Q}_{ZZ}^{-1/2-}\Psi_{Z\mathbf{u}}\sim N\left(\boldsymbol{Q}_{ZZ}^{-1/2}\mathbf{C},\sigma^2_{\mathbf{u}}\right).$$

Next, note that

$$\begin{aligned}
&\frac{1}{N-K}(\mathbf{y}-\boldsymbol{Y}\beta_0)'M_Z(\mathbf{y}-\boldsymbol{Y}\beta_0)\\
&=\frac{1}{N-K}\mathbf{u}'M_Z\mathbf{u}\\
&=\frac{1}{N-K}\mathbf{u}'\mathbf{u}-\frac{1}{N-K}\mathbf{u}'_{PZ}\mathbf{u}
\end{aligned}$$

By part (a) in Assumption 2, the first term converges in probability to $\sigma_{\mathbf{u}}^2$, and the last term tends to zero. We have

$$\frac{1}{N-K}(\mathbf{y}-\boldsymbol{Y}\beta_0)'M_Z(\mathbf{y}-\beta_0)\xrightarrow{p}\sigma_{\mathbf{u}}^2.$$

So

$\mathrm{AR}(\beta_0)\xrightarrow{d}\chi_K^2\left(\mathbf{C}'\Omega^{-1}\mathbf{C}/2\right)$, where
$\Omega=\sigma_{\mathbf{u}}^2\boldsymbol{Q}_{ZZ}$ Q.E.D.

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