# STUDY OF THE REACTION $n p \rightarrow n p \pi^{+} \pi^{-}$AT INTERMEDIATE <br> ENERGIES 

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#### Abstract

The reaction $n p \rightarrow n p \pi^{+} \pi^{-}$was studied at the various momenta of incident neutrons. It was shown that the characteristics of the reaction at the momenta above $3 \mathrm{GeV} / \mathrm{c}$ could be described by the model of reggeized $\pi$ exchange (OPER). At the momenta below $3 \mathrm{GeV} / \mathrm{c}$, it was necessary to use additionally the mechanism of one baryon exchange (OBE).


## 1 Introduction: study of inelastic np interactions at accelerator facility of LHEP JINR

The data about inelastic np interactions were obtained due to irradiation of 1 m hydrogen bubble chamber ( $4 \pi$ geometry) by quasimonochromatic neutron beam ( $\delta P<2.5 \%$ ) at the following incident momenta:
$P_{0}=1.25,1.43,1.73,2.23,3.10,3.83,4.10$ and $5.20 \mathrm{GeV} / \mathrm{c}$
The unique of fullness and precision data are obtained [1]. It permits to carry out the detailed study of inelastic $n p$ interactions in a wide region of energies.

## 2 The reaction $n p \rightarrow n p \pi^{+} \pi^{-}$at $P_{0}>3 \mathrm{GeV} / \mathrm{c}$

This reaction is characterized by:

- plentiful production of the $\Delta$-resonance (see Fig.2),
- large peripherality of the secondary nucleons.

To study the mechanism of the reaction it was chosen the model of reggeized $\pi$ exchange (OPER), developed in ITEP [2].


Figure 1: Cross-sections of some inelastic np interactions (black squares our data)

The advantages of OPER model are:

- small number of free parameters (3 in our case),
- wide region of the described energies $(2 \div 200 \mathrm{GeV})$,
- calculated values are automatically normalized to the reaction cross-section.

The following main diagrams correspond to the reaction $n p \rightarrow n p \pi^{+} \pi^{-}$ within the framework of OPER model:

Matrix element for the diagrams a, b and c from Fig. 3 is written in the following form:

$$
M_{1}=T_{\pi N \rightarrow \pi N} F_{2} T_{\pi N \rightarrow \pi N} /\left(t-m_{\pi}^{2}\right)
$$

where $T_{\pi N \rightarrow \pi N}$ - amplitude of elastic $\pi N \rightarrow \pi N$ scattering off mass shell,
$F_{2}$ - form-factor, going away off mass shell of $T_{\pi N \rightarrow \pi N}$ amplitudes, $1 /\left(t-m_{\pi}^{2}\right)-\pi$-meson propagator.
The data of elastic $\pi N \rightarrow \pi N$ were taken from PWA [3].
The analysis shows, that interference between diagrams $3 \mathrm{a}, 3 \mathrm{~b}$ and 3 c is negligible [4].

The study has shown that it is not necessary to take into account the contribution of the "hanged" diagrams (Fig.)into the reaction cross-sections at $P_{0}<10 \mathrm{GeV} /$ :

It was shown in [5] that the use of some specific cuts permits to select the kinematic region of the reaction $n p \rightarrow n p \pi^{+} \pi^{-}$in which the contribution of the diagrams 3a, 3b and 3c consists up to $95 \%$ at $P_{0}>3 \mathrm{GeV} / \mathrm{c}$.


Figure 2: The distributions of $M_{p \pi^{+}}$and $M_{n \pi^{-}}$from the reaction $n p \rightarrow$ $n p \pi^{+} \pi^{-}$at $P_{0}=3 \mathrm{GeV} / \mathrm{c}$

Fig. 5 shows some distributions for the reaction $n p \rightarrow n p \pi^{+} \pi^{-}$for this region at $P_{0}=5.20 \mathrm{GeV} / \mathrm{c}$ (solid curves - results of calculations using OPER model).

But the diagrams shown in Fig. 3 are insufficient to describe totally the characteristics of the reaction $n p \rightarrow n p \pi^{+} \pi^{-}$. It is necessary to take into account the diagrams of the following type:
the matrix element for which is written in the following form:

$$
M_{3}=G \bar{u}\left(q_{N}\right) \gamma_{5} u\left(Q_{N}\right) F_{1} T_{\pi N \rightarrow \pi \pi N} /\left(t-m_{\pi}^{2}\right),
$$

where $T_{\pi N \rightarrow \pi \pi N}$ - off mass shell amplitudes of inelastic $\pi N \rightarrow \pi \pi N$ - scattering that are known much worse than elastic $T_{\pi N \rightarrow \pi N}$ amplitudes. Therefore it is necessary to do a parametrization of the inelastic $\pi N \rightarrow \pi \pi N$-scattering (see Appendix).

It permits to get a good description of the experimental characteristics of the reaction $n p \rightarrow n p \pi^{+} \pi^{-}$at $P_{0}=5.20 \mathrm{GeV} / \mathrm{c}$ (Fig.7) taking into account OPER diagrams shown in Fig. 3 and Fig. 6 :


Figure 3: OPER diagrams $2 \times 2$ for the reaction $n p \rightarrow n p \pi^{+} \pi^{-}$


Figure 4: "Hanged" OPER diagram for the reaction $n p \rightarrow n p \pi^{+} \pi^{-}$

## 3 The reaction $\bar{p} p \rightarrow \bar{p} p \pi^{+} \pi^{-}$at $P_{0}=7.23$ GeV/c

Using OPER model we try to describe the experimental distributions from the reaction $\bar{p} p \rightarrow \bar{p} p \pi^{+} \pi^{-}$at $P_{0}=7.23 \mathrm{GeV} / \mathrm{c}$ [5]

It is observed a good agreement between experimental data and theory in Fig. 8.


Figure 5: Distributions for the reaction $n p \rightarrow n p \pi^{+} \pi^{-}$at $P_{0}=5.20 \mathrm{GeV} / \mathrm{c}$ obtained due to specific cuts.

## 4 The reaction $n p \rightarrow n p \pi^{+} \pi^{-}$at $P_{0}<\mathbf{3} \mathbf{G e V} / \mathrm{c}$

The study of effective mass spectra of $n p$ - combinations at $P_{0}=1.73$ and 2.23 $\mathrm{GeV} / \mathrm{c}$ (Fig.9) shows the clear peack close the threshold $\left(M_{n p}=m_{n}+m_{p}\right)$ that can not be described within the framework of OPER-model using the diagrams from Fig. 3 and Fig. 6.

The model of Regge poles with baryon exchange and nonlinear trajectories, suggested in [6] was used to describe these features. The following diagrams of one baryon exchange (OBE) were taken into account within the framework of this model:

The vertex function of elastic $n p \rightarrow n p$ scattering was calculated using the data from [7].

The vertex functions of $\Delta N \rightarrow n p, N N \rightarrow \Delta N$ and $\Delta N \rightarrow \Delta N$ scattering were calculated corresponding to [8]. In result one can get the good description of the experimental distribution from the reaction $n p \rightarrow n p \pi^{+} \pi^{-}$ at $P_{0}=1.73$ and $2.23 \mathrm{GeV} / \mathrm{c}$ (Fig. 9 and Fig.11).


Figure 6: OPER diagrams $1 \times 3$ for the reaction $n p \rightarrow n p \pi^{+} \pi^{-}$

## 5 OPER model and other reactions

The other reactions of $n p$ interactions are scheduled to study by means of OPER model:

$$
\begin{array}{ll}
n p \rightarrow p p \pi^{-} & \text {vertex functions } 1 \times 2 \\
n p \rightarrow p p \pi^{-} \pi^{0} & \text { vertex functions } 2 \times 2 \text { and } 1 \times 3 \\
n p \rightarrow p p \pi^{+} \pi^{-} \pi^{-} & \text {vertex functions } 2 \times 3 \\
n p \rightarrow p p \pi^{+} \pi^{-} \pi^{-} \pi^{0} & \text { vertex functions } 3 \times 3 \\
n p \rightarrow n p \pi^{+} \pi^{-} \pi^{+} \pi^{-} & \text {vertex functions } 3 \times 3
\end{array}
$$

Similar reactions of $p p, \bar{p} p$ and $\pi N$ interactions also can be described by OPER model. The following reactions were simulated for HADES experiment:

$$
p p \rightarrow p p \pi^{+} \pi^{-} \text {at } T_{k i n}=3.5 \mathrm{GeV}
$$

$n p \rightarrow n p \pi^{+} \pi^{-}$at $T_{k i n}=1.25 \mathrm{GeV}$
$n p \rightarrow n p e^{+} e^{-}$at $T_{k i n}=1.25 \mathrm{GeV}$ with vertex function of $\gamma N \rightarrow$ $N e+e^{-}$.

Since the $\pi N \rightarrow \pi N$ and $\pi N \rightarrow \pi \pi N$ vertex functions are taken in helicity representation it seems to be perspective to use OPER model for description of the reaction with polarized particles.


Figure 7: Distributions for the reaction $n p \rightarrow n p \pi^{+} \pi^{-}$at $P_{0}=5.20 \mathrm{GeV} / \mathrm{c}$

## 6 Conclusion

Reaction $n p \rightarrow n p \pi^{+} \pi^{-}$is characterized by the plentiful production of the $\Delta$ resonance and the large peripherality of the secondary particles. The experimental data are successfully described by the further development of OPER model.

However at $P_{0}<3 \mathrm{GeV} / \mathrm{c}$ it is necessary to take into account another mechanism of the reaction (such as OBE).

OPER model permits to describe another $N(\bar{N})-N$ reactions with the production of some $\pi$-mesons. The further development of OPER-model can be very promising to describe the production of $e^{+} e^{-}$-pairs in hadronic interactions.

OPER model can be used as an effective tool to simulate various reactions of hadronic interactions.


Figure 8: Distributions for the reaction $\bar{p} p \rightarrow \bar{p} p \pi^{+} \pi^{-}$at $P_{0}=7.23 \mathrm{GeV} / \mathrm{c}$


Figure 9: The distributions of $M_{n p}$ for treaction $n p \rightarrow n p \pi^{+} \pi^{-}$at $P_{0}=2.23$ $\mathrm{GeV} / \mathrm{c}$ (left) and $1.73 \mathrm{GeV} / \mathrm{c}$ (right).

## 7 Appendix: Parametrization of $\pi N \rightarrow \pi \pi N$ reactions

Within the framework of Generalized Isobar Model (GIM) [9] $\pi N \rightarrow \pi \pi N$ reactions are described as quasi-two body ones $(a+b \rightarrow c+d)$ :

$$
\begin{aligned}
& \pi N \rightarrow N^{*}\left(\Delta^{*}\right) \rightarrow \Delta \pi, \\
& \pi N \rightarrow N^{*}\left(\Delta^{*}\right) \rightarrow N \rho \\
& \pi N \rightarrow N^{*}\left(\Delta^{*}\right) \rightarrow N \epsilon \\
& \pi N \rightarrow N^{*}\left(\Delta^{*}\right) \rightarrow N_{1440}^{*} \pi
\end{aligned}
$$

with the consequent decays:

$$
\Delta \rightarrow N \pi,
$$



Figure 10: OBE diagrams for the reaction $n p \rightarrow n p \pi^{+} \pi^{-}$

$$
\begin{aligned}
& \rho \rightarrow \pi \pi \\
& \rho \rightarrow \pi \pi \\
& N_{1440}^{*} \rightarrow N \pi
\end{aligned}
$$

The parameters of the following resonances ( ${ }^{* * * *}$ and ${ }^{* * *}$ ) were taken from Review of Particle Properties:

$$
\begin{array}{ll}
N^{*}(1440) P 11 & D^{*}(1600) P 33 \\
N^{*}(1520) D 13 & D^{*}(1620) S 31 \\
N^{*}(1675) D 15 & D^{*}(1700) D 33 \\
N^{*}(1680) F 15 & D^{*}(1900) S 31 \\
N^{*}(1720) P 13 & D^{*}(1905) F 35 \\
N^{*}(2000) F 15 & D^{*}(1910) P 31 \\
N^{*}(2080) D 13 & D^{*}(1920) P 33 \\
N^{*}(2190) G 17 & D^{*}(1940) D 33 \\
& D^{*}(1950) F 37
\end{array}
$$

The spin and isospin relations were taken account.


Figure 11: Distributions for the reaction $n p \rightarrow n p \pi^{+} \pi^{-}$at $P_{0}=1.73 \mathrm{GeV} / \mathrm{c}$

For quasi two-body reactions like $a+b \rightarrow c+d$ one can write

$$
\begin{aligned}
& d \sigma=\frac{1}{\left(2 S_{a}+1\right)\left(2 S_{b}+1\right)}\left(\frac{2 \pi}{p}\right)^{2} \sum_{\lambda_{i}}\left|<\lambda_{d} \lambda_{c}\right| T\left|\lambda_{b} \lambda_{a}>\right|^{2} \times d P S \\
& <\lambda_{d} \lambda_{c}|T| \lambda_{b} \lambda_{a}>=\frac{1}{4 \pi} \sum_{j}(2 j+1)<\lambda_{d} \lambda_{c}\left|T_{j}\right| \lambda_{b} \lambda_{a}>e^{i(\lambda-\mu) \varphi} d_{\lambda \mu}^{j}(\theta)
\end{aligned}
$$

where $\lambda=\lambda_{a}-\lambda_{b}, \mu=\lambda_{c}-\lambda_{d}$ - helicity variables,

$$
d_{\lambda \mu}^{j}(\theta) \text { - rotation matrixe, }
$$

$d P S$ - phase space element.
The polarization components of the particles c and d from the reaction $a+b \rightarrow c+d$ is suiteable to express through the elements of the spin density matrix (for example, for particle d):

$$
\rho_{m m^{\prime}}^{d}=\frac{1}{N} \sum_{\lambda_{c} \lambda_{b} \lambda_{a}}<m \prime \lambda_{c}|T| \lambda_{b} \lambda_{a}>^{*}<m \lambda_{c}|T| \lambda_{b} \lambda_{a}>
$$

where normalization factor for $\operatorname{Sp} \rho=1$ :

$$
N=\sum_{m \lambda_{c} \lambda_{b} \lambda_{a}}<m \lambda_{c}|T| \lambda_{b} \lambda_{a}>^{2} .
$$

## Example:

$$
\begin{aligned}
\pi+N & \rightarrow N_{1680}^{*} \rightarrow \Delta+\pi \rightarrow(N+\pi)+\pi \\
<\lambda_{\Delta}|T| \lambda_{N}> & =C_{3,0 ; \frac{1}{2},-\lambda_{\Delta}}^{\frac{5}{2},-\lambda_{N}} C_{1,0 ; \frac{3}{2},-\lambda_{\Delta}}^{\frac{5}{2},-\lambda_{\Delta}} d_{-\lambda_{N},-\lambda_{\Delta}}^{\frac{5}{2}}(\theta) \times R_{J},
\end{aligned}
$$

where $R_{J}$ is taken in Breight-Wigner form.
Then it is easy to get the angular distribution of $\Delta$ (in CMS):

$$
\frac{d \sigma(s, t)}{d \Omega} \sim\left(1+2 \cos ^{2} \theta_{\Delta}\right)\left|R_{J}\right|^{2}=\left(1+2 \cos ^{2} \theta_{\Delta}\right) \times B W\left(\sqrt{s}, M_{R}, \Gamma_{R}\right)
$$

If particle d is unstable: $d \rightarrow \alpha+\beta(d \rightarrow \Delta+\pi)$ then in the rest system of the particle d:
$W_{\Delta}=\frac{3}{4 \pi}\left\{\rho_{33} \sin ^{2} \theta+\frac{1}{3} \rho_{11}\left(1+3 \cos ^{2} \theta\right)-\frac{2}{\sqrt{3}} R e \rho_{3-1} \sin ^{2} \theta \cos 2 \varphi-\frac{2}{\sqrt{3}} R e \rho_{31} \sin 2 \theta \cos \varphi\right\}$

- is the normalized angular distribution of the decay products.

To compare with experimental data the following cross-sections were calculated using GIM (Fig.12):

One can see a satisfactory description of cross-sections, except $\pi^{+} p \rightarrow$ $n \pi^{+} \pi^{+}$. May be it is necessary to take into account S-wave of $\pi^{+} \pi^{+}$scattering with $\mathrm{I}=2$ in GIM.

Some distributions of the reaction $\pi^{-} p \rightarrow n \pi^{+} \pi^{-}$were calculated at various energies to study a quality of the application of GIM (Fig.13):

It is observed a good agreement between experimental data and theory.

## References

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Figure 12: Cross-sections of the $\pi n \rightarrow \pi \pi N$ reactions.
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Figure 13: Some distributions from the reaction $\pi^{-} p \rightarrow \pi^{+} \pi^{-} n$ at $T_{k i n}=1.0$ GeV [10]

