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Relating B_s Mixing and $B_s \to \mu^+\mu^-$ with New Physics

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Abstract

We perform a study of the Standard Model (SM) fit to the mixing quantities ΔM_{B_s}, and $\Delta\Gamma_{B_s}/\Delta M_{B_s}$ in order to bound contributions of New Physics to B_s mixing. We then use this to explore the branching fraction of $B_s \to \mu^+\mu^-$ in certain models of New Physics (NP). In most cases, this constrains NP amplitudes for $B_s \to \mu^+\mu^-$ to lie below the SM component.

I. INTRODUCTION

We report here on a study of New Physics (NP) predictions for $B_s \to \mu^+\mu^-$. The Standard Model (SM) prediction for $B_s \to \mu^+\mu^-$ is currently smaller than the experimental branching fraction limit [1] of $\mathcal{B}^{(\text{expt})}_{B_s\to\mu^+\mu^-}$ by about a factor of 15. This presents a window of opportunity for observing New Physics (NP) effects in this mode.

This topic is particularly timely in view of experimental indications of NP effects in both the exclusive decay $B_s \to J/\Psi + \Phi$ [2] (for recent CDF results, also see Ref. [3]) as well as the inclusive like-sign dimuon asymmetry observed in $p\bar{p} \to \mu\mu + X$ [4]. Moreover, future work at LHC-B, e^+e^- Super B-factories and ongoing CDF & D0 measurements at Fermilab (see the discussion following Eq. (6)) is expected to markedly improve the current branching fraction bound.

Our strategy in this paper is somewhat reminiscent of our recent study [5] noting that in some NP models the D^0 mixing and $D^0 \to \mu^+\mu^-$ decay amplitudes have a common dependence on the NP parameters. If so, one can predict the $D^0 \to \mu^+\mu^-$ branching fraction in terms of the observed ΔM_D provided that much or all of the mixing is attributed to NP. This is a viable possibility for D^0 mixing because the Standard Model (SM) signal has large theoretical uncertainties and because many NP models can produce the observed mixing [6].

For ΔM_{B_s} the situation is very different. Here, the SM prediction is in accord with the observed value (*e.g.* see Refs. [7, 8] and papers cited therein). In fact, the analysis described below (*cf.* see Eqs. (11),(12)) gives $|\Delta M^{(\text{NP})}_{B_s}/\Delta M^{(\text{SM})}_{B_s}| \leq 0.20$, which demonstrates just how well the SM prediction agrees with the experimental value of ΔM_{B_s}. In view of this, our SM expression for ΔM_{B_s} will be given at NLO [9, 10] whereas LO results will suffice for NP models. As regards the corresponding width difference $\Delta\Gamma_{B_s}$, the experimental and theoretical uncertainties are still rather significant (*viz* Sect. II-C).

In those NP models where mixing and $B_s \to \mu^+\mu^-$ arise from a common set of parameters, the severe constraint on any NP signal to B_s mixing places strong bounds on its contribution to $\mathcal{B}_{B_s\to\mu^+\mu^-}$.[1] In fact, we shall find the constraint can be so strong that for some NP models the predicted $B_s \to \mu^+\mu^-$ branching fraction lies well below the SM prediction.

[1] In particular, Ref. [7] considers the possibility, not covered here, on effects of so-called minimal flavor violation which affect the quark mixing-matrix elements.

The first step in our study (*cf* Section II) will be to revisit the SM predictions for mixing in the b-quark system by using up-to-date inputs. We carry this out for the two mixing quantities ΔM_{B_s} and $\Delta\Gamma_{B_s}/\Delta M_{B_s}$. The former in turn yields phenomenological bounds on NP mixing contributions which in certain models can be used to bound the magnitude of the $B_s \to \mu^+\mu^-$ decay mode. We also update the SM branching fraction for $B_s \to \mu^+\mu^-$ by using the observed B_s mixing as input. Then, in Section III we discuss general properties of NP models with tree-level amplitudes. In Section IV, we explore various NP models such as extra Z' bosons, family symmetry, R-parity violating supersymmetry, flavor-changing Higgs models, and models with the fourth sequential generation. Our concluding remarks appear in Section V, and some technical details are relegated to the Appendix.

II. UPDATE OF B_s MIXING AND $B_s \to \mu^+\mu^-$ IN THE STANDARD MODEL

We begin by considering the SM predictions for B_s mixing. This step is crucial to obtaining bounds on NP contributions. We also use the B_s mixing signal as input to a determination of the branching fraction for $B_s \to \mu^+\mu^-$.

A. Inputs to the Analysis

The work in this Section takes advantage of recent progress made in determining several quantities used in the analysis. We summarize our numerical inputs in Table I, along with corresponding references. Included in Table I is an updated determination of the top quark

$M_{B_s} = 5366.3 \pm 0.6$ MeV [1]	$\tau_{B_s} = (1.425 \pm 0.041) \times 10^{-12}$ s [1]
$\Delta M_{B_s} = (117.0 \pm 0.8) \times 10^{-13}$ GeV	$\Delta\Gamma_{B_s}/\Gamma_{B_s} = 0.092^{+0.051}_{-0.054}$ [1]
$x_{B_d} = 0.776 \pm 0.008$ [1]	$x_{B_s} = 26.2 \pm 0.5$ [1]
$m_t^{(\rm pole)} = 173.1 \pm 1.3$ [11]	$\alpha_s(M_Z) = 0.1184 \pm 0.0007$ [12]
$f_{B_s} = 0.2388 \pm 0.0095$ GeV [13]	$f_{B_s}\sqrt{\hat{B}_{B_s}} = 275 \pm 13$ MeV [13]
$\lvert V_{ts}\rvert = 0.0403^{+0.0011}_{-0.0007}$ [1]	$\lvert V_{tb}\rvert = 0.999152^{+0.000030}_{-0.000045}$ [1]

TABLE I: List of Input Parameters

pole mass [11] $m_t^{(\mathrm{pole})}$ which in turn is used to determine the corresponding running mass $\bar{m}_t(\bar{m}_t)$ [14] along with several decay constants and B-factors as evaluated in lattice QCD. For definiteness, we have used values appearing in Ref. [13]. This area is, however, constantly evolving and one anticipates further developments in the near future [15]. Our values for the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $|V_{ts}|$ and $|V_{tb}|$ are taken from Ref. [1]. Similar values occur for the global fits cited elsewhere (*e.g.* Refs. [16, 17]).

B. ΔM_{B_s}

The PDG value for ΔM_{B_s},

$$\Delta M_{B_s}^{(\mathrm{expt})} = (117.0 \pm 0.8) \times 10^{-13}\ \mathrm{GeV} \quad , \tag{1}$$

is a very accurate one – the uncertainty amounts to about 0.7%. The NLO SM formula,

$$\Delta M_{B_s}^{(\mathrm{SM})} = 2\frac{G_{\mathrm{F}}^2 M_{\mathrm{W}}^2 M_{B_s} f_{B_s}^2 \hat{B}_{B_s}}{12\pi^2} |V_{\mathrm{ts}}^* V_{\mathrm{tb}}|^2 \eta_{B_s} S_0(\bar{x}_t) \quad , \tag{2}$$

is arrived at from an operator product expansion of the mixing hamiltonian. The short-distance dependence in the Wilson coefficient appears in the scale-insensitive combination $\eta_{B_s} S_0(\bar{x}_t)$, where the factor $S_0(\bar{x}_t)$ is an Inami-Lin function [18] (with $\bar{x}_t \equiv \bar{m}_t^2(\bar{m}_t)/M_{\mathrm{W}}^2$) and $\bar{m}_t(\bar{m}_t)$ is the running top-quark mass parameter in $\overline{\mathrm{MS}}$ renormalization. In particular, we have $\bar{m}_t(\bar{m}_t) = (163.4 \pm 1.2)$ GeV which leads to $S_0(\bar{x}_t) = 2.319 \pm 0.028$. Using the same matching scale, we obtain $\eta_{B_s} = 0.5525 \pm 0.0007$ for the NLO QCD factor.

Our evaluation for $\Delta M_{B_s}^{(\mathrm{SM})}$ then gives

$$\Delta M_{B_s}^{(\mathrm{SM})} = \left(117.1^{+17.2}_{-16.4}\right) \times 10^{-13}\ \mathrm{GeV} \quad , \tag{3}$$

which is in accord with the experimental value of Eq. (1). The theoretical uncertainty in the SM prediction of Eq. (3) is roughly a factor of twenty larger than the experimental uncertainty of Eq. (2). The largest source of error occurs in the nonperturbative factor $\hat{B}_{B_s} f_{B_s}^2$, followed by that in the CKM matrix element V_{ts}. The asymmetry in the upper and lower uncertainties in $\Delta M_{B_s}^{(\mathrm{SM})}$ arises from the corresponding asymmetry in the value of V_{ts} cited in Ref. [1].

Finally, we note in passing that for the ratio $\Delta M_{B_d}/\Delta M_{B_s}$ the experimental value is 0.02852 ± 0.00034 whereas the SM determination gives 0.02714 ± 0.00193. This good agreement is not surprising since the ratio $\Delta M_{B_d}/\Delta M_{B_s}$ contains less theoretical uncertainty than ΔM_{B_d} or ΔM_{B_s} separately.

C. The Ratio $\Delta\Gamma_{B_s}/\Delta M_{B_s}$

The above work on $\Delta M_{B_s}^{(\mathrm{SM})}$ sets the stage for analyzing NP contributions to $B_s \to \mu^+\mu^-$. There is, in principle, a second approach which instead utilizes $\Delta\Gamma_{B_s}$. The PDG value for the B_s width difference is $\Delta\Gamma_{B_s}^{(\mathrm{expt})} = 0.062^{+0.034}_{-0.037} \times 10^{12}\mathrm{s}^{-1}$. Together with Eq. (1), this gives[2]

$$r^{(\mathrm{expt})} \equiv \frac{\Delta\Gamma_{B_s}^{(\mathrm{expt})}}{\Delta M_{B_s}^{(\mathrm{expt})}} = \frac{0.062^{+0.034}_{-0.037} \times 10^{12}\ \mathrm{s}^{-1}}{(17.77 \pm 0.12) \times 10^{12}\ \mathrm{s}^{-1}} = (34.9 \pm 20.0) \times 10^{-4} \quad . \tag{4}$$

whereas the corresponding SM prediction from Ref. [8] is $r^{(\mathrm{SM})} = (49.7 \pm 9.4) \times 10^{-4}$. In contrast to the mass splitting ΔM_{B_s}, the theoretical uncertainty in the ratio $\Delta\Gamma_{B_s}/\Delta M_{B_s}$ is much smaller than in the current experimental determination. Nonetheless, this situation is expected to change once LHCb gathers sufficient data. As such, we would expect a highly accurate value of $\Delta\Gamma_{B_s}^{(\mathrm{expt})}$ to eventually become available. We propose that it could be applied to the kind of analysis used in this paper as follows. We define a kind of mass difference $\mathcal{D}M_{B_s}$ as

$$\mathcal{D}M_{B_s} \equiv \frac{\Delta M_{B_s}^{(\mathrm{thy})}}{\Delta\Gamma_{B_s}^{(\mathrm{thy})}}\ \Delta\Gamma_{B_s}^{(\mathrm{expt})} \quad . \tag{5}$$

The point is that if NP contributions are neglected in $\Delta B = 1$ transitions, then $\Delta\Gamma_{B_s}^{(\mathrm{thy})}$ is purely a SM effect. In addition, the ratio $\Delta M_{B_s}^{(\mathrm{SM})}/\Delta\Gamma_{B_s}^{(\mathrm{SM})}$ will be less dependent on hadronic parameters than either factor separately. At the very least, it would be of interest to analyze the NP issue using both quantities ΔM_{B_s} and the above $\mathcal{D}M_{B_s}$.

D. $B_s \to \mu^+\mu^-$

PDG entries for $\mathcal{B}_{B_s\to\ell^+\ell^-}$ are

$$\mathcal{B}^{(\mathrm{expt})}_{B_s\to\mu^+\mu^-} < 4.7 \times 10^{-8} \qquad \text{and} \qquad \mathcal{B}^{(\mathrm{expt})}_{B_s\to e^+e^-} < 5.4 \times 10^{-5} \quad , \tag{6}$$

[2] Using instead the recent CDF evaluation $\Delta\Gamma_{B_s}^{(\mathrm{CDF})} = 0.075 \pm 0.035 \pm 0.01 \times 10^{12}\ \mathrm{s}^{-1}$ implies $r^{(\mathrm{expt})} = (42.2 \pm 20.5) \times 10^{-4}$, consistent with the value in Eq. (4).

with no experimental limit currently for the $B_s \to \tau^+\tau^-$ transition. Data collected by the D0 and CDF collaborations will improve the above brancing fraction limit. For example, the D0 collaboration reports $\mathcal{B}^{(\rm D0)}_{B_s\to\mu^+\mu^-} < 5.1 \times 10^{-8}$, with an anticipated limit of eleven times the SM prediction and similarly for the CDF collaboration [19].

Since the LD estimate for the branching fraction of $B_s \to \mu^+\mu^-$ in the SM gives $\mathcal{B}^{(\rm LD)}_{B_s\to\mu^+\mu^-} \sim 6 \times 10^{-11}$, we consider only the SD component in the following. Using Eq. (2) as input to the SD-dominated $B_s \to \mu^+\mu^-$ transition (see also Ref. [7]) we arrive at

$$\mathcal{B}^{(\rm SM)}_{B_s\to\mu^+\mu^-} = \Delta M_{B_s}\ \tau_{B_s} \frac{3G_{\rm F}^2 M_{\rm W}^2 m_\mu^2}{4\eta_{B_s}\hat{B}_{B_s}\pi^3}\left[1-4\frac{m_\mu^2}{M_{B_s}^2}\right]^{1/2}\frac{Y^2(\bar{x}_t)}{S_0(\bar{x}_t)} \quad , \tag{7}$$

where $Y(\bar{x}_t)$ is another Inami-Lin function [18]. Expressing $\mathcal{B}^{(\rm SM)}_{B_s\to\mu^+\mu^-}$ in this manner serves to remove some of the inherent model dependence. Numerical evaluation gives

$$\mathcal{B}^{(\rm SM)}_{B_s\to\mu^+\mu^-} \simeq 3.3 \times 10^{-9} \quad . \tag{8}$$

III. STUDY OF NEW PHYSICS MODELS

In this section, we first obtain a numerical (1σ) bound on any possible New Physics contribution to ΔM_{B_s}. We then use this to constrain couplings in a variety of NP models and thereby learn something about the $B_s \to \mu^+\mu^-$ transition.

A. Constraints on NP Models from B_s Mixing

As shown in Ref. [32], New Physics in $\Delta B = 1$ interactions can *in principle* markedly affect $\Delta\Gamma_s$. The logic is similar to that used in Ref. [33] regarding the possible impact of NP on $\Delta\Gamma_D$. Since, however, in B_s mixing such models are not easy to come up with, one can simply assume that $\Delta B = 1$ processes are dominated by the SM interactions. Thus we can write

$$\Delta M_{B_s} = \Delta M^{(\rm SM)}_{B_s} + \Delta M^{(\rm NP)}_{B_s} \quad . \tag{9}$$

If the $\Delta B = 1$ sector were to contain significant NP contributions, then the above relation would no longer be valid due to interference between the SM and NP components.
Accounting for NP as an additive contribution,

$$\Delta M^{(\rm expt)}_{B_s} = \Delta M^{(\rm SM)}_{B_s} + \Delta M^{(\rm NP)}_{B_s} \quad , \tag{10}$$

we have from Eqs. (1),(3),

$$\Delta M_{B_s}^{(\mathrm{NP})} = \left(-0.1^{+17.2}_{-16.4}\right) \times 10^{-13}\ \mathrm{GeV} \quad . \tag{11}$$

The error in $\Delta M_s^{(\mathrm{expt})}$ has been included, but it is so small compared to the theoretical error in $\Delta M_s^{(\mathrm{SM})}$ as to be negligible. The 1σ range for the NP contribution is thus

$$\Delta M_{B_s}^{(\mathrm{NP})} = (-17.3 \to +16.5) \times 10^{-13}\ \mathrm{GeV} \quad . \tag{12}$$

To proceed further without ambiguity, we would need to know the relative phase between the SM and NP components. Lacking this, we employ the absolute value of the largest possible number,

$$|\Delta M_{B_s}^{(\mathrm{NP})}| \leq 17.3 \times 10^{-13}\ \mathrm{GeV} \quad , \tag{13}$$

to constrain the NP parameters.

B. Generic NP Models with tree-level amplitudes

New Physics can affect both B_s mixing and rare decays like $B_s \to \mu^+\mu^-$ by engaging in these two transitions at tree level. In this section we will, for generality, consider a generic spin-1 boson V or a spin-0 boson S with flavor-changing and flavor-conserving neutral current interactions that couple both to quarks and leptons. The bosons V and S can be of either parity. This situation is frequently realized, as in the interactions of a heavy Z' boson or in multi-Higgs doublet models without natural flavor conservation.

Spin-1 Boson V: Assuming that the spin-1 particle V has flavor-changing couplings, the most general Lagrangian can be written as[3]

$$\mathcal{H}_V = g'_{V1}\overline{\ell}'_L\gamma_\mu\ell_L V^\mu + g'_{V2}\overline{\ell}'_R\gamma_\mu\ell_R V^\mu + g_{V1}\overline{b}_L\gamma_\mu s_L V^\mu + g_{V2}\overline{b}_R\gamma_\mu s_R V^\mu + \mathrm{h.c.} \quad . \tag{14}$$

Here V_μ is the vector field and the flavor of the lepton ℓ' might or might not coincide with ℓ. It is not important whether the field V_μ corresponds to an abelian or non-abelian gauge symmetry group. Using methods similar to those in Ref. [5], we obtain

$$\Delta M_{\mathrm{B}_s}^{(\mathrm{V})} = \frac{f_{B_s}^2 M_{B_s}}{3M_V^2}\,\mathcal{R}e\left[C_1(\mu)B_1 + C_6(\mu)B_6 - \frac{5}{4}C_2(\mu)B_2 + \frac{7}{8}C_3(\mu)B_3\right] \quad , \tag{15}$$

[3] Throughout, our convention for defining chiral projections for a field $q(x)$ will be $q_{L,R}(x) \equiv (1\pm\gamma_5)q(x)/2$.

where the superscript on $\Delta M_{\rm B_s}^{\rm (V)}$ denotes propagation of a vector boson in the tree amplitude. The Wilson coefficients evaluated at a scale μ are related to the couplings g_{V1} and g_{V2} as

$$
\begin{aligned}
\mathrm{C}_1(\mu) &= r(\mu, M_V)\ g_{V1}^2\ , & \mathrm{C}_3(\mu) &= \tfrac{4}{3}\left[r(\mu, M_V)^{1/2} - r(\mu, M_V)^{-4}\right] g_{V1}g_{V2}\ ,\\
\mathrm{C}_2(\mu) &= 2\ r(\mu, M_V)^{1/2} g_{V1}g_{V2}\ , & \mathrm{C}_6(\mu) &= r(\mu, M_V)\ g_{V2}^2\ \ ,
\end{aligned}
$$

where (presuming that $M > m_t$ and $\mu \geq m_b$),

$$
r(\mu, M) = \left(\frac{\alpha_s(M)}{\alpha_s(m_t)}\right)^{2/7} \left(\frac{\alpha_s(m_t)}{\alpha_s(\mu)}\right)^{6/23} \quad . \tag{16}
$$

Similar calculations can be performed for the $B_s^0 \to \ell^+\ell^-$ decay. The effective Hamiltonian in this case is

$$
\mathcal{H}^{\rm (V)}_{b\to q\ell^+\ell^-} = \frac{1}{M_V^2}\left[g_{V1}g'_{V1}\widetilde{Q}_1 + g_{V1}g'_{V2}\widetilde{Q}_7 + g'_{V1}g_{V2}\widetilde{Q}_2 + g_{V2}g'_{V2}\widetilde{Q}_6\right]\ , \tag{17}
$$

where the operators $\{\widetilde{Q}_i\}$ can be read off from those in Ref. [5] with the label changes $c \to s$ and $u \to b$. This leads to the branching fraction,

$$
\mathcal{B}^{\rm (V)}_{B_s^0\to\ell^+\ell^-} = \frac{f_{B_s}^2 m_\ell^2 M_{B_s}}{32\pi M_V^4 \Gamma_{B_s}} \sqrt{1 - \frac{4m_\ell^2}{M_{B_s}^2}}\ |g_{V1} - g_{V2}|^2 |g'_{V1} - g'_{V2}|^2\ . \tag{18}
$$

Clearly, Eqs. (15),(18) can be related to each other only for a specific set of NP models.

Spin-0 Boson S: Analogous procedures can be followed if now the FCNC is generated by quarks interacting with spin-0 particles. Again, the most general Hamiltonian can be written as

$$
\mathcal{H}_S = g'_{S1}\overline{\ell}_L\ell_R S + g'_{S2}\overline{\ell}_R\ell_L S + g_{S1}\overline{b}_L s_R S + g_{S2}\overline{b}_R s_L S + \text{h.c.}\ \ . \tag{19}
$$

Evaluation of $\Delta M_{\rm B_s}^{\rm (S)}$ at scale $\mu = m_b$ gives

$$
\Delta M_{\rm B_s}^{\rm (S)} = \frac{5 f_{B_s}^2 M_{B_s}}{24 M_S^2}\ \mathcal{R}e\left[\frac{7}{5}C_3(\mu)B_3 - (C_4(\mu)B_4 + C_7(\mu)B_7) + \frac{12}{5}\left(C_5(\mu)B_5 + C_8(\mu)B_8\right)\right] \tag{20}
$$

with the Wilson coefficients defined as

$$
\begin{aligned}
C_3(\mu) &= -2r(\mu, M_S)^{-4}\ g_{S1}g_{S2} \equiv \overline{C}_3(\mu)\ g_{S1}g_{S2}\\
C_4(\mu) &= -\left[\left(\frac{1}{2} - \frac{8}{\sqrt{241}}\right) r_+(\mu, M_S) + \left(\frac{1}{2} + \frac{8}{\sqrt{241}}\right) r_-(\mu, M_S)\right] g_{S2}^2 \equiv \overline{C}_4(\mu)\ g_{S2}^2
\end{aligned}
$$

$$C_5(\mu) = \frac{1}{8\sqrt{241}}\left[r_+(\mu, M_S) - r_-(\mu, M_S)\right] g_{S2}^2 \equiv \overline{C}_5(\mu)\ g_{S2}^2 \tag{21}$$
$$C_7(\mu) = -\left[\left(\frac{1}{2} - \frac{8}{\sqrt{241}}\right) r_+(\mu, M_S) + \left(\frac{1}{2} + \frac{8}{\sqrt{241}}\right) r_-(\mu, M_S)\right] g_{S1}^2 \equiv \overline{C}_7(\mu)\ g_{S1}^2$$
$$C_8(\mu) = \frac{1}{8\sqrt{241}}\left[r_+(\mu, M_S) - r_-(\mu, M_S)\right] g_{S1}^2 \equiv \overline{C}_8(\mu)\ g_{S1}^2 \quad ,$$

where for notational simplicity we have defined $r_\pm \equiv r^{(1\pm\sqrt{241})/6}$. Note that Eq. (20) is true only for the real spin-0 field S. If S is a complex field, then only operator Q_3 will contribute to Eq. (20).

The effective Hamiltonian for the $B_s^0 \to \ell^+\ell^-$ decay via a heavy scalar S with FCNC interactions is then

$$\mathcal{H}^{(\mathrm{S})}_{b\to s\ell^+\ell^-} = -\frac{1}{M_S^2}\left[g_{S1}g'_{S1}\widetilde{Q}_9 + g_{S1}g'_{S2}\widetilde{Q}_8 + g'_{S1}g_{S2}\widetilde{Q}_3 + g_{S2}g'_{S2}\widetilde{Q}_4\right] \ , \tag{22}$$

and from this, it follows that the branching fraction is

$$\mathcal{B}^{(\mathrm{S})}_{B_s^0\to\ell^+\ell^-} = \frac{f_B^2 M_{B_s}^5}{128\pi m_b^2 M_S^4 \Gamma_{B_s}}\sqrt{1 - \frac{4m_\ell^2}{M_{B_s}^2}}\ |g_{S1} - g_{S2}|^2$$
$$\times \ \left[|g'_{S1} + g'_{S2}|^2\left(1 - \frac{4m_\ell^2}{M_{B_s}^2}\right) + |g'_{S1} - g'_{S2}|^2\right]. \tag{23}$$

Note that if the spin-0 particle S only has *scalar* FCNC couplings, *i.e.* $g_{S1} = g_{S2}$, no contribution to $B_s^0 \to \ell^+\ell^-$ branching ratio is generated at tree level; the non-zero contribution to rare decays is instead produced at one-loop level. This follows from the *pseudoscalar* nature of the B_s-meson.

Let us now consider specific models where the correlations between the $B_s - \overline{B_s}$ mixing rates and (in particular) the $B_s \to \mu^+\mu^-$ rare decay can be found.

C. Z' Boson

B_s Mixing: The B_s mixing arising from the Z' pole diagram has the same form as in D^0 mixing [6],

$$\Delta M^{(\mathrm{Z}')}_{B_s} = \frac{M_{B_s} f_{B_s}^2 B_{B_s} r_1(m_b, M_{Z'})}{3}\cdot\frac{g^2_{Z's\bar{b}}}{M^2_{Z'}} \ , \tag{24}$$

where $r_1(m_b, M_{Z'})$ is a QCD factor which we take to be

$$r_1(m_b, M_{Z'}) \simeq 0.79 \ . \tag{25}$$

This is a compromise between $r_1(m_b, 1\text{ TeV}) = 0.798$ and $r_1(m_b, 2\text{ TeV}) = 0.783$. Solving for the Z' parameters, we have

$$\frac{g^2_{Z's\bar{b}}}{M^2_{Z'}} = \frac{3|\Delta M^{(\mathrm{NP})}_{B_s}|}{M_{B_s} f^2_{B_s} B_{B_s} r_1(m_b, M_{Z'})} \leq 2.47 \times 10^{-11}\ \mathrm{GeV}^{-2} \tag{26}$$

upon using the constraint from B_s mixing.

$B_s \to \mu^+\mu^-$ *Decay:* This has already been calculated for $D^0 \to \mu^+\mu^-$ decay in Ref. [5]. Inserting obvious modifications for $D^0 \to B_s$, we have from the branching fraction relation Eq. (39) of Ref. [5],

$$\mathcal{B}^{(\mathrm{Z}')}_{B_s\to\mu^+\mu^-} = \frac{G_F f^2_{B_s} m^2_\mu M_{B_s}}{16\sqrt{2}\pi\Gamma_{B_s}} \sqrt{1 - \frac{4m^2_\mu}{M^2_{B_s}}}\ \frac{g^2_{Z's\bar{b}}}{M^2_{Z'}} \cdot \frac{M^2_Z}{M^2_{Z'}} \quad . \tag{27}$$

Upon inserting numbers, we obtain

$$\mathcal{B}^{(\mathrm{Z}')}_{B_s\to\mu^+\mu^-} \ \leq\ 0.25 \times 10^{-9} \ \cdot \left(\frac{1\ \mathrm{TeV}}{M_{Z'}}\right)^2 \quad . \tag{28}$$

This value is already below the corresponding SM prediction ($\mathcal{B}^{(\mathrm{SM})}_{B_s\to\mu^+\mu^-} = 3.3 \times 10^{-9}$) even if we take a Z' mass as light as $M_{Z'} \simeq 1$ TeV.

D. R Parity Violating Supersymmetry

One of the models of New Physics that has a rich flavor phenomenology is R-parity violating (RPV) SUSY. The crucial difference between studies of RPV SUSY contributions to phenomenology of the up-quark (see [5]) and down-type quark sectors is the possibility of tree-level diagrams contributing to B_s-mixing[4] and $B_s \to \ell^+\ell^-$ decays [21–24]. If one allows for R-parity violation, the following terms should be added to the superpotential,

$$\mathcal{W}_{\not{R}} = \frac{1}{2}\lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \frac{1}{2}\lambda''_{ijk} U^c_i D^c_j D^c_k. \tag{29}$$

Here Q and L denote $SU(2)_L$ doublet quark and lepton superfields, and U, D and E stand for the $SU(2)_L$ singlet up-quark, down-quark and charged lepton superfields. Also, $\{i, j, k\} = 1, 2, 3$ are generation indices. We shall require baryon number symmetry by setting λ'' to

[4] We assume that there is no strong hierarchy between the RPV SUSY couplings that favors possible box diagrams.

zero. Also, we will assume CP-conservation, so all couplings λ_{ijk} and λ'_{ijk} are treated as real.

$B_s^0 - \overline{B}_s^0$ *Mixing*: Neglecting the baryon-number violating contribution, the Lagrangian describing RPV SUSY contribution to $B_s^0 - \overline{B}_s^0$ mixing can be written as

$$\mathcal{L}_{\not R} = -\lambda'_{i32}\widetilde{\nu}_{i_L}\overline{b}_R s_L - \lambda'_{i23}\widetilde{\nu}_{i_L}\overline{s}_R b_L + h.c. \quad , \tag{30}$$

where $i = 1, 2, 3$ is a generational index for the sneutrino. Matching to Eq. (19) implies that the only non-zero contribution comes from the operator Q_3. Taking into account renormalization group running, we obtain for ΔM_s from the R-parity violating terms,

$$\Delta M_{\mathrm{B_s}}^{(\not R)} = \frac{5}{24} f_{B_s}^2 M_{B_s} F(C_3, B_3) \sum_i \frac{\lambda'^*_{i23}\lambda'_{i32}}{M_{\tilde{\nu}_i}^2}, \tag{31}$$

where $M_{\tilde{\nu}_i}$ denotes the mass of the sneutrino of ith generation and the function

$$F(C_3, B_3) = \frac{7}{5}\overline{C}_3(\mu, M_{\tilde{\nu}_i}) B_3, \tag{32}$$

is defined in terms of reduced Wilson coefficient of Eq. (21) and the B-factor is defined in Table II of the Appendix.

$B_s \to \mu^+\mu^-$ *Decay*: In RPV-SUSY, the underlying transition for $B_s \to \mu^+\mu^-$ is $s + \bar{b} \to \mu^+ + \mu^-$ via tree-level u-squark or sneutrino exchange. In order to relate the rare decay to the mass difference contribution from RPV SUSY $\Delta M_{\mathrm{B_s}}^{(\not R)}$, we need to assume that the up-squark contribution is negligible. This can be achieved in models where sneutrinos are much lighter than the up-type squarks, which are phenomenologically viable. Employing this assumption leads to the predicted branching fraction

$$\begin{aligned} \mathcal{B}_{B_s\to\mu^+\mu^-}^{(\not R)} = & \frac{f_{B_s}^2 M_{B_s}^3}{64\,\pi\,\Gamma_{B_s}} \left(\frac{M_{B_s}}{m_b}\right)^2 \left(1 - \frac{2m_\mu^2}{M_{B_s}^2}\right) \sqrt{1 - \frac{4m_\mu^2}{M_{B_s}^2}} \\ & \times \left(\left| \sum_i \frac{\lambda^*_{i22}\lambda'_{i32}}{M_{\tilde{\nu}_i}^2} \right|^2 + \left| \sum_i \frac{\lambda_{i22}\lambda'^*_{i23}}{M_{\tilde{\nu}_i}^2} \right|^2 \right). \end{aligned} \tag{33}$$

In order to relate $B_s \to \mu^+\mu^-$ to ΔM_s in the framework of RPV SUSY, we need to make additional assumptions. In particular, we shall assume that the sum is dominated by a single sneutrino state, which we shall denote by $\tilde{\nu}_k$. In addition, we will assume that $\lambda'_{k23} = \lambda'_{k32}$, which will reduce the number of unknown parameters. This assumption is not needed, however, if one wishes to set a bound on a combination of coupling constants directly from

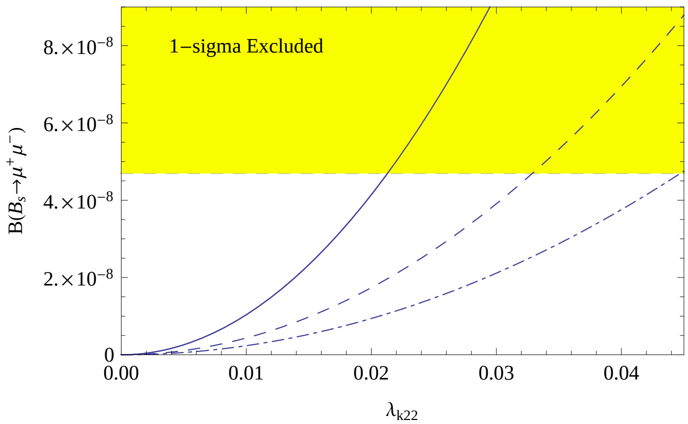

FIG. 1: Branching ratio of $\mathcal{B}_{B_s^0 \to \mu^+\mu^-}$ as a function of RPV leptonic coupling λ_{k22} and sneutrino mass $M_{\tilde{\nu}_i} = 100$ GeV, 150 GeV, and 200 GeV (solid, dashed, and dash-dotted lines). The yellow shaded area represents excluded parameter space.

the experimental bound on $\mathcal{B}_{B_s \to \mu^+\mu^-}$. Then, neglecting CP-violation,

$$\mathcal{B}^{(\not{R})}_{B_s \to \mu^+\mu^-} = k \frac{f_{B_s}^2 M_{B_s}^3}{64\pi \ \Gamma_{B_s}} \left(\frac{\lambda_{i22} \lambda'_{i32}}{M_{\tilde{\nu}_i}^2} \right)^2 \left(\frac{M_{B_s}}{m_b} \right)^2 \left(1 - \frac{2m_\mu^2}{M_{B_s}^2} \right) \sqrt{1 - \frac{4m_\mu^2}{M_{B_s}^2}}, \tag{34}$$

where $k = 2$ if an assumption that $\lambda'_{k23} = \lambda'_{k32}$ is made, and $k = 1$ otherwise.

Since no $B_s \to \mu^+\mu^-$ signal has yet been seen, we can use the experimental bound to obtain an updated constraint on the RPV couplings,

$$\lambda_{k22} \lambda'_{k32} \leq 5.5 \times 10^{-6} \ \left(\frac{M_{\tilde{\nu}_k}}{100 \text{ GeV}} \right)^2 \quad . \tag{35}$$

Now, assuming $\lambda'_{k23} = \lambda'_{k32}$, one can relate the branching ratio $\mathcal{B}_{B_s \to \mu^+\mu^-}$ to $x_{B_s}^{(\not{R})}$,

$$\mathcal{B}^{(\not{R})}_{B_s \to \mu^+\mu^-} = \frac{3}{20\pi} \frac{M_{B_s}^2}{F(C_3, B_3)} \left(\frac{M_{B_s}}{m_b} \right)^2 \left(1 - \frac{2m_\mu^2}{M_{B_s}^2} \right) \sqrt{1 - \frac{4m_\mu^2}{M_{B_s}^2}} \ x_{B_s}^{(\not{R})} \ \frac{\lambda_{k22}^2}{M_{\tilde{\nu}_i}^2}. \tag{36}$$

It is possible to plot the dependence of $\mathcal{B}_{B_s \to \mu^+\mu^-}$ on λ_{k22} for different values of $M_{\tilde{\nu}_i}$, which we present in Fig. 1.

E. Family (Horizontal) Symmetries

The gauge sector in the Standard Model has a large global symmetry which is broken by the Higgs interaction [25]. By enlarging the Higgs sector, some subgroup of this symmetry can be imposed on the full SM lagrangian and the symmetry can be broken spontaneously.

This family symmetry can be global [26] as well as gauged [27]. If the new gauge couplings are very weak or the gauge boson masses are large, the difference between a gauged or global symmetry is rather difficult to distinguish in practice [28]. In general there would be FCNC effects from both the gauge and scalar sectors. Here we study the gauge contribution. Consider the family gauge symmetry group $SU(3)_G$ acting on the three left-handed families. Spontaneous symmetry breaking renders all the gauge bosons massive. If the SU(3) is broken first to SU(2) before being completely broken, we may have an effective 'low' energy symmetry $SU(2)_G$. This means that the gauge bosons $\mathbf{G} \equiv \{G_i\}$ $(i = 1, \ldots, 3)$ are much lighter than the $\{G_k\}$ $(k = 4, \ldots, 8)$. For simplicity we assume that after symmetry breaking the gauge boson mass matrix is diagonal to a good approximation. If so, the light gauge bosons $\mathbf{G}$ are mass eigenstates with negligible mixing.

The LH doublets

$$\begin{pmatrix} u^0 \\ d^0 \end{pmatrix}_L , \quad \begin{pmatrix} c^0 \\ s^0 \end{pmatrix}_L , \quad \begin{pmatrix} t^0 \\ b^0 \end{pmatrix}_L , \tag{37}$$

transform as $I_G = 1/2$ under $SU(2)_G$, as do the lepton doublets

$$\begin{pmatrix} \nu_e^0 \\ e^0 \end{pmatrix}_L , \quad \begin{pmatrix} \nu_\mu^0 \\ \mu^0 \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau^0 \\ \tau^0 \end{pmatrix}_L . \tag{38}$$

and the right-handed fermions are singlets under $SU(2)_G$. In the above, the superscript 'o' refers to the fact that these are weak eigenstates and not mass eigenstates. The couplings of fermions to the light family gauge bosons $\mathbf{G}$ is given by

$$L = f\left[\bar{\psi}_{d^0,L}\gamma_\mu \boldsymbol{\tau} \cdot \mathbf{G}^\mu \psi_{d^0,L} + \bar{\psi}_{u^0,L}\gamma_\mu \boldsymbol{\tau} \cdot \mathbf{G}^\mu \psi_{u^0,L} + \bar{\psi}_{\ell^0,L}\gamma_\mu \boldsymbol{\tau} \cdot \mathbf{G}^\mu \psi_{\ell^0,L}\right] , \tag{39}$$

where f denotes the coupling strength and $\boldsymbol{\tau}$ are the generators of $SU(2)_G$

The fermion mass eigenstates are given by, first for quarks,

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L = U_d \begin{pmatrix} d^0 \\ s^0 \\ b^0 \end{pmatrix}_L \quad and \quad \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L = U_u \begin{pmatrix} u^0 \\ c^0 \\ t^0 \end{pmatrix}_L \tag{40}$$

and then for leptons,

$$\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L = U_\ell \begin{pmatrix} u^0 \\ \mu^0 \\ \tau^0 \end{pmatrix}_L \quad \text{and} \quad \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L = U_\nu \begin{pmatrix} \nu_e^0 \\ \nu_\mu^0 \\ \nu_\tau^0 \end{pmatrix}_L \quad . \tag{41}$$

The four matrices U_d, U_u, U_ℓ and U_ν are unknown, except for

$$U_u^\dagger U_d = V_{\rm CKM} \qquad \text{and} \qquad U_\nu^\dagger U_\ell = V_{\rm MNSP} \quad . \tag{42}$$

where $V_{\rm MNSP}$ is the Maki-Nakagawa-Sakata-Pontcorvo lepton mixing matrix. The couplings of the gauge bosons relevant for the B_s system in the mass basis are:

$$\begin{aligned} L = f\Big[& G_1^\mu . \left(U_{b1}U_{s2}^* \bar{b}_L \gamma_\mu s_L \ + \ U_{s1}U_{b2}^* \bar{s}_L \gamma_\mu b_L + U_{b2}U_{s1}^* \bar{b}_L \gamma_\mu s_L + U_{s2}U_{b1}^* \bar{s}_L \gamma_\mu b_L \right) \\ & + i G_2^\mu \left(-U_{b1}U_{s2}^* \bar{b}_L \gamma_\mu s_L \ - \ U_{s1}U_{b2}^* \bar{s}_L \gamma_\mu b_L + \ U_{b2}U_{s1}^* \bar{b}_L \gamma_\mu s_L \ + \ U_{s2} \ U_{b1}^* \bar{s}_L \gamma_\mu b_L \right) \\ & + G_3^\mu \left(U_{b1}U_{s1}^* \bar{b}_L \gamma_\mu s_L \ + \ U_{s1}U_{b1}^* \bar{s}_L \gamma_\mu b_L - \ U_{b2}U_{s2}^* \bar{b}_L \gamma_\mu \bar{s}_L \ - \ U_{s2}U_{b2}^* \bar{s}_L \ \gamma_\mu \ b_L \right) \Big] \end{aligned} \tag{43}$$

The contribution to $B_s^0 - \bar{B}_s^0$ mixing is given by

$$\Delta M_{B_s}^{(\rm FS)} = \frac{2 M_{B_s} f_{B_s}^2 B_{B_s} r(m_{B_s,M})}{3} \ f^2 \left[\frac{A}{m_1^2} \ + \frac{C}{m_3^2} + \frac{B}{m_2^2} \right] \tag{44}$$

where

$$\begin{aligned} A &= \mathrm{Re}\left[(U_{b1} \ U_{s2}^* + U_{b2} \ U_{s1}^*)^2 \right] \\ B &= -\mathrm{Re}\left[(U_{b1} \ U_{s2}^* - \ U_{b2} \ U_{s1}^*)^2 \right] \\ C &= \mathrm{Re}\left[(U_{b1}U_{s1}^* - U_{b2}U_{s2}^*)^2 \right] \end{aligned} \tag{45}$$

In a simple scheme of symmetry breaking [29], one obtains $m_1 = m_3$ and the square bracket in Eq. (44) becomes

$$\left[\frac{A + C}{m_1^2} \ + \frac{B}{m_2^2} \right] \quad . \tag{46}$$

Although the matrices U_i $(i = d, u, \ell)$ in principle are unknown, it has been argued that a reasonable ansatz [30], which is incorporated in many models is $U_u = I, \ U_d^\dagger = V_{\rm CKM}$. In this case[5] one can simplify A, B and C further:

$$A, B \ll C \simeq 1.6 \times 10^{-3} \quad . \tag{47}$$

[5] Here, we use values listed in Ref. [1].

Thus the B_s mixing becomes

$$\Delta M_{B_s}^{(\mathrm{FS})} \simeq \frac{2M_{B_S} f_{B_s}^2 B_{B_s} r(m_b, M)}{3} \; \frac{f^2}{m_1^2} \; 1.6 \times 10^{-3} \quad , \tag{48}$$

so that, substituting experimental bound $\Delta M_{B_s}^{(\mathrm{FS})} = \Delta M_{B_s}^{(\mathrm{NP})}$,

$$\frac{f^2}{m_1^2} \leq \frac{3|\Delta M_{B_s}^{(\mathrm{NP})}|}{2M_{B_S} f_{B_s}^2 B_{B_s} r(m_b, M) 1.6 \times 10^{-3}} \quad . \tag{49}$$

The same above ansatz also implies that $U_\ell^\dagger = U_{\mathrm{MNSP}}$ and $U_\nu = 1$. Then the coupling of the gauge bosons to muon pairs is given by

$$\begin{aligned} \mathcal{L}_{\mathrm{G}\mu^+\mu^-} &= f \Big[\left(U_{\mu 1}^* U_{\mu 2} + U_{\mu 1} \; U_{\mu 2}^* \right) G_1^\lambda \\ &+ i \left(-U_{\mu 1} \; U_{\mu 2}^* + U_{\mu 1}^* U_{\mu 2} \right) G_2^\lambda \; + \left(U_{\mu 1} \; U_{\mu 1}^* - U_{\mu 2} \; U_{\mu 2}^* \right) G_3^\lambda \Big] \bar{\mu}_L \gamma_\lambda \mu_L \quad . \end{aligned} \tag{50}$$

The branching ratio for $B_s \to \mu^+\mu^-$ is given by

$$\begin{aligned} \mathcal{B}_{B_s \to \mu^+\mu^-} &= \frac{M_{B_S} f_{B_s}^2 \; m_\mu^2}{32\pi \Gamma_{B_s}} \; f^4 \Bigg| \frac{(U_{b1} U_{s2}^* \; + U_{b2} U_{s1}^*) \left(U_{\mu 1} U_{\mu 2}^* + U_{\mu 1}^* \; U_{\mu 2} \right)}{m_1^2} \\ &- \frac{(U_{b1} U_{s2}^* - U_{b2} U_{s1}^*) \left(U_{\mu 1} U_{\mu 2}^* - U_{\mu 2} U_{\mu 1}^* \right)}{m_2^2} + \frac{(U_{b1} U_{s1}^* - U_{b2} U_{s2}^*) \left(U_{\mu 1} U_{\mu 1}^* - U_{\mu 2} U_{\mu 2}^* \right)}{m_3^2} \Bigg|^2 \end{aligned} \tag{51}$$

Next we employ the approximation (well-supported empirically) that $U_{\mathrm{MNSP}} \simeq U_{\mathrm{TBM}}$, where U_{TBM} is the tri-bi-maximal matrix [31]. Then Eq. (50) becomes

$$\mathcal{L}_{\mathrm{G}\mu^+\mu^-} = -f \left[\frac{\sqrt{2}}{3} \; G_1^\mu + \frac{1}{6} \; G_3^\mu \right] \bar{\mu}_L \gamma_\mu \mu_L \quad . \tag{52}$$

With this, the contribution to the branching ratio for $B_s \to \mu^+\mu^-$ becomes

$$\begin{aligned} B_{B_s \to \mu^+\mu^-} &= \frac{M_{B_s} f_{B_s}^2 m_\mu^2 f^4}{32\pi \Gamma_{B_s}} \left[\frac{\sqrt{2}}{3} \left(1.1 \times 10^{-2} \right) \; + \; \frac{1}{6} \times 0.04 \right]^2 \frac{1}{m_1^4} \\ &\simeq \frac{M_{B_s} f_{B_s}^2 m_\mu^2 f^4}{32\pi \Gamma_{B_s}} \; \frac{1.4 \times 10^{-4}}{m_1^4} \quad . \end{aligned} \tag{53}$$

The dependence on unknown factors in Eq. (53) (*i.e.* $(f/m_1)^4$) can be entirely removed by using the bound in Eq. (49) to yield

$$\mathcal{B}_{B_s \to \mu^+\mu^-}^{(\mathrm{FS})} \leq \frac{3.85 m_\mu^2}{\pi M_{B_S} f_{B_s}^2 \Gamma_{B_s} B_{B_s}^2 r^2(m_b, m_1)} \; |\Delta M_{B_s}^{(\mathrm{NP})}|^2 \quad . \tag{54}$$

From the bounds of Eqs. (11),(12), we obtain Family Symmetry branching fractions in the range

$$\mathcal{B}_{B_s \to \mu^+\mu^-}^{(\mathrm{FS})} \leq 0.5 \times 10^{-12} \quad . \tag{55}$$

F. FCNC Higgs interactions

Many extensions of the Standard Model contain multiple scalar doublets, which increases the possibility of FCNC mediated by flavor non-diagonal interactions of neutral components. While many ideas exist on how to suppress those interactions (see, *e.g.* [35–37]), the ultimate test of those ideas would involve direct observation of scalar-mediated FCNC.

Consider a generic Yukawa interaction consisting of a set of N Higgs doublets H_n ($n = 2, .., N$) with SM fermions,

$$\mathcal{H}_Y = \lambda^U_{ijn} \overline{Q}_{Li} U_{Rj} \widetilde{H}_n + \lambda^D_{ijn} \overline{Q}_{Li} D_{Rj} H_n + \lambda^E_{ijn} \overline{L}_{Li} E_{Rj} H_n + \text{h.c.} \quad , \tag{56}$$

where $\widetilde{H}_n = i\sigma_2 H_n^*$ and Q_{Li} (L_{Li}) are respectively the left-handed weak doublets of an ith-generation of quarks (leptons). Restricting the discussion to B_s Mixing and $B_s \to \mu^+\mu^-$ decay, we find that Eq. (56) reduces to

$$\mathcal{H}_Y^H = \lambda^D_{23n} \overline{s}_L b_R \Phi_n^0 \ + \lambda^D_{32n} \overline{b}_L s_R \Phi_n^0 + \lambda^E_{22n} \overline{\mu}_L \mu_R \Phi_n^0 + \text{h.c.}, \tag{57}$$

where $\Phi_n^0 \equiv \left(\phi_n^0 + i a_n^0\right)/\sqrt{2}$. Bringing this to the form of Eq. (19) and confining the discussion only to the contribution of the lightest ϕ_n^0 and a_n^0 states, we obtain

$$\begin{aligned}\mathcal{H}_Y^H = & \frac{\lambda^{D\dagger}_{23}}{\sqrt{2}} \, \overline{b}_R s_L \phi^0 \ + \frac{\lambda^D_{32}}{\sqrt{2}} \, \overline{b}_L s_R \phi^0 + \frac{\lambda^E_{22}}{\sqrt{2}} \, \overline{\mu}_L \mu_R \phi^0 \\ & - i\frac{\lambda^{D\dagger}_{23}}{\sqrt{2}} \, \overline{b}_R s_L a^0 \ + i\frac{\lambda^D_{32}}{\sqrt{2}} \, \overline{b}_L s_R a^0 + \ i\frac{\lambda^E_{22}}{\sqrt{2}} \, \overline{\mu}_L \mu_R a^0 + ... \ + \text{h.c.} \quad ,\end{aligned} \tag{58}$$

where ellipses stand for the terms containing heavier ϕ_n^0 and a_n^0 states whose contributions to ΔM_{B_s} and $\mathcal{B}_{B_s \to \mu^+\mu^-}$ will be suppressed.

If the matrix of coupling constants in Eq. (58) is Hermitian, *e.g.* $\lambda^{D\dagger}_{23} = \lambda^D_{32}$, then we can identify the couplings of Eq. (19) as

$$g_{S_1} = g_{S_2} = \frac{\lambda^D_{32}}{\sqrt{2}}, \qquad g'_{S_1} = g'_{S_2} = \frac{\lambda^E_{22}}{\sqrt{2}} \tag{59}$$

for *scalar* interactions and

$$g_{S_1} = -g_{S_2} = \frac{i\lambda^D_{32}}{\sqrt{2}}, \quad g'_{S_1} = -g'_{S_2} = \frac{i\lambda^E_{22}}{\sqrt{2}} \tag{60}$$

for *pseudoscalar* interactions.

To proceed, we need to separate two cases: (i) the lightest FCNC Higgs particle is a scalar, and (ii) the lightest FCNC Higgs particle is pseudoscalar.

1. *Light scalar FCNC Higgs*

The case of relatively light scalar Higgs state is quite common, arising most often in Type-III two-Higgs doublet models (models without natural flavor conservation) [38, 39, 41].

B_s^0-$\bar{B}_s^0$ *Mixing*: Given the general formulas of Eq. (20), it is easy to compute the contribution to $\Delta M_{\mathrm{B}_s}^{(\phi)}$ of an intermediate scalar (ϕ) with FCNC couplings,

$$\Delta M_{\mathrm{B}_s}^{(\phi)} = \frac{5 f_{B_s}^2 M_{B_s} f_\phi(\overline{C}_i, m_b)}{48} \left(\frac{\lambda_{32}^D}{M_\phi}\right)^2 \quad , \tag{61}$$

$$f_\phi(\overline{C}_i, m_b) \equiv \frac{7}{5}\overline{C}_3(m_b)B_3 - \left(\overline{C}_4(m_b)B_4 + \overline{C}_7(m_b)B_7\right) + \frac{12}{5}\left(\overline{C}_5(m_b)B_5 + \overline{C}_8(m_b)B_8\right) \ ,$$

with 'reduced' Wilson coefficients $\{\overline{C}_i(\mu)\}$ given in Eq. (21).

$B_s^0 \to \mu^+\mu^-$ *Decay*: Comparing Eq. (59) to Eq. (23), we can easily see that the branching fraction for the rare decay $B_s^0 \to \ell^+\ell^-$ is zero for the intermediate scalar Higgs,

$$\mathcal{B}^{(\phi)}_{B_s^0 \to \ell^+\ell^-} = 0 \quad . \tag{62}$$

This is consistent with what was already discussed in Sec. III B and implies that the FCNC Higgs model does not produce a contribution to $B_s^0 \to \mu^+\mu^-$ at tree level. The non-zero contribution to $B_s^0 \to \mu^+\mu^-$ decay is produced at one-loop level [40].

2. *Light pseudoscalar FCNC Higgs*

The case of a lightest *pseudoscalar* Higgs state can occur in the non-minimal supersymmetric standard model (NMSSM) [42, 43, 45, 46]. or related models [44]. In NMSSM, a singlet pseudoscalar is introduced to dynamically solve the μ problem. The resulting pseudoscalar can be very light with a mass as light as tens of GeV. This does not mean, however, that it necessarily gives the dominant contribution to both $B_s^0 - \overline{B}_s^0$ mixing and the $B_s^0 \to \mu^+\mu^-$ decay rate since there can be loop contributions from other Higgs states. In the following, we shall work in the region of the parameter space where it does.

B_s^0-$\bar{B}_s^0$ *Mixing*: The contribution to $\Delta M_{\mathrm{B}_s}^{(\mathrm{a})}$ due to intermediate pseudoscalar with flavor-changing couplings can be computed using the general formula in Eq. (20) along with the identification given in Eq. (60),

$$\Delta M_{\mathrm{B}_s}^{(\mathrm{a})} = \frac{5 f_{B_s}^2 M_{B_s} f_a(\overline{C}_i, m_b)}{48} \left(\frac{\lambda_{32}^D}{M_a}\right)^2 \quad , \tag{63}$$

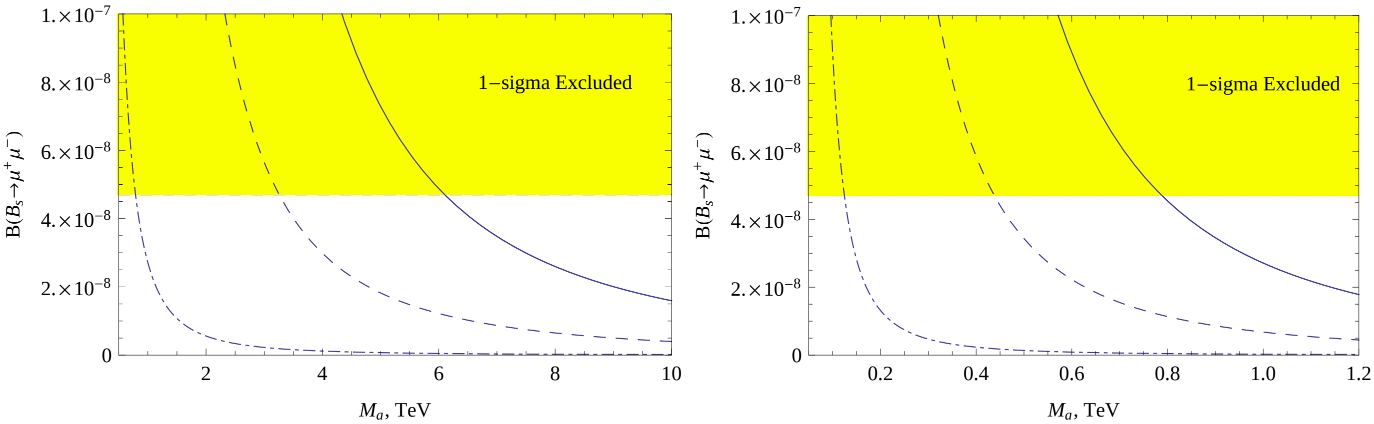

FIG. 2: Branching ratio of $\mathcal{B}_{B_s^0 \to \mu^+\mu^-}$ as a function of pseudoscalar Higgs mass M_a. Left: $\lambda_{22}^E = 1, 0.5, 0.1$ (solid, dashed, dash-dotted lines). Right: $\lambda_{22}^E = 0.1, 0.05, 0.01$ (solid, dashed, dash-dotted lines). In each figure, the yellow shaded area represents excluded parameter space.

$$f_a(\overline{C}_i, m_b) = \left[\frac{7}{5}\overline{C}_3(m_b)B_3 + \left(\overline{C}_4(m_b)B_4 + \overline{C}_7(m_b)B_7\right) - \frac{12}{5}\left(\overline{C}_5(m_b)B_5 + \overline{C}_8(m_b)B_8\right)\right]$$

with 'reduced' Wilson coefficients $\overline{C}_i(\mu)$ again being defined in Eq. (21).

$B_s^0 \to \mu^+\mu^-$ *Decay*: The branching ratio for rare decay can be computed with the help of the general formula of Eq. (23),

$$\mathcal{B}^{(a)}_{B_s^0 \to \ell^+\ell^-} = \frac{1}{32\pi} \frac{f_B^2 M_{B_s}^5}{m_b^2 \Gamma_{B_s}} \left(1 - \frac{4m_\ell^2}{M_{B_s}^2}\right)^{1/2} \left(\frac{\lambda_{32}^D\, \lambda_{22}^E}{M_a^2}\right)^2 \quad . \tag{64}$$

We can now eliminate one of the three unknown parameters (λ_{32}^D, λ_{22}^E, and M_a) which appear in Eqs.(63) and (64). We choose to eliminate λ_{32}^D, so

$$\mathcal{B}^{(a)}_{B_s^0 \to \ell^+\ell^-} = \frac{3}{10\pi} \cdot \frac{M_{B_s}^4 x_s^{(a)}}{m_b^2 f_a(\overline{C}_i, m_b)} \left(1 - \frac{4m_\ell^2}{M_{B_s}^2}\right)^{1/2} \left(\frac{\lambda_{22}^E}{M_a}\right)^2 \quad , \tag{65}$$

where $x_s^{(a)} = \Delta M_{\mathrm{B}_s}^{(a)}/\Gamma_{B_s}$. As one can see, the unknown factors enter Eq. (65) in the combination λ_{22}^E/M_a. It is, however, more convenient to plot the dependence on M_a for different values of λ_{22}^E, which we present in Fig. 2.

G. Fourth generation models

One of the simplest extensions of the Standard Model involves addition of the sequential fourth generation of chiral quarks [47–49], denoted for the lack of the better names by t' and

b'. The addition of the sequential fourth generation of quarks leads to a 4×4 CKM quark mixing matrix [50]. This implies that the parameterization of this matrix requires six real parameters and three phases. Besides providing the new sources of CP-violation, the two additional phases can affect the branching ratios considered in this paper due to interference effects [51].

There are many existing constraints on the parameters related to the fourth generation of quarks. In particular, a fit of precision electroweak data (S and T parameters) [52–54] implies that the masses of the new quarks are strongly constrained to be [55]

$$m_{t'} - m_{b'} \simeq \left(1 + \frac{1}{5}\frac{m_H}{(115\ \text{GeV})}\right) \times 50\ \text{GeV}, \tag{66}$$

with $m_{t'} > 400$ GeV. Here m_H is the SM Higgs mass, which we take for simplicity to be 120 GeV. We also used updated constraints on CKM matrix elements [56].

The relationship between ΔM_{B_s} and $\mathcal{B}_{B_s\to\mu^+\mu^-}$ in the model with four generations of quarks, which is the main focus of this paper, has been previously studied in detail in [57]. Here we update their result. The branching ratio of $B_s \to \mu^+\mu^-$ can be related to the experimentally-measured[6] x_{B_s} as [57]

$$\mathcal{B}_{B_s\to\mu^+\mu^-} = \frac{3\alpha^2 m_\mu^2 x_{B_s}}{8\pi \hat{B}_{B_s} M_W^2}\sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}}\frac{|C_{10}^{tot}|^2}{|\Delta'|}, \tag{67}$$

where the parameter Δ' is a B_s-mixing loop parameter [57],

$$\Delta' = \eta_t S_0(x_t) + \eta_{t'} R_{t't}^2 S_0(x_{t'}) + 2\eta_{t'} R_{t't} S_0(x_t, x_{t'}), \tag{68}$$

and $R_{t't} = V_{t's}V_{t'b}^*/V_{ts}V_{tb}^*$. $\hat{B}_{B_s}$ can be obtained from Table I. The definition of the function $S_0(x_t, x_{t'})$ can be found in Ref. [57]. The Wilson coefficient C_{10}^{tot} is defined as

$$C_{10}^{tot}(\mu) = C_{10}(\mu) + R_{t't} C_{10}^{t'}(\mu) \tag{69}$$

with $C_{10}^{t'}$ obtained by substituting $m_{t'}$ into the SM expression for C_{10} [58]. The results can be found in Fig. 3. As one can see, the resulting branching ratios are still lower than the current experimental bound of Eq. (6), but for the values of the four-generation CKM matrix $\lambda_{bs}^{t'} = |V_{t's}V_{t'b}^*|$ of about 0.01, disfavored by [56], but still favored by [59], can be quite close to it.

[6] Here we use ΔM_{B_s} from Table I, as the separation of NP and SM contributions used in the rest of this paper, $x_{B_s} = x_{SM3} + x_{SM4}$, is not possible due to loops with both t' and t, c, or u quarks.

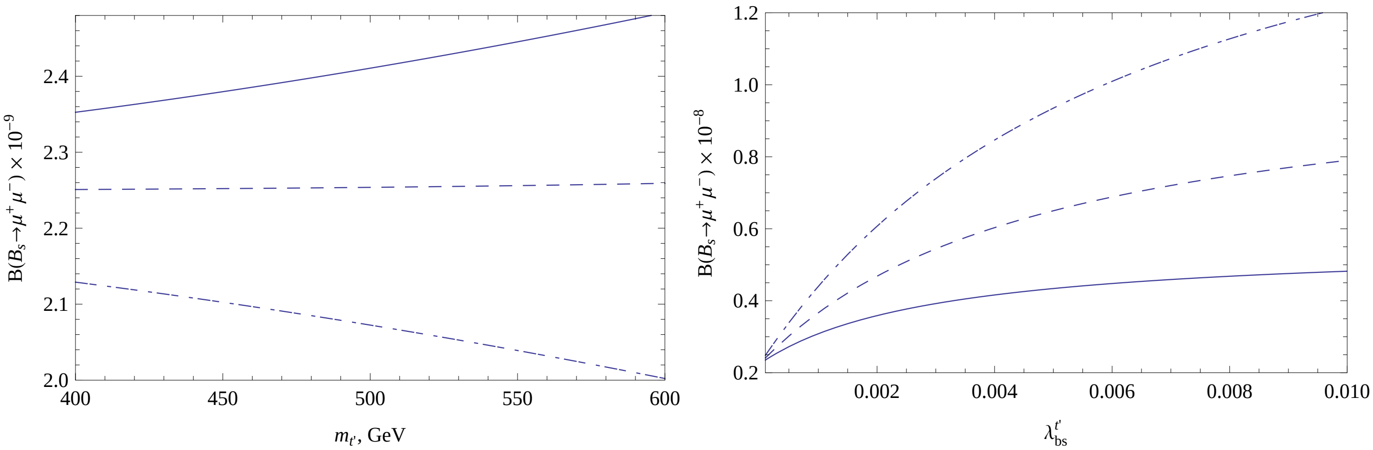

FIG. 3: Left: branching ratio of $\mathcal{B}_{B_s^0\to\mu^+\mu^-}$ as a function of the top-prime mass $m_{t'}$ for different values of the phase $\phi_{t's} = 0, \pi/2, \pi$ (solid, dashed, dash-dotted lines) and $\lambda_{bs}^{t'} = |V_{t's}V_{t'b}^*| \simeq 10^{-4}$ [56] (see also [59]). Right: branching ratio of $\mathcal{B}_{B_s^0\to\mu^+\mu^-}$ as a function of the CKM parameter combination $\lambda_{bs}^{t'}$ with $\phi_{t's} = 0$ and different values of $m_{t'} = 400$ GeV (solid), 500 GeV (dashed), and 600 GeV (dash-dotted).

IV. CONCLUSION

Experiment has determined ΔM_{B_s} exceedingly well. The Standard Model determination provides a consistent value, although with a markedly greater uncertainty (due mainly to the dependence on the nonperturbative quantity $f_{B_s}^2 \hat{B}_{B_s}$ and to a lesser extent on the CKM mixing element V_{ts}). The theoretical SM uncertainties notwithstanding, the good agreement of SM theory with experiment places nontrivial constraints on any possible New Physics parameters. We have argued that these can in turn impact NP predictions for other processes, such as the $B_s \to \mu^+\mu^-$ transition considered here.

We expect this kind of correlation to be a rather general feature of New Physics models, provided there is an overlap between the NP parameters which describe ΔM_{B_s} and (for our purposes here) $B_s \to \mu^+\mu^-$. However, given the abundance of New Physics scenarios, each with its particular structure, it is not reasonable to expect any *universal* correlation between B_s-mixing and $B_s \to \mu^+\mu^-$. Instead, what we have done in this paper is to analyze several NP models in detail. In each case, we have first determined the set of unknown NP parameters and then, using dynamical assumptions, have been able to reduce (or entirely eliminate) the arbitrariness. Analyzing specific NP models this way has two purposes:

to serve as an instructive example for further study and to see what kinds of numerical predictions these particular models yield.

Not surprisingly, the simplest model (with a single Z' boson) provides a strong correlation between ΔM_{B_s} and $B_s \to \mu^+\mu^-$ in which the latter is determined in terms of $M_{Z'}$. An even stronger prediction occurs in the particular version of the Family Symmetry model discussed earlier, where a clean determination of $B_s \to \mu^+\mu^-$ is obtained. In this instance, a set of reasonable assumptions allows for the initial presence of unknown parameters to be totally overcome. A similar, but not quite as fortunate, situation occurs for R-parity violating supersymmetry, wherein a reasonable assumption partially reduces the NP parameter set. In this case, $B_s \to \mu^+\mu^-$ can be expressed in terms of a ratio of a coupling constant and sneutrino mass $M_{\tilde{\nu}}$. The flavor-changing Higgs model turns out to be less accommodating in that no set of assumptions known to us can reduce the original set of three unknown parameters. Thus, the constraint from B_s mixing still leaves one with two unknowns (see Fig. 2). We also updated constraints on the models with fourth sequential generation of quarks.

The numerical results in our study lead us to suspect that in many (if not most) NP models, it will be difficult to generate a NP $B_s \to \mu^+\mu^-$ as large as that in the SM. In our approach, this is a consequence of the SM predicting the measured value of ΔM_{B_s} so well. Of course, additional NP models are available for study, *e.g.* R-parity conserving supersymmetry, and work proceeds on these.

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Appendix A: Choice of the basis and mixing matrix elements

There are eight $\Delta b = 2$ effective operators that can contribute to B_s-mixing. The operator basis we shall employ is

$$\begin{aligned}
Q_1 &= (\overline{b}_L \gamma_\mu s_L)\ (\overline{b}_L \gamma^\mu s_L)\ , & Q_5 &= (\overline{b}_R \sigma_{\mu\nu} s_L)\ (\overline{b}_R \sigma^{\mu\nu} s_L)\ , \\
Q_2 &= (\overline{b}_L \gamma_\mu s_L)\ (\overline{b}_R \gamma^\mu s_R)\ , & Q_6 &= (\overline{b}_R \gamma_\mu s_R)\ (\overline{b}_R \gamma^\mu s_R)\ , \\
Q_3 &= (\overline{b}_L s_R)\ (\overline{b}_R s_L)\ , & Q_7 &= (\overline{b}_L s_R)\ (\overline{b}_L s_R)\ , \\
Q_4 &= (\overline{b}_R s_L)\ (\overline{b}_R s_L)\ , & Q_8 &= (\overline{b}_L \sigma_{\mu\nu} s_R)\ (\overline{b}_L \sigma^{\mu\nu} s_R)\ \ ,
\end{aligned} \tag{A1}$$

where quantities enclosed in parentheses are color singlets, *e.g.* $(\overline{b}_L \gamma_\mu s_L) \equiv \overline{b}_{L,i} \gamma_\mu s_{L,i}$. These operators are generated at a scale M where the NP is integrated out. A non-trivial operator mixing then occurs via renormalization group running of these operators between the heavy scale M and the light scale μ at which hadronic matrix elements are computed.

We need to evaluate the B_s^0-to-$\overline{B}_s^0$ matrix elements of these eight dimension-six basis operators. This introduces eight *non-perturbative* B-parameters $\{B_i\}$ that require evaluation by means of QCD sum rules or QCD-lattice simulation. We express these in the form

$$\begin{aligned}
\langle Q_1 \rangle &= \tfrac{2}{3} f_{\mathrm{B}_s}^2 M_{\mathrm{B}_s}^2 B_1\ , & \langle Q_5 \rangle &= f_{\mathrm{B}_s}^2 M_{\mathrm{B}_s}^2 B_5\ , \\
\langle Q_2 \rangle &= -\tfrac{5}{6} f_{\mathrm{B}_s}^2 M_{\mathrm{B}_s}^2 B_2\ , & \langle Q_6 \rangle &= \tfrac{2}{3} f_{\mathrm{B}_s}^2 M_{\mathrm{B}_s}^2 B_6\ , \\
\langle Q_3 \rangle &= \tfrac{7}{12} f_{\mathrm{B}_s}^2 M_{\mathrm{B}_s}^2 B_3\ , & \langle Q_7 \rangle &= -\tfrac{5}{12} f_{\mathrm{B}_s}^2 M_{\mathrm{B}_s}^2 B_7\ , \\
\langle Q_4 \rangle &= -\tfrac{5}{12} f_{\mathrm{B}_s}^2 M_{\mathrm{B}_s}^2 B_4\ , & \langle Q_8 \rangle &= f_{\mathrm{B}_s}^2 M_{\mathrm{B}_s}^2 B_8\ \ ,
\end{aligned} \tag{A2}$$

where f_{B_s} is the B_s meson decay constant and $\langle Q_i \rangle \equiv \langle \bar{B}_s^0 | Q_i | B_s^0 \rangle$.

Ref. [34] has performed a QCD-lattice determination (quenched approximation) of the B-parameters in an operator basis $\{O_i\}$ which is distinct from the $\{Q_i\}$ of Eq. (A1),

$$\begin{aligned}
O_1 &= \overline{b}^i \gamma_\mu (1+\gamma_5) s^i\ \overline{b}^j \gamma^\mu (1+\gamma_5) s^j\ , \\
O_2 &= \overline{b}^i (1+\gamma_5) s^i\ \overline{b}^j (1+\gamma_5) s^j\ , & O_4 &= \overline{b}^i (1+\gamma_5) s^i\ \overline{b}^j (1-\gamma_5) s^j\ , \\
O_3 &= \overline{b}^i (1+\gamma_5) s^j\ \overline{b}^j (1+\gamma_5) s^i\ , & O_5 &= \overline{b}^i (1+\gamma_5) s^j\ \overline{b}^j (1-\gamma_5) s^i\ .
\end{aligned} \tag{A3}$$

Three more operators O_i $(i = 6, 7, 8)$ can be obtained by substituting right-handed chiral projection operators with the left-handed ones O_i $(i = 1, 2, 3)$ in Eq. (A3). The B_s^0-to-$\overline{B}_s^0$

matrix elements of these operators have been parameterized in Ref. [34] as

$$\begin{aligned}
&\langle O_1\rangle = \tfrac{8}{3} f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} \widetilde{B}_1 \ , && \\
&\langle O_2\rangle = -\tfrac{5}{3} R_s^2 f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} \widetilde{B}_2 \ , && \langle O_4\rangle = 2 R_s^2 f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} \widetilde{B}_4 \ , \\
&\langle O_3\rangle = \tfrac{1}{3} R_s^2 f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} \widetilde{B}_3 \ , && \langle O_5\rangle = \tfrac{2}{3} R_s^2 f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} \widetilde{B}_5 \ .
\end{aligned} \tag{A4}$$

Also, the chiral structure of QCD requires that $\langle O_6\rangle = \langle O_1\rangle$, $\langle O_7\rangle = \langle O_2\rangle$, and $\langle O_8\rangle = \langle O_3\rangle$.

Several of the quantities introduced above are scale dependent, *i.e.* $\{B_i(\mu)\}$, $\{\widetilde{B}_i(\mu)\}$ and $R_s^2(\mu)$. Throughout this paper, we shall understand all these quantities to be renormalized at a common scale $\mu = m_b$ and to simplify notation, we shall denote them simply as $\{B_i\}$, $\{\widetilde{B}_i\}$ and R_s^2. In particular, our evaluation at scale $\mu = m_b$ of the quantity $R_s(\mu) \equiv M_{B_s}/(m_b(\mu) + m_s(\mu))$ yields

$$R_s^2 = M^2_{B_s} / \left(\bar{m}_b(\bar{m}_b) + \bar{m}_s(\bar{m}_b)\right)^2 = 1.57^{+0.04}_{-0.10} \ , \tag{A5}$$

where we have used the input values $\bar{m}_b(\bar{m}_b) = 4.2^{+0.17}_{-0.07}$ GeV [1] and $\bar{m}_s(\bar{m}_b) = 0.085 \pm 0.017$ GeV [8].

The two bases $\{Q_i\}$ and $\{O_i\}$ can be related via Fierz rearrangement,

$$\begin{aligned}
&O_1 = 4\ Q_1 \ , && \\
&O_2 = 4\ Q_4 \ , && O_4 = 4\ Q_3 \ , \\
&O_3 = -2\ Q_4 - \tfrac{1}{2}\ Q_5 \ , && O_5 = -2\ Q_2 \ .
\end{aligned} \tag{A6}$$

from which we find

$$\begin{aligned}
&B_1 = \widetilde{B}_1 \ , && B_5 = -\tfrac{1}{3} R_s^2 \left(2\widetilde{B}_3 - 5\widetilde{B}_2\right) \ , \\
&B_2 = \tfrac{2}{5} \widetilde{B}_5 R_s^2 \ , && B_6 = \widetilde{B}_1 \ , \\
&B_3 = \tfrac{6}{7} \widetilde{B}_4 R_s^2 \ , && B_7 = \tfrac{6}{7} \widetilde{B}_4 R_s^2 \ , \\
&B_4 = \widetilde{B}_2 R_s^2 \ , && B_8 = \tfrac{1}{3} R_s^2 \left(2\widetilde{B}_3 - 5\widetilde{B}_2\right) \ .
\end{aligned} \tag{A7}$$

Alternatively, the B-parameters can be estimated using the 'modified vacuum saturation' (MVS) approach, wherein all matrix elements in Eq. (A2) are written in terms of (known)

List of $\{B_i\}$ (in $\{Q_i\}$ Basis)	$\{B_i\}$ from lattice QCD (from Ref. [34])	B_i in MVS (from Eq. (A8))
$B_1 = B_6$	0.87	0.87
B_2	$0.70 R_s^2$	$0.87 \left[\frac{3}{5} + \frac{2}{5} R_s^2\right]$
B_3	$0.99 R_s^2$	$0.87 \left[\frac{1}{7} + \frac{6}{7} R_s^2\right]$
$B_4 = B_7$	$0.80 R_s^2$	$0.87 R_s^2$
$B_5 = B_8$	$0.71 R_s^2$	$0.87 R_s^2$

TABLE II: Numerical estimates of the B-parameters. The determination from lattice QCD is done in $\overline{\text{MS}}$(NDR).

matrix elements of $(V-A)\times(V-A)$ and $(S-P)\times(S+P)$ matrix elements $B_{\rm B}$ and $B_{\rm B}^{(\rm S)}$,

$$
\begin{aligned}
\langle Q_1 \rangle &= \frac{2}{3} f_{\rm B_s}^2 M_{\rm B_s}^2 B_{B_s} \ , & \langle Q_5 \rangle &= \frac{3}{N_c} f_{\rm B_s}^2 M_{\rm B_s}^2 B_{B_s}\ \eta \ , \\
\langle Q_2 \rangle &= f_{\rm B_s}^2 M_{\rm B_s}^2 B_{B_s} \left[-\frac{1}{2} - \frac{\eta}{N_c}\right] \ , & \langle Q_6 \rangle &= \langle Q_1 \rangle \ , \\
\langle Q_3 \rangle &= f_{\rm B_s}^2 M_{\rm B_s}^2 B_{B_s} \left[\frac{1}{4N_c} + \frac{\eta}{2}\right] \ , & \langle Q_7 \rangle &= \langle Q_4 \rangle \ , \\
\langle Q_4 \rangle &= -\frac{2N_c - 1}{4N_c} f_{\rm B_s}^2 M_{\rm B_s}^2 B_{B_s}\ \eta \ , & \langle Q_8 \rangle &= \langle Q_5 \rangle \ ,
\end{aligned}
\tag{A8}
$$

where we take $N_c = 3$ as the number of colors and define

$$
\eta \equiv \frac{B_{\rm B_s}^{(\rm S)}}{B_{B_s}} \cdot \frac{M_{\rm B_s}^2}{(\bar m_b(\bar m_b) + \bar m_s(\bar m_b))^2} \to R_s^2 \quad \text{for} \quad B_{\rm B_s}^{(\rm S)} = B_{B_s} \ . \tag{A9}
$$

It is instructive to compare how well the MVS approximation estimates the recent lattice results. We provide such a comparison in Table II.

[1] K. Nakamura *et al.* [Particle Data Group Collaboration], J. Phys. G **G37**, 075021 (2010).

[2] M. Bona *et al.* [UTfit Collaboration], PMC Phys. A **3**, 6 (2009) [arXiv:0803.0659 [hep-ph]].

[3] 'Measurement of β_s at CDF', talk by Louise Oakes at Flavor Physics and CP Violation COn-ference 2010, Turin, Italy (May 25-29, 2010); 'New Measurement of B_s Mixing Phase at CDF', talk by Gavril Giurgiu talk delivered at ICHEP 2010 (Paris, France).

[4] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **82**, 032001 (2010) [arXiv:1005.2757 [hep-ex]].

[5] E. Golowich, J. Hewett, S. Pakvasa and A. A. Petrov, Phys. Rev. D **79**, 114030 (2009) [arXiv:0903.2830 [hep-ph]].

[6] E. Golowich, J. Hewett, S. Pakvasa and A. A. Petrov, Phys. Rev. D **76**, 095009 (2007) [arXiv:0705.3650 [hep-ph]].

[7] A. J. Buras, Phys. Lett. B **566**, 115 (2003) [arXiv:hep-ph/0303060].

[8] A. Lenz and U. Nierste, JHEP **0706**, 072 (2007) [arXiv:hep-ph/0612167].

[9] A. J. Buras, M. Jamin and P. H. Weisz, Nucl. Phys. B **347**, 491 (1990).

[10] J. Urban, F. Krauss, U. Jentschura and G. Soff, Nucl. Phys. B **523**, 40 (1998) [arXiv:hep-ph/9710245].

[11] "Top Quark Physics", plenary talk by Mousumi Datta at the 2009 meeting of the Division of Particles and Fields of the American Physical Society, Wayne State University Detroit, MI (7/26/09- 7/31/09).

[12] S. Bethke, Eur. Phys. J. C **64**, 689 (2009) [arXiv:0908.1135 [hep-ph]].

[13] J. Laiho, E. Lunghi and R. S. Van de Water, Phys. Rev. D **81**, 034503 (2010) [arXiv:0910.2928 [hep-ph]].

[14] K. Melnikov and T. v. Ritbergen, Phys. Lett. B **482**, 99 (2000) [arXiv:hep-ph/9912391].

[15] Aida El-Khadra, private communication.

[16] M. Antonelli *et al.*, Phys. Rept. **494**, 197 (2010) [arXiv:0907.5386 [hep-ph]]; see also the web sites for CKMfitter and UTfit [17].

[17] See also the latest results in the CKMfitter web site *http://ckmfitter.in2p3.fr* as well as those for UTfit at *http://www.utfit.org*.

[18] T. Inami and C.S. Lim, Prog. Theor. Phys. **65**, 297 (1981) [Erratum-ibid. **65**, 1772 (1981)].

[19] B.C.K. Casey, *New upper limit on the decay* $B_s \to \mu^+\mu^-$ *from D0*, talk delivered at ICHEP 2010 (Paris, France); see also V. M. Abazov *et al.* [D0 Collaboration], Phys. Lett. B **693**, 539 (2010) [arXiv:1006.3469 [hep-ex]].

[20] G. Burdman, E. Golowich, J. Hewett and S. Pakvasa, Phys. Rev. D **66**, 014009 (2002) [arXiv:hep-ph/0112235].

[21] A. Kundu and J. P. Saha, Phys. Rev. D **70**, 096002 (2004) [arXiv:hep-ph/0403154].

[22] Y. Kao and T. Takeuchi, arXiv:0910.4980 [hep-ph].

[23] H. K. Dreiner, M. Kramer and B. O'Leary, Phys. Rev. D **75**, 114016 (2007) [arXiv:hep-ph/0612278].

[24] J. P. Saha and A. Kundu, Phys. Rev. D **66**, 054021 (2002) [arXiv:hep-ph/0205046]; we concur with a conclusion of Ref. [23] regarding the missing factor of 4 in this paper.

[25] M. Sher, Phys. Rept. **179**, 273 (1989).

[26] S. Pakvasa and H. Sugawara, Phys. Lett. B **73**, 61 (1978).

[27] T. Maehara and T. Yanagida, Lett. Nuovo Cim. **19**, 424 (1977); M. A. B. Beg and A. Sirlin, Phys. Rev. Lett. **38**, 1113 (1977); C. L. Ong, Phys. Rev. D **19**, 2738 (1979); F. Wilczek and A. Zee, Phys. Rev. Lett. **42**, 421 (1979); A. Davidson, M. Koca and K. C. Wali, Phys. Rev. D **20**, 1195 (1979), Phys. Rev. Lett. **43**, 92 (1979).

[28] S. Weinberg, UTTG-05-91, Proceedings *High Energy Physics and Cosmology* (Islamabad, Pakistan), M.A.B. Beg Memorial Volume.

[29] V. A. Monich, B. V. Struminsky and G. G. Volkov, Phys. Lett. B **104**, 382 (1981) [JETP Lett. **34**, 213.1981 ZETFA,34,222 (1981 ZETFA,34,222-225.1981)].

[30] J.D. Bjorken, S. Pakvasa and S.F. Tuan, Phys. Rev. D **66**, 053008 (2002) [arXiv:hep-ph/0206116].

[31] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002) [arXiv:hep-ph/0202074].

[32] A. Badin, F. Gabbiani and A. A. Petrov, Phys. Lett. B **653**, 230 (2007) [arXiv:0707.0294 [hep-ph]]; see also A. Badin, F. Gabbiani and A. A. Petrov, arXiv:0909.4897 [hep-ph].

[33] E. Golowich, S. Pakvasa and A.A. Petrov, Phys. Rev. Lett. **98**, 181808-1 (2007) [arXiv:hep-ph/0610039].

[34] D. Becirevic, V. Gimenez, G. Martinelli, M. Papinutto and J. Reyes, JHEP **0204**, 025 (2002) [arXiv:hep-lat/0110091].

[35] L. J. Hall and S. Weinberg, Phys. Rev. D **48**, 979 (1993) [arXiv:hep-ph/9303241].

[36] T. P. Cheng and M. Sher, Phys. Rev. D **35**, 3484 (1987).

[37] A. Pich and P. Tuzon, Phys. Rev. D **80**, 091702 (2009) [arXiv:0908.1554 [hep-ph]].

[38] V. D. Barger, J. L. Hewett and R. J. N. Phillips, Phys. Rev. D **41**, 3421 (1990).

[39] D. Atwood, L. Reina and A. Soni, Phys. Rev. D **55**, 3156 (1997) [arXiv:hep-ph/9609279].

[40] R. A. Diaz, R. Martinez and C. E. Sandoval, Eur. Phys. J. C **41**, 305 (2005) [arXiv:hep-ph/0406265].

[41] A. E. Blechman, A. A. Petrov and G. Yeghiyan, JHEP **1011**, 075 (2010) [arXiv:1009.1612 [hep-ph]].

[42] H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B **120**, 346 (1983); J. M. Frere, D. R. T. Jones and S. Raby, Nucl. Phys. B **222**, 11 (1983); J. P. Derendinger and C. A. Savoy, Nucl. Phys. B **237**, 307 (1984).

[43] J. R. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. D **39**, 844 (1989).

[44] B. A. Dobrescu, Phys. Rev. D **63**, 015004 (2001) [arXiv:hep-ph/9908391].

[45] U. Ellwanger, M. Rausch de Traubenberg and C. A. Savoy, Nucl. Phys. B **492**, 21 (1997) [arXiv:hep-ph/9611251].

[46] G. Hiller, Phys. Rev. D **70**, 034018 (2004) [arXiv:hep-ph/0404220].

[47] B. Holdom, W. S. Hou, T. Hurth *et al.*, PMC Phys. **A3**, 4 (2009) [arXiv:0904.4698 [hep-ph]].

[48] A. J. Buras, B. Duling, T. Feldmann *et al.*, JHEP **1009**, 106 (2010) [arXiv:1002.2126 [hep-ph]].

[49] W. -S. Hou, C. -Y. Ma, Phys. Rev. **D82**, 036002 (2010) [arXiv:1004.2186 [hep-ph]].

[50] M. S. Chanowitz, Phys. Rev. **D79**, 113008 (2009) [arXiv:0904.3570 [hep-ph]].

[51] M. Bobrowski, A. Lenz, J. Riedl and J. Rohrwild, Phys. Rev. D **79**, 113006 (2009) [arXiv:0902.4883 [hep-ph]].

[52] V. A. Novikov, L. B. Okun, A. N. Rozanov *et al.*, Mod. Phys. Lett. **A10**, 1915-1922 (1995).

[53] V. A. Novikov, L. B. Okun, A. N. Rozanov *et al.*, Phys. Lett. **B529**, 111-116 (2002). [hep-ph/0111028].

[54] J. Erler and P. Langacker, Phys. Rev. Lett. **105**, 031801 (2010) [arXiv:1003.3211 [hep-ph]].

[55] G. D. Kribs, T. Plehn, M. Spannowsky and T. M. P. Tait, Phys. Rev. D **76**, 075016 (2007) [arXiv:0706.3718 [hep-ph]].

[56] A. K. Alok, A. Dighe and D. London, arXiv:1011.2634 [hep-ph].

[57] A. Soni, A. K. Alok, A. Giri, R. Mohanta and S. Nandi, Phys. Rev. D **82**, 033009 (2010) [arXiv:1002.0595 [hep-ph]].

[58] A. J. Buras, M. Munz, Phys. Rev. **D52**, 186-195 (1995). [hep-ph/9501281].

[59] S. Nandi and A. Soni, arXiv:1011.6091 [hep-ph].