

## **Analysis of activation of active double-clad optical fibers**

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The paper presents the analysis of double-clad optical fibers activation, used in fiber laser construction. The main focus was the research related to active fiber geometry influence on the value of an effective absorption efficiency coefficient – the parameter which is one of the main determinants of the total fiber laser efficiency. The whole analysis was conducted in geometrical optics approximation and wave-optical approach. The computer programs studied are a very helpful tool in selection of optimal parameters of active double-clad optical fibers.

Keywords: high power fiber laser, double-clad optical fiber, pump absorption efficiency.

### **1. Introduction**

Optical engineering, making use of fiber lasers and amplifiers, gains increasingly wide applications in many disciplines of life, science and technology [1]. The dynamic development of fiber lasers, observed at the turn of the last fifteen years, has been connected with the significant progress in manufacturing optical fibers. They also owe their existence to the rapid development of high-power semiconductor laser diodes which ensure effective pumping. Currently, *e.g.*, Limo Lissotschenko Mikrooptik Ltd offers single laser diode modules delivering even 200 W of cw output power commercially [2]. The turning point of the development of active optical fiber devices was the specific double-clad optical fiber construction [3, 4]. This construction, using high-power laser diodes, guaranteed the development of high-power fiber lasers and amplifiers.

The active double-clad fiber consists of:

- a rare-earth doped core (small in diameter), which ensures propagation of waves (usually in basic mode  $TEM_{00}$ ),
- an inner clad (directly sticking to the core),
- a polymer outer layer (outer clad).

The glass outer layer is used both as a layer limiting the laser radiation inside a single-mode core and a multimode optical wave-guide (with high numerical aperture)

to ensure propagation of the pump light. The pump radiation is propagated in the inner clad. Every time the pump light crosses the core of the fiber it is absorbed and thereby the active dopant of the core is activated.

Such a system has two fundamental advantages: firstly, it does not require precise focusing of the radiation of pump sources and secondly, the active core is pumped homogeneously along the whole length of the fiber. Initially, this design had circular symmetry. However, after some time scientists departed from the circular symmetry starting to use D-shaped or rectangle-shaped geometry. This change causes purposeful disturbance of the distribution of pump radiation inside the fiber. This leads to more effective activation of the core [5]. Such a solution also has many advantages, among the most important ones are: the simplicity of the system and easy launching of a pump beam into a fiber.

## 2. Pump power absorption efficiency in double-clad optical fibers

The idea of fiber lasers operation and their typical constructions have been widely described in literature, *e.g.*, [3, 6–8]. However, according to our best knowledge, there is a lack of generally accepted theory concerning pump power propagation in active double-clad optical fibers. In connection with the foregoing, the calculation and simulation methods concerning absorption efficiency of fibers with different shapes of inner clad cross-section will be proposed.

Pump power absorption mechanism of individual modes by the active core is a very important issue for power scaling of double-clad fiber lasers. Literature, *e.g.* [9, 10], reports that the use of double-clad fibers allows us to launch relatively high pumping power into the fiber, but the efficiency of energy delivery into an active ions-doped core is rather low. Moreover, the fundamental influence on homogeneity and efficiency of an active core activation has the shape of inner clad cross-section. For fibers of circular symmetry only 10–30% of pump power are delivered into the core, independently on its length [11, 12]. The weak absorption of helical modes of pump light (which does not cross an active core) is responsible for that. However, the efficiency rapidly increases if symmetry of the fiber cladding is not circular. Rectangular and D-shaped claddings are most efficient [13]. For these fibers almost full pumping power could be absorbed, and an effective coefficient of pump radiation absorption can be expressed as [14]:

$$\alpha_{\text{eff}} \approx \alpha \frac{A_{\text{core}}}{A_{\text{clad}}} \quad (1)$$

where  $\alpha$  is the absorption coefficient for the core region,  $A_{\text{core}}$  and  $A_{\text{clad}}$  are the area of the core and cladding, respectively.

Equation (1) shows that pump energy transmission from cladding modes to an active dopant of the core is only dependent on proportions between individual dimensions of the core and the inner clad. Equation (1) is an empirical and approximate expression and therefore, in order to find an expression describing the value of an effective absorption efficiency coefficient, special numerical algorithms were worked out. The value of effective absorption efficiency can be found by means of:

- geometrical optics approximation,
- wave-optical approximation.

The shortcoming of the first method is high complexity of numerical calculations and longer time of computer simulation. However, by means of it (using suitable assumptions), it is possible to analyse fibers with different shapes of inner clad cross-section. The second method is less complicated numerically. The main problem here is focused on the solution of the characteristic equation of a fiber. However, by means of it, it is possible to analyse effectively only the fibers for which the set of eigenfunctions is known – practically there are only fibers with circular symmetry.

### 3. Geometrical optics approximation

#### 3.1. Method description

The basic problem of a theory concerning propagation in optical fibers consists in describing the mechanism of transmission by which the energy of the electromagnetic field is bounded to the dielectric guiding structure. The easiest way of describing this issue is based on geometrical optics approach, where propagation of light is approximated by means of rays. In a particular case of propagation in homogeneous mediums these rays consist of intervals of straight lines. In fact, such a description is only an approximation, the more accurate, the smaller value of ratio of the pump radiation wavelength considered to the transverse dimensions of the structure in which the pump light propagates.

Such an approach seems to be correct in case of double-clad optical fibers. The following assumptions support it:

- a typical active fiber length does not exceed 50 m. For such a fiber length there is no yet fully stabilized mode distributions of propagating (in multimode inner clad) pump radiation. It is also assumed that energy transfer does not exist between modes;
- in case of double-clad optical fibers, a diameter of an active core ranges from 5 to 7  $\mu\text{m}$  while the diameter of an inner clad is 200–400  $\mu\text{m}$ . The difference between these dimensions makes it possible to neglect (from the point of view of propagation mechanism in the inner clad) the existence of the active core inside the fiber construction. The number of ray transitions through the core area is proportional to absorbed pump power in this region;
- the lifetime of particles activated to the upper laser level, *e.g.*, for neodymium dopant, is 0.3–0.9 ms [15]. Through that period of time the pump light is capable of

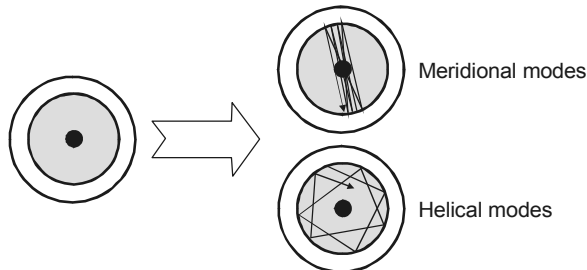


Fig. 1. Diagrammatic representation of modes in double-clad optical fiber (geometrical approach).

travelling a distance over 60 km. For a typical fiber length (up to 50 m) the laser action will appear later than the time of pump absorption in this section. Therefore, problems of laser action, in this case, can be omitted.

Taking into account the above-mentioned assumptions, the analysis of activation of double-clad fibers for different inner clad cross-sections is sufficiently justified.

Two kinds of modes (rays in geometrical optics approach) exist in an optical fiber: meridional modes and helical modes, also called slanted modes (see Fig. 1). Meridional rays are characterized by the fact that they cross the centre of symmetry of an optical fiber with a circle-shaped inner clad, therefore they can be absorbed by active dopant ions of the centrally located core. However, helical modes propagate along the fiber and they do not cross the centre of symmetry of an optical fiber and thereby they are not absorbed by the core dopant. Such behaviour of helical rays is very unfavourable because it decreases effective pump absorption in case of circular double-clad fibers and forces the use of longer pieces of fibers. It rises the costs of the whole fiber laser system.

Ray propagation along the fiber can be presented by means of analytical expressions. However, in order to carry out the full analysis of radiation distribution in a fiber, it is necessary to consider a few hundred of rays which reflect hundreds or even thousands times inside the inner clad. Hence, analyzing pump propagation in fibers with noncircular symmetry, the need of the use of numerical methods and algorithms is indispensable.

Based on the above comments, a computer program (written in Borland C++ program language) allowing us to visualize a light ray path was developed. The results obtained by means of that cannot be used for quantitative analysis of the issue, but they are useful for qualitative analysis of light propagation in fibers discussed. This program visualizes the path of light rays along the optical fiber and the final result is a graphic presentation of light ray trajectory in a cross-section plane of a fiber. The user sees a crosswise fiber core, he has a possibility to determine the angle of rotation of the fiber and the input angle of pump radiation (angle between the plane, including axis of rotation and the first reflected ray) (Fig. 2).

The user can also adjust the number of pump light reflections in the fiber which simulates the change of the fiber length.

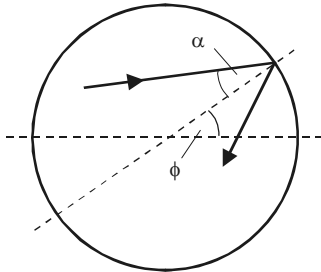


Fig. 2. Illustration of location of angles defined in computer program for the case of geometrical method.

On the basis of the input data the next  $\phi$  and  $\alpha$  angles are calculated. For a fiber with circular symmetry  $\alpha$  angle is constant, but  $\phi$  angle changes according to the following relation:

$$\phi_n = 180^\circ + \phi_p - 2\alpha \tag{2}$$

where:  $\phi_p$  – the value of  $\phi$  in the previous program step,  $\phi_n$  – the new value of  $\phi$  angle.

In fibers with the inner clad cross-section in the form of square and rectangle the relationship between the angles defined is much more complicated. The  $\phi$  and  $\alpha$  angles are different every time and do not only depend on its previous value, but also on which wall of the fiber the pump ray was reflected. Angles computation algorithm for fibers with D-shaped inner clad is very similar to that applied for cylindrical fibers. The reflection from cutting plane of the fiber is additionally taken into consideration here.

The computer program algorithm is shown in Fig. 3. It consists of two parts. The first one visualizes the trajectory of light rays in all shapes defined and for chosen by the user  $\phi$  and  $\alpha$  angles. In the second part of the program, fiber cross-section with the shape defined by the user was split into elementary cells (this division resulted

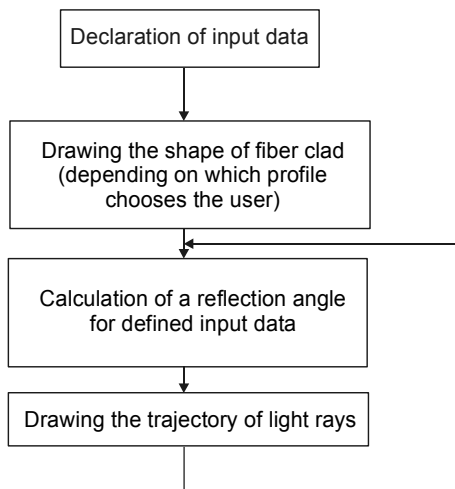


Fig. 3. Algorithm of computer program visualizing light ray trajectory along the fiber.

from the cross-section area of the fiber and its shape). Each elementary cell has its equivalent in special designed representation matrix. The light ray crossing the cell causes an increase in the value in its corresponding element of representation matrix. The information about the number of ray crossing through the particular cell enables us to calculate the probability of ray transitions through the elementary cell, which corresponds the ray transition through the active core. Making use of the information about the core dimensions and an incidence angle of particular rays onto the input plane of the fiber, the computer program may calculate absorption efficiency of pump rays in the core. Thanks to that, it is possible to determine which inner clad shapes are better as far as applications in fiber lasers and amplifiers are concerned.

### 3.2. Simulation results

Based on the program mentioned in the prior section, the following pump radiation distribution in a fiber were obtained (Fig. 4).

It can be easily seen that pump rays trajectory depends fundamentally on the shape of the inner clad. For the input date assumed, for cylindrical fibers, the light rays completely omit the centrally located core. The probability of pump light absorption can be increased applying the off-set of the core. However, in fibers with noncircular symmetry (square, rectangle, D-shape) the probability of light ray transition through the core is much higher. These shapes were examined for the case of the pencil of rays (several thousand rays) propagating in a fiber of the length known. On the basis of that, the characteristics of absorption efficiency as a function of fiber shape were calculated.

Absorption efficiency was defined as:

$$\eta_{\text{abs}} = 1 - \exp(-l\alpha\kappa) \quad (3)$$

where  $l$  is the fiber length,  $\alpha$  is the absorption coefficient of the core region and  $\kappa$  is the crossing frequency of light rays through the active core.

During the computing the following values of fiber parameters were assumed (for all the fibers):

- numerical aperture  $NA = 0.4$ ,
- fiber length  $l = 2$  m,
- core diameter  $\phi_{\text{core}} = 12$   $\mu\text{m}$ ,
- inner clad diameter  $\phi_{\text{clad}} = 400$   $\mu\text{m}$ ,
- absorption coefficient of the core region  $\alpha = 100$  dB/m,
- absorption coefficient of the inner clad region equals 0 dB/km,
- the input rays fall from fifteen coaxial rings situated on the fiber end face,
- the rays fill all the fiber numerical aperture.

For fibers with circular symmetry the absorption efficiency of pumping rays by the centrally located core was less than 10%. Figure 5 shows that pumping rays cross at certain distance from the center of symmetry of the fiber. Therefore, the use of fibers characterized by off-set is justified. It is also important to remember that optimal

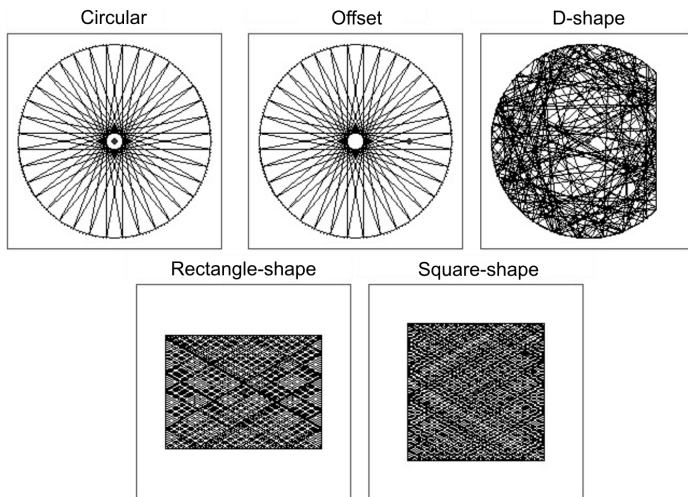


Fig. 4. Projection of launched light rays for different fiber cross-sections. Number of reflection  $N = 200$ ,  $\phi = 35^\circ$ ,  $\alpha = 5^\circ$ .

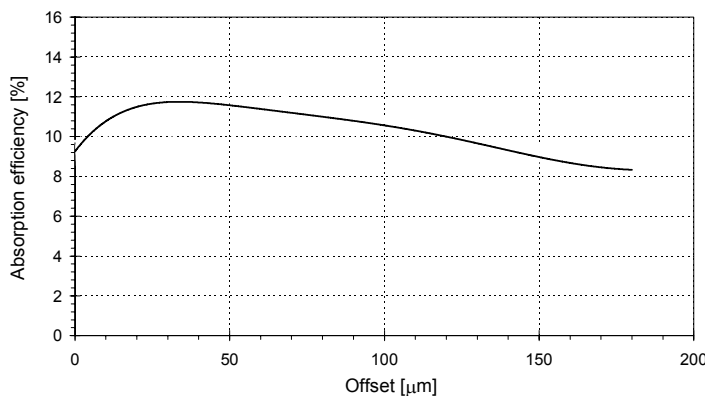


Fig. 5. Absorption efficiency for cylindrical fiber.

offset of the core is dependent on many factors, such as numerical aperture, fiber length or inner clad diameter.

In the case examined, the place of the active core location was shifted from the fiber center symmetry of  $32 \mu\text{m}$  and the pump rays absorption efficiency at this point equaled about 12%.

The simulation conducted for different fiber lengths showed that along with an increase in ray reflections, the place of optimal core location to greater extent shifts towards inner clad edge.

For D-shaped optical fiber (for five different cut sizes of the inner clad and for  $\phi_{\text{core}} = 12 \mu\text{m}$ ,  $\phi_{\text{clad}} = 400 \mu\text{m}$ ) simulations were done. They are depicted in Fig. 6.

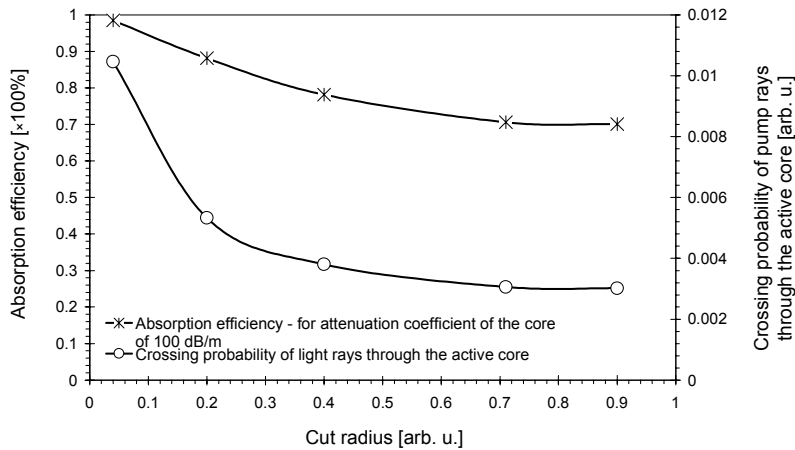


Fig. 6. Absorption efficiency and probability of rays crossing through the active core for D-shaped fiber vs. cut radius of the fiber.

The cut radius was expressed here in relative units with reference to inner clad radius. The optical fiber with the smallest cut radius is characterized by the highest absorption efficiency. The increase of cut radius from 0.05 to 0.9 corresponds to over a threefold decrease of the probability of rays crossing through the active core. The probability of rays crossing through the core is expressed as the number of pump light crossing in a particular core section to all ray reflections in this section. Better results are obtained for lower value of the cut radius. However, it decreases the effective surface of the fiber end-face, thereby it forces the focusing of pump beam to smaller dimensions. Therefore, the selection of cut radius should take the above fact into consideration.

For rectangle-shaped optical fiber, the absorption coefficient (of the core) of 4.5 dB/m was additionally assumed (for comparison). The cut radius was defined as the ratio of the distance from the center of the fiber to the cut plane to inner clad radius. The diameter of the core and inner clad was 12 and 400  $\mu\text{m}$ , respectively. The simulation results were shown in Fig. 7. As can be easily seen the highest absorption efficiency appears when the cut radius is smaller. The fiber with an inner clad in the form of a square (cut radius  $R_c = 0.71$ ) is characterized by the smallest absorption efficiency. As the cut radius value decreases, the absorption efficiency increases and for low values of the cut radius can even reach 100%.

On the basis of the characteristics and the results obtained, the following conclusions were drawn:

- in optical fibers with circular symmetry the probability of appearance of the meridional modes (rays) is very small;
- for certain values of  $\alpha$  and  $\phi$  angles for fibers with the circle, square and rectangle cross-section symmetry a stable, independent on the number of light reflection, mode (ray) distribution may appear. In this case these rays do not cross the active core;



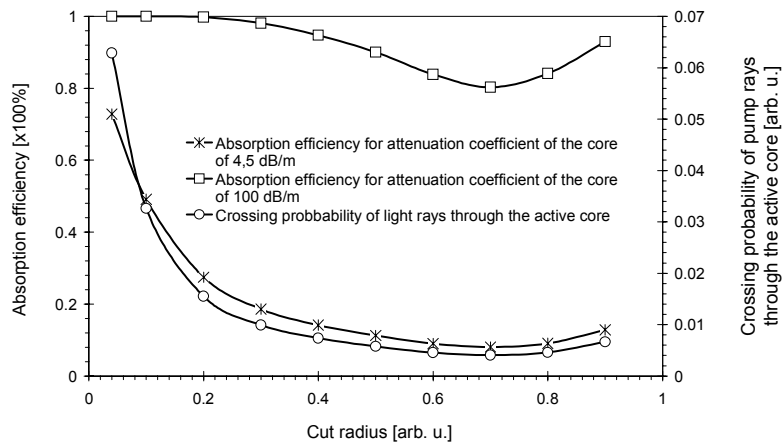


Fig. 7. Absorption efficiency and probability of rays crossing through the active core for rectangle-shaped fiber vs. cut radius of the fiber.

- for fibers with non-circular shape of an inner clad, meridional and helical modes are mixed together. For these fibers the absorption efficiency rises along with the fiber length;
- for fibers with circular symmetry the maximum position of the crossing probability curve depends on the number of ray reflections – it shifts from the core along with the fiber length increase;
- the absorption efficiency for rectangle-shaped fibers is higher than for D-shaped fibers;
- the calculation results obtained are in good agreement with experimental results, presented in literature, *e.g.*, [16, 17].

#### 4. Analysis with the use of wave-optical method

The wave-optical method presented in this section relatively well describes the physical effects occurring in optical fibers, including double-clad optical fibers. During the simulations the following assumptions were applied:

- the pump radiation is launched into the fiber using its whole numerical aperture,
- the pump radiation has a “top hat” distribution,
- a stable mode distribution of pump light is analysed,
- the mode distribution of the pump radiation (propagating in an active fiber) does not depend on the laser generation process,
- there is no mode coupling. Every mode carries the same power equaled  $P_{WE}/N$  ( $P_{WE}$  – input pump power,  $N$  – the number of modes propagating in a fiber). In commonly used fiber laser setups the typical fiber length is ca. 20 m. For this length the mode distribution is stationary.

On the basis of equations describing radiation propagation in cylindrical optical fibers, a special computer program was developed. This program calculates and

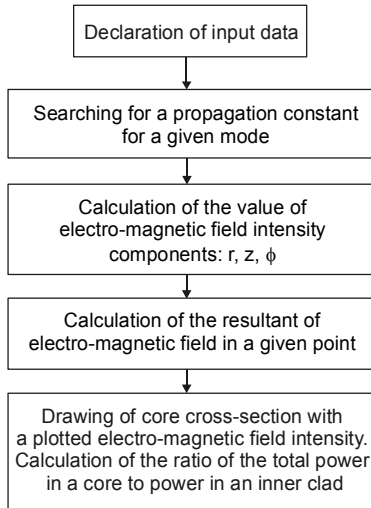


Fig. 8. Simplified computer program algorithm used for wave-optical method.

visualizes the distribution of power radiation for individual modes propagating in a fiber. It also calculates how much power of a given mode is absorbed by the active core dopant. The shortened algorithm of the program functioning is shown in Fig. 8.

One of the most important procedure of this program is the procedure which allows us to determine the propagation constant for a given mode. In order to calculate its value, it is necessary to solve the characteristic equation which is described by means of the following expression:

$$m^2(x_m + y_m)(x_m + sy_m) = (X_m + Y_m)(X_m + sY_m) \quad (4)$$

where:

$$X_m = \frac{J'_m(ua)}{uaJ_m(ua)}, \quad (5)$$

$$Y_m = \frac{K'_m(wa)}{waK_m(wa)}, \quad (6)$$

$$x_m = \frac{1}{(ua)^2}, \quad (7)$$

$$y_m = \frac{1}{(wa)^2}, \quad (8)$$

$$s = \frac{\varepsilon_{c2}}{\varepsilon_{c1}} = \frac{n_{c2}^2}{n_{c1}^2}, \quad (9)$$

while  $m$  – azimuth mode number,  $u$  – propagation constant of electro-magnetic wave in an inner clad,  $w$  – propagation constant of electro-magnetic wave in an outer clad,  $a$  – inner clad radius,  $n_{c1}$  – refractive index of an inner clad,  $n_{c2}$  – refractive index of an outer clad,  $J_m$  – Bessel function of the first kind,  $J'_m$  – derivative of  $J_m$  function in a given point,  $K_m$  – Bessel function of the second kind,  $K'_m$  – derivative of  $K_m$  function in a given point,  $\epsilon_{c1}$  – permittivity in a core,  $\epsilon_{c2}$  – permittivity in an inner clad.

Equation (3) is “strongly” nonlinear and it is dependent on the Bessel function of  $m$  order. Therefore, the most serious problem in this case is the precise determining of the value of Bessel functions and their derivatives. This task is not an easy one because Bessel functions are series dependent on Gamma function. The standard way of solving them, even by means of numerical algorithm, is long-lasting and gives serious errors of calculation method. Therefore, in the program (presented on the basis of [18]) a fast algorithm allowing us to solve Bessel functions and their derivatives for all kinds of modes was applied. For small arguments and for  $m = 0$  and  $m = 1$ , the algorithm mentioned calculates the searching function directly from tabled dependences [19]. For the case when  $m > 1$  the searching function value is calculated using recurrent dependences. The calculated values of Bessel function are sent to the suitable function allowing us to estimate the propagation constant for a given mode. Such a procedure is simpler because the only zero-place of the equations calculated must be determined. Then using the calculated propagation constant and the input date defined by the user, the program calculates the power density distribution of pumping beam and gives us the information about the absorption efficiency in a double-clad fiber.

Figure 9 depicts the hypothetical characteristic equation solution in the form of graphic presentation (the right-hand side of the figure) and cross-section of the given mode (the left-hand side of the figure). The lighter colour of a given point in the mode cross-section means the higher value of electric field intensity. The user can choose

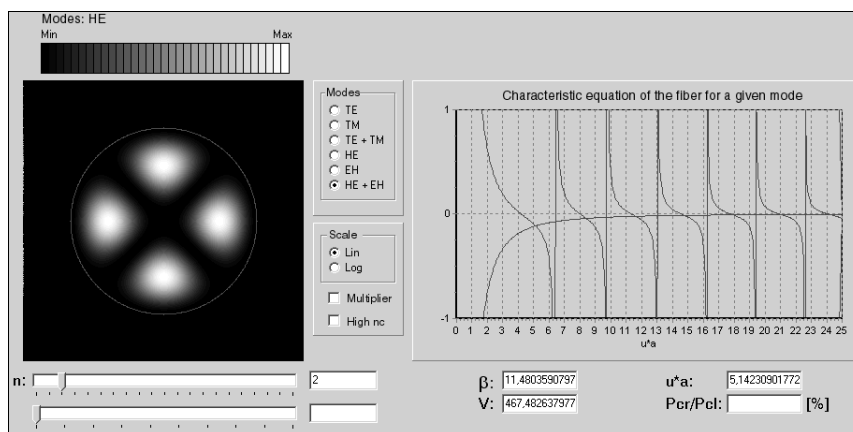


Fig. 9. Electro-magnetic field intensity for mode group  $HE_{31} + EH_{21}$  for the circular symmetry fiber characterized by:  $n_{c1} = 1.48$ ,  $n_{c2} = 1.449$ ,  $\lambda = 808$  nm and  $a = 200$   $\mu$ m.

whether he wants to see cross-section of an individual mode, the group of chosen modes or the complete electro-magnetic field intensity distribution after summing up all the modes. Every time for each of above cases the ratio of the total power carried by a mode (or a group of modes) to the total power occurring in an inner clad is calculated. Having this information, the program calculates the effective absorption efficiency of pump power.

The next computer simulations were shown in Figs. 10, 11 and the Table. From the computer calculations, the following conclusions can be drawn:

- the modes  $HE_{1p}$  (where  $p$  is the number of radiational mode) transfer most energy to the active core. The higher value of  $p$  index corresponds with the higher absorption efficiency coefficient;
- along with the inner-clad diameter increase the absorption efficiency decreases;
- wave-optical method, like geometrical method, revealed that the location of the active core in symmetry axis of the cylindrical fiber is not an optimal solution. In order to obtain better pump light absorption, the core must be shifted towards the

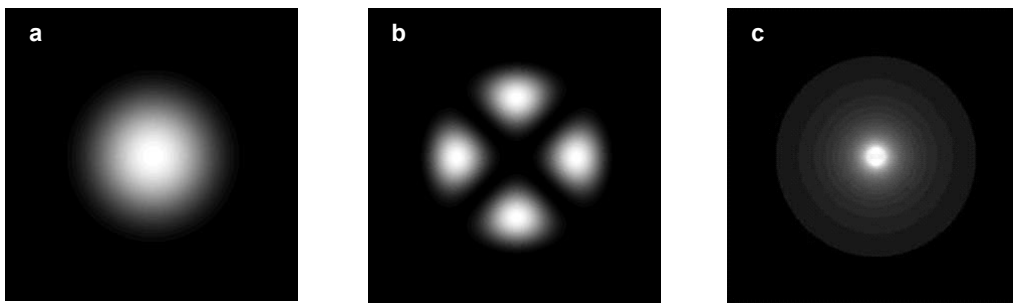


Fig. 10. Power density distribution for  $HE_{11}$  ( $LP_{01}$ ) modes (a),  $LP_{21}$  modes (b), and for all the modes (c) propagating in a fiber characterized by:  $n_{c1} = 1.48$ ,  $n_{c2} = 1.449$ ,  $\lambda = 808$  nm,  $a = 200$   $\mu\text{m}$ .

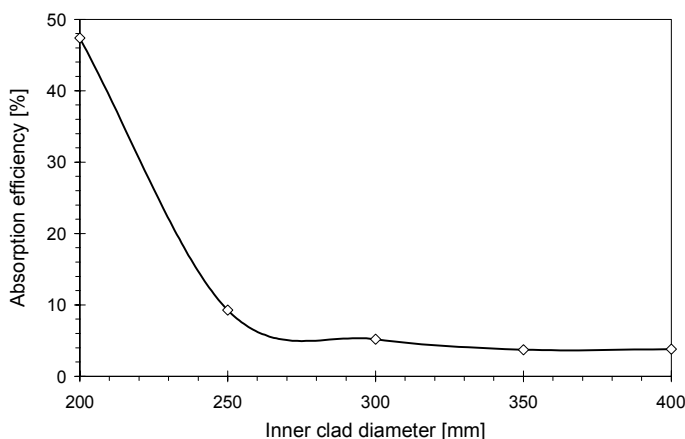


Fig. 11. Effective coefficient of absorption efficiency vs. inner clad diameter for the fiber of circular symmetry characterized by:  $n_{c1} = 1.48$ ,  $n_{c2} = 1.449$ ,  $\lambda = 808$  nm,  $a = 200$   $\mu\text{m}$ .

Table. Absorption efficiency of pump light for selected modes and mode groups.

Mode	$\eta_{\text{abs}}$ [%]
HE <sub>11</sub>	3.769
TE <sub>01</sub> + TM <sub>01</sub> + HE <sub>21</sub>	0.355
HE <sub>31</sub> + EH <sub>11</sub>	0.003
HE <sub>41</sub> + EH <sub>21</sub>	$2 \times 10^{-6}$
HE <sub>15</sub>	5.69
TE <sub>05</sub> + TM <sub>05</sub> + HE <sub>25</sub>	2.671
HE <sub>35</sub> + EH <sub>15</sub>	0.988
HE <sub>45</sub> + EH <sub>25</sub>	0.119

outer clad. The optimal size of this offset depends on numerical aperture, fiber length, core and clad diameter as well as the pump wavelength;

- the disadvantage of wave optical method is the fact that it can be used only for fibers with circular symmetry;

- the wave-optical and geometrical methods give similar results for cylindrical fibers.

## 5. Conclusions

In conclusion, the analysis of the pump power absorption efficiency in active optical fibers was done. The problem was examined by means of geometrical and wave-optical method. The double-clad optical fibers with the inner clad shape of circle, square, rectangle and D-letter were considered. The technical effect of this study is the computer software allowing us to determine the optimal fiber geometry – as the pump light absorption is the highest.

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