Optica Applicata, Vol. XXXV, No. 1, 2005

Application of Wigner transform for characterization of aberrated laser beams*

JAROSŁAW JAGUŚ, JAN K. JABCZYŃSKI, WALDEMAR ŻENDZIAN, JACEK KWIATKOWSKI

Institute of Optoelectronics, Military University of Technology, ul. Kaliskiego 2,
00-908 Warszawa, Poland; e-mail: jjabczynski@wat.edu.pl

The slit scan method was implemented for registration of intensity profiles along the caustics of a laser beam. The inverse Radon transform of spatially normalized intensity profiles enables direct computation of Wigner transform of real laser beam. The Rayleigh range, divergence angle, beam quality factor, global degree of coherence can be determined in such a simple way. A procedure enabling derivation of the shape of aberrated wavefornt and spherical aberration content was elaborated. This method was applied for investigation of the aberrated laser beams generated by cw and pulsed diode pumped lasers.

Keywords: laser beams, laser optics, beam quality, Wigner transform, aberrations.

1. Introduction

Quantitative characterization of spatial structure of a laser beam has been of vital interest to opticians and laser physicists since the advent of lasers. Because of inherent uncertainty of parameters of such a type of light source caused by its spatial and temporal fluctuations as well as the state of coherence and polarization, it has been an attractive subject of intensive theoretical research as well as measurement and experimental works. The well established simplest parameter describing the spatial properties of laser radiation, *i.e.*, the beam propagation factor M^2 introduced by SIEGMAN [1], was accepted by ISO [2] as a measure of beam quality. The measurements of M^2 parameter can lead to some ambiguities, especially for untypical, asymmetric beams, moreover it can be easily shown that quite different light beams can have the same value of M^2 parameter. Thus, additional parameters describing the state of coherence and wavefront aberrations should be defined.

To completely describe the properties of partially coherent light, the formalism of mutual coherence function or cross-spectral density function can be applied [3]. The

*This work was presented at XIV Slovak-Czech-Polish Optical Conference in Nitra (Slovakia), 13–17 September 2004.

alternative approach (however, basically connected with the previous ones via Fourier transform) is known as a Wigner distribution method (WDM), see [4–8]. The main advantage of WDM is the simultaneous access to intensity and phase distributions in the far field (defined over *u*-angular frequency space) and in the near field (defined over *x*-spatial coordinate space). Moreover, there exist at least two, well-established experimental procedures [6, 7] enabling direct access to Wigner distribution (WD) from experimental data. In the first one, Sagnac interferometer with inversion of field by means of Dove's prism is applied [6], Wigner signal being the autocorrelation of incidence electric field is collected on a wide area detector for the given *x*-spatial and *u*-angular positions of input mirror. In the second method, a typical set-up for measurements of intensity distributions in the caustics can be applied. Basically, both methods require the measurements of intensity distributions in 2D space for 1D geometry of incident beam and 4D for 2D geometry. It was shown by EPPICH and RENG [7] that the Wigner distribution can be found as the inverse Radon transform of intensity distribution in the caustics. Knowing Wigner distribution of a laser beam, the beam quality parameter M^2, the spatial coherence degree K^2 and the coherence length can be calculated [9, 10]. The WDM can also be applied to derive deterministic wavefront aberrations of a laser beam [8, 11, 12].

The goal of this work is to implement the Wigner transform for characterization of aberrated beams generated by diode pumped lasers. The main properties of Wigner transform and method of wavefront retrieval are described in Sec. 2. In Section 3, the experimental set-up for intensity measurements in caustics, based on the slit scan method is presented. The procedure of WDM was tested on several beams generated by cw and *Q*-switched diode pumped lasers operating at 1064-nm and 1340-nm wavelengths.

2. Theory

2.1. Properties of the Wigner transform

For simplicity, our analysis will be limited to 1D geometry. The Wigner distribution (WD) function $F(x, u)$ for partially coherent beam is defined as follows:

$$F(u, x) = \lambda^{-1}\int \Gamma(x, s)\exp(-iksu)\mathrm{d}s \qquad (1)$$

where $\Gamma(x, s)$ is the mutual coherence function defined in the following way:

$$\Gamma(x, s) = \langle E(x+s/2)\,E^*(x-s/2)\rangle = \langle E(x_1)\,E^*(x_2)\rangle \qquad (2)$$

$\langle ... \rangle$ is the statistical averaging over time or ensemble, $E(x)$ – the amplitude of electric field at point x, s – the correlation spatial variable ($x_1 = x + s/2$, $x_2 = x - s/2$), u – the angular frequency, $k = 2\pi/\lambda$ is the wavenumber and λ is the wavelength. A very useful property of WD is the simple transformation rule in the first order systems described by *ABCD* matrix:

$$F_{\mathrm{out}}(x, u) = F_{\mathrm{inp}}(Dx - Bu, -Cx + Au). \tag{3}$$

Thus, to completely describe propagation of partially coherent beam it is necessary and sufficiently to know the WD in one, arbitrarily chosen, incidence plane. The projections of WD on x and u subspaces give the intensities in near and far fields, respectively:

$$\begin{aligned} I_{\mathrm{nf}}(x) &= \int_{-\infty}^{\infty} F(x, u)\,\mathrm{d}u, \\ I_{\mathrm{ff}}(u) &= \int_{-\infty}^{\infty} F(x, u)\,\mathrm{d}x. \end{aligned} \tag{4}$$

The global coherence degree K^2 can be defined as follows:

$$K^2 = P^{-2}\iint |\Gamma(x, s)|^2\,\mathrm{d}x\,\mathrm{d}s \tag{5}$$

where P is the beam power given by $P = \int |\Gamma(x, 0)|\,\mathrm{d}x = \int I(x)\,\mathrm{d}x$. The parameter M^2 can be defined in WDM as a product of beam radii in near and far fields [1]:

$$M^2 = \frac{\pi}{\lambda} 4\sigma_x \sigma_u, \qquad \sigma_x = \sqrt{\langle x - \langle x \rangle \rangle^2}, \qquad \sigma_u = \sqrt{\langle u - \langle u \rangle \rangle^2} \tag{6}$$

where $\langle x^n \rangle = P^{-1}\int x^n I(x)\,\mathrm{d}x$ is the n-th moment of intensity distribution.

2.1.1. Properties of Gauss–Schell model beam

For Gauss–Schell model (GSM) of partially coherent beam the mutual coherence function is given as follows:

$$\Gamma_{\mathrm{GS}}(x, s) = \exp\left[-2\left(\frac{x}{W_{\mathrm{GS}}}\right)^2 - \frac{1}{2}\left(\frac{s}{\rho_{\mathrm{GS}}}\right)^2\right] \tag{7}$$

where W_{GS} is the radius of beam, ρ_{GS} is the coherence radius of beam. The parameters M^2 and K^2 for GSM beam are given as follows:

$$M^2_{\mathrm{GS}} = \frac{W_{\mathrm{GS}}}{\rho_{\mathrm{GS}}}, \qquad K^2_{\mathrm{GS}} = \frac{\rho_{\mathrm{GS}}}{W_{\mathrm{GS}}}. \tag{8}$$

The GSM beam is an "eigen"-function in transformation in the first order systems. The propagation law of GSM beam in free space is given by:

$$W_{\mathrm{GS}}^2(z) = W_{0,\,\mathrm{GS}}^2\left(1+\left(\frac{z}{Z_{\mathrm{R,\,GS}}}\right)^2\right),$$

$$\rho_{\mathrm{GS}}^2(z) = \rho_{0,\,\mathrm{GS}}^2\left(1+\left(\frac{z}{Z_{\mathrm{R,\,GS}}}\right)^2\right) \tag{9}$$

where $W_{0,\,\mathrm{GS}}$ and $\rho_{0,\,\mathrm{GS}}$ are the beam and coherence radii in waist plane, respectively, $Z_{\mathrm{R,\,GS}}$ is the Rayleigh range given by:

$$Z_{\mathrm{R,\,GS}} = \frac{W_{0,\,\mathrm{GS}}}{\theta_{\mathrm{GS}}} = \frac{\pi W_{0,\,\mathrm{GS}}^2}{M_{\mathrm{GS}}^2\lambda} \tag{10}$$

where θ_{GS} is the divergence half angle of GSM beam ($\theta_{\mathrm{GS}} = M_{\mathrm{GS}}^2\lambda/\pi W_{0,\,\mathrm{GS}}$). The WD function for GSM beam is given as:

$$F_{\mathrm{GS}}(x, u) = \exp\left[-2\left(\frac{x}{W_{\mathrm{GS}}}\right)^2 - 2\left(\frac{u}{\theta_{\mathrm{GS}}}\right)^2\right]. \tag{11}$$

For fully coherent GSM beam, *i.e.*, $M_{\mathrm{GS}}^2 = 1$ we have the formulae describing the properties of a Gaussian beam in terms of WDM. The GSM beam minimizes the product of beam quality M^2 and coherence degree parameter K^2:

$$M_{\mathrm{GS}}^2 K_{\mathrm{GS}}^2 = 1. \tag{12}$$

2.1.2. Wavefront analysis in WDM

For real laser beams the following "laser beam optics principle" can be formulated [8]:

$$M^2 K^2 \geq 1. \tag{13}$$

The laser source having for the given M^2 the lowest value of the coherence parameter K^2 is the most "randomly" ordered and it has the smoothest profiles. In such a case, the radiation is close to GSM beam and no deterministic phase deviations occur. On the other hand, when the M^2K^2 product is high, this means that some level of deterministic amplitude or phase modulation exists, which can be basically removed. Thus, from basic as well as practical points of view, it is important to determine the deterministic wavefront aberration content for the given laser beam. The methods of wavefront measurements can be divided, with respect to the principle, into two groups: interferometric and direct methods. The one of the well established representatives of the latter group, Hartmann–Shack method (see, *e.g.*, [13]) gives the simultaneous determination of phase and amplitude distribution for a single shot beam. The WDM offers an alternative way to wavefront analysis [8, 11, 12]. The basic principles of this approach, valid for 1D geometry of an incident beam are presented below. The transversal Poynting vector component $S_t(x)$ for the given WD is defined as follows:

$$S_t(x) = \int F(x, u) u \, du. \tag{14}$$

The ray aberration, *i.e.*, the angle of ray with respect to the propagation axis $U(x) \cong \sin(U(x))$ can be found as a ratio of the Poynting vector $S_t(x)$ to intensity in the near field $I_{\mathrm{nf}}(x)$ as follows:

$$U(x) = \frac{S_t(x)}{I_{\mathrm{nf}}(x)} = \frac{\int F(x, u) u \, du}{\int F(x, u) \, du.}. \tag{15}$$

Knowing the ray aberration vector $U(x)$ we can calculate (see, *e.g.*, [14]) small angle approximation of the wavefront aberration $\phi_{\mathrm{aber}}(x)$ as follows:

$$U(x) \cong \frac{\partial \phi_{\mathrm{aber}}}{\partial x}, \qquad \phi_{\mathrm{aber}}(x) \cong \int_{-\infty}^{x} U(t) \, dt. \tag{16}$$

The feasibility of this approach was examined for a laser beam with *a priori* known aberration by NEUBERT *et al.* [11, 12].

2.2. Eppich's method of Wigner transform measurements

Firstly, we have to note that the analysis presented below is valid for 1D geometry (*i.e.*, axially symmetric beam), however, the generalization to 2D geometry is straightforward [8]. The main concept of Eppich's method consists in application of the properties of Fourier and Radon transforms. He has shown [7] that the WD is the inverse Radon transform of specifically transformed 1D intensity distributions collected for several z_k locations in the caustics of a laser beam. The main idea of the procedure is presented in Fig. 1. In the first step, the 1D intensity profiles as functions of x-coordinate are measured for several z_k locations along the propagation axis in the vicinity of caustics. The effective parameters of the beam, *i.e.*, Rayleigh range, waist location and the M^2 parameter are calculated according to ISO procedure [2]. For the set of intensity plots, the transformation to Gouy's space is realized as follows:

$$I(x; z_k) \to \tilde{I}(\tilde{x}, \alpha_k), \quad \tilde{x} = \frac{x}{w_k}, \quad \alpha_k = \operatorname{atan}\left(\frac{z_k - z_0}{Z_R}\right), \quad w_k = \frac{w_0}{\cos \alpha_k} \tag{17}$$

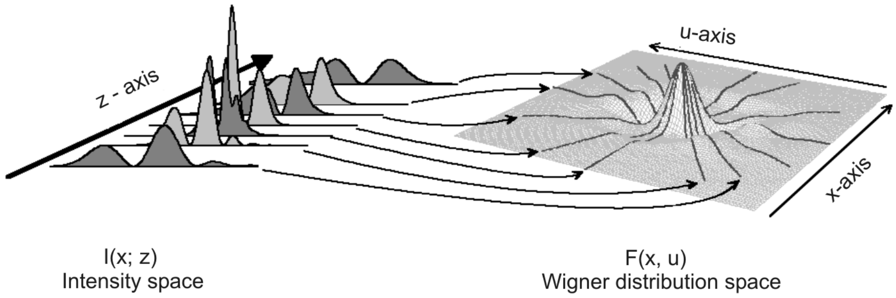

Fig. 1. Idea of Wigner transform derivation from intensity in caustics measurements.

where Z_R is the Rayleigh range, z_0 is the waist location, w_0 is the beam radius in the waist plane. It can be shown that the *k*-th normalized intensity plot corresponds to the WD radial section in direction inclined at an angle α_k to the *x*-axis (see Fig. 1). Thus, we have direct access to the values of WD data points defined in cylindrical system of coordinates. The return to the Cartesian system of coordinates (x, u) is performed by means of the inverse Radon transform. Because of numerical implementation of inverse Radon transform algorithm, the intensity data rows in a normalized Gouy's space have to be equidistant with respect to Gouy's angles α_k. Thus, the appropriate values of z_k locations should be chosen in the process of intensity registrations, or the additional interpolation in Gouy's space has to be done. Let us note that we have no access to intensity in the far field ($z/Z_R \to \infty$ or $\alpha \to 90°$) without transformation through additional lens. It was found in experimental practice that to ensure the sufficient accuracy in WD calculations, the number of z_k sections should be greater than 20, and the range of Gouy's angles should be at least 135° corresponding to the range of $\pm 3Z_R$ in the distance along caustics.

3. Experiment

3.1. Laboratory set-up for WDM

The laboratory set-up is described in detail in paper [15]. Some brief information is given below. To ensure a satisfactory accuracy and reasonable size of laboratory set-up, the laser beam under examination was focused to the 0.2–0.5 mm width applying thin, ideal lens of long focal length (typically, f = 300–500 mm). The beam widths which we have to measure ranged from 0.2 up to 3 mm, the length of *z*-scan was of a few dozens cm. We have decided to apply a slit scan method to measure the 1D intensity plots, assuming axial symmetry of the beam examined. The slit with a variable width (10–20 μm) attached to large area detector was moved across the beam by means of a step motor with 2.5 μm resolution. The digitized signal with 12-bit resolution was sent via 841-Optel controller to PC computer. The knife edge definition of a beam width with 10% clip level was applied to find the effective parameters of convergent laser beam [2]. The typical values of Rayleigh ranges were of a few dozens mm. The numerical procedure of WDM for experimental (as well as theoretical) series of 1D intensity data vectors was implemented in *MATLAB* v.5.3.

3.2. Measurements of laser beams

The main task of WDM set-up was to examine parameters of the beams generated by diode pumped lasers. We have tested it at 1064-nm and 1340-nm wavelengths for cw and pulsed regimes of operation. Two examples of the near diffraction limited but aberrated laser beams are shown in Figs. 2, 3. The left contour map corresponds to 2D intensity distributions in caustics as functions of horizontal x-axis and vertical z-axis. The middle picture presents the same beam after normalization and transformation to Gouy's space: x – horizontal axis, α – Gouy's angle vertical axis, the right-hand picture

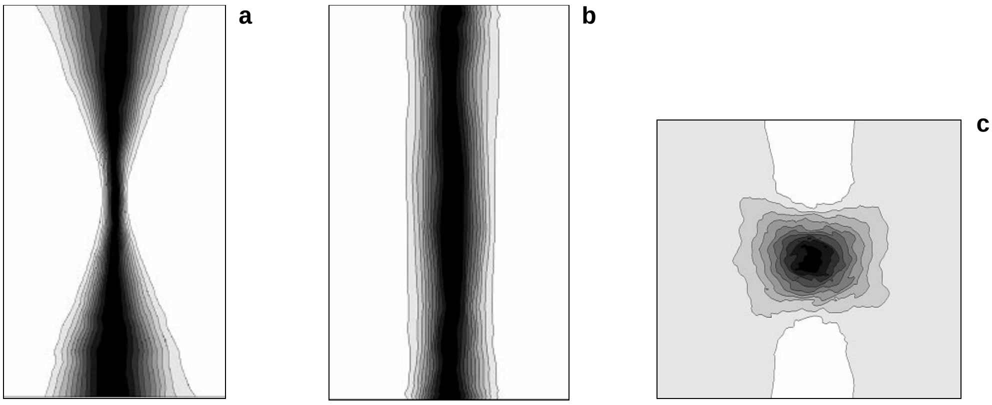

Fig. 2. Intensity plot $I(x{:}z_k)$ in the caustics of laser beam, $M^2 = 1.16$ (**a**). Normalized intensity plot $I(x,\ \alpha_k)$ of laser beam in figure **a** (**b**). Wigner distribution $F(x,\ u)$ of $I(x{:}z_k)$; inverse Radon transform of $I(x,\ \alpha_k)$ (**c**).

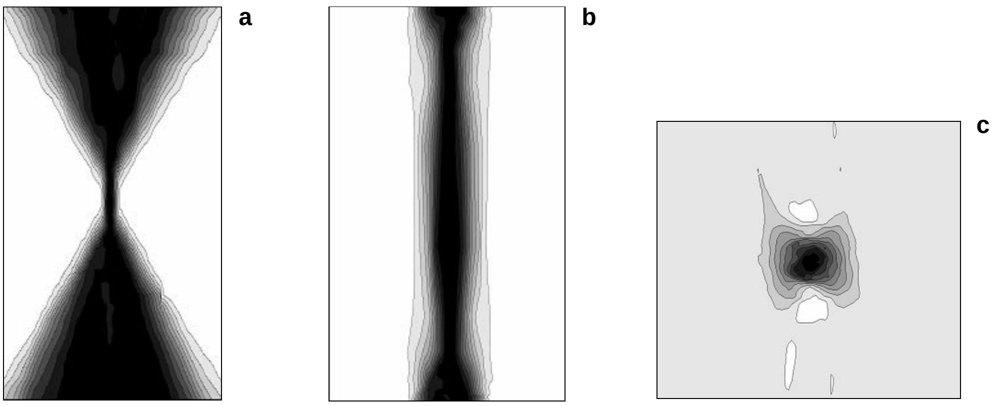

Fig. 3. Intensity plots $I(x{:}z_k)$ in the caustics of laser beam, $M^2 = 1.52$ (**a**). Normalized intensity plot $I(x,\ \alpha_k)$ of laser beam in figure **a** (**b**). Wigner distribution $F(x,\ u)$ of $I(x{:}z_k)$; inverse Radon transform of $I(x,\ \alpha_k)$ (**c**).

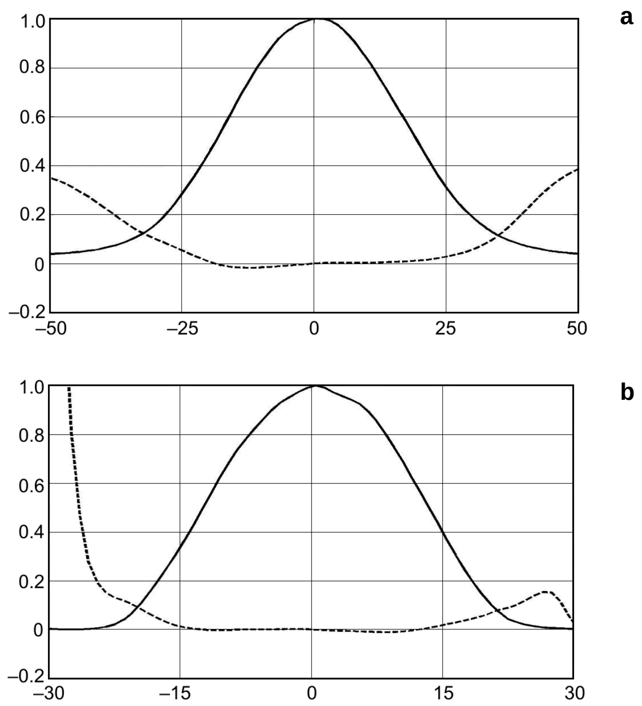

Fig. 4. Near field intensity (solid line), wavefront aberration (dashed line) *vs*. *x*, for the same beam as in: Fig. 2 (**a**) and in Fig. 3 (**b**).

presents the Wigner distribution of this beam (vertical axis corresponds to *u*-axis, horizontal to *x*-axis). The laser beam ($M^2 = 1.16$, $w_0 = 0.105$ mm, $Z_R = 28.2$ mm) generated by Nd:YVO_4 laser in *V*-type cavity at pump power $P_p = 6.2$ W is presented in Fig. 2**a**–**c**. A different laser beam ($M^2 = 1.52$, $w_0 = 0.082$ mm, $Z_R = 13.9$ mm), generated by the same laser at 18-W pump power is presented in Fig. 3**a**–**c**. The corresponding plots of intensity and wavefront aberration are given in Fig. 4**a**, **b**. As a result of negative or negligible values of the calculated near field intensity, the values of ray and wavefornt aberrations can be undetermined at certain points in the wings of a beam. A reliable method of a wavefront aberration derivation in WDM requires higher accuracy in measurements and improvement in inverse Radon transform algorithm.

4. Conclusions

The presented method and experimental set-up enable qualitative and quantitative characterization of aberrated laser beams. It was tested successfully on several beams generated by diode pumped lasers. The shape of Wigner distribution can give some intuitive information on the deviation of the beam examined apart from Gauss–Schell model. The wavefront analysis can lead to some ambiguities and requires further investigation. The WDM can offer an additional valuable tool for characterization of laser beams.

Acknowledgements – This work was supported by the Polish Committee for Scientific Research (KBN) under the project 4T11B02724, PBZ-MIN-009/T11/2003. We would like to thank Dr. Z. Zawadzki for his fruitful discussion on the results.

References

[1] SIEGMAN A., *New developments in laser resonators*, Proceedings of the SPIE **1224**, 1990, pp. 2–14.

[2] International Organization for Standardization ISO/FDIS 11146, ISO document ISO/TC172/SC9/WG1, ISO Geneva 1999.

[3] MANDEL L., WOLF E., *Optical Coherence and Quantum Optics*, 2-nd Ed., Cambridge University Press, England 1994.

[4] WIGNER E., *On the quantum correction for thermodynamic equilibrium*, Physical Review **40**, 1932, pp. 749–59.

[5] BAASTIANS M.J., *Wigner distribution function and its application to first-order optics*, Journal of the Optical Society of America **69**(12), 1979, pp. 1710–16.

[6] MUKAMEI E., BANASZEK K., WAMSLEY I.A., DORRER C., *Direct measurement of the spatial Wigner function with area-integrated detection*, Optics Letters **28**(15), 2003, 1317–19.

[7] EPPICH B., RENG N., *Measurement of the Wigner distribution function based on the inverse Radon transformation*, Proceedings of the SPIE **2375**, 1995, pp. 261–8.

[8] EPPICH B., JOHANSSON S., LAABS H., WEBER H., *Measuring laser beam parameters, phase and spatial coherence using the Wigner function*, Proceedings of the SPIE **3930**, 2000, pp. 76–86.

[9] EPPICH B., *Definition, meaning and measurement of coherence parameters*, Proceedings of the SPIE **4270**, 2001, pp. 71–9.

[10] EPPICH B., MANN G., WEBER H., *Spatial coherence: comparison of interferometric and non-interferometric measurements*, Proceedings of the SPIE **4969**, 2003, pp. 137–48.

[11] NEUBERT B.J., HUBER G., SCHARFE W.-D., [In] *Instruments and Standard Test Procedures for Laser Beam and Optics Characterization*, 2003, p. 44.

[12] NEUBERT B.J., *Measurements of the Wigner Distribution of Aberrated and Partially Coherent Laser Beams*, Cuvillier Verlag, Goettingen 2004.

[13] SCHAFER B., MANN K., *Determination of beam parameters and coherence properties of laser radiation by use of an extended Hartmann-Shack wave-front sensor*, Applied Optics **41**(15), 2002, pp. 2809–17.

[14] BORN M., WOLF E., *Principles of Optics*, Pergamon Press, Oxford 1965.

[15] JAGUŚ J., M.Sc. Thesis, Military University of Technology, Warszawa 2004 (in Polish).

Received September 29, 2004
in revised form November 23, 2004