# Possible $2 S$ and $1 D$ charmed and charmed-strange mesons 

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#### Abstract

Possible $2 S$ and $1 D$ excited $D$ and $D_{s}$ states are studied, the charmed states $D(2550)^{0}, D^{*}(2600)$, $D(2750)^{0}$ and $D^{*}(2760)$ newly observed by the BaBar Collaboration are analyzed. The masses of these states are explored within the Regge trajectory phenomenology, and the strong decay widths are computed through the method proposed by Eichten et al. [1]. Both the mass and the decay width indicate that $D(2550)^{0}$ is a good candidate of $2^{1} S_{0} . D^{*}(2600)$ and $D_{s 1}^{*}(2700)^{ \pm}$are very possible the admixtures of $2^{3} S_{1}$ and $1^{3} D_{1}$ with $J^{P}=1^{-}$and a mixing angle $\phi \approx 19^{0} . D^{*}(2760)$ and $D_{s J}^{*}(2860)^{ \pm}$ are possible the $1^{3} D_{3} D$ and $D_{s}$, respectively. $D(2750)^{0}$ and $D^{*}(2760)$ seem two different states, and $D(2750)^{0}$ is very possible the $1 D\left(2^{-}, \frac{5}{2}\right)$ though the possibility of $1 D\left(2^{-}, \frac{3}{2}\right)$ has not been excluded. There may exist an unobserved meson $D_{s J}(2850)^{ \pm}$corresponding to $D_{s J}^{*}(2860)^{ \pm}$.


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## I. INTRODUCTION

The properties of $2 S$ and $1 D Q \bar{q}$ mesons have been studied for a long time. However, no such higher excited $Q \bar{q}$ state has been established for lack of experimental data. In the past years, some higher excited charmed or charmed-strange states were reported though most of them have not yet been pinned down [2]. It will be useful to systemically study the possible $2 S$ and $1 D$ charmed and charmed-strange mesons in time.

The first possible charmed radial excitation, $D^{* \prime}(2640)$, was reported by DELPHI [3]. This state is difficult to be understood as a charmed radially excited state for the observed decaying channel $D^{*+} \pi^{+} \pi^{-}$and decay width $<15 \mathrm{MeV}$ [4]. Its existence has not yet been confirmed by other collaboration. $D_{s J}(2632)^{+}$is another puzzling state firstly observed by SELEX [5]. It has not been observed by other collaboration either. $D_{s J}(2632)^{+}$seems impossible a conventional $c \bar{s}$ meson for its narrow decay width and anomalous branching ratio $\Gamma\left(D^{0} K^{+}\right) / \Gamma\left(D_{s}^{+} \eta\right)=0.14 \pm 0.06$ [6] even if it does exist. In an early analysis of the spectrum within Regge trajectories phenomenology [7], it is pointed out that $D^{* \prime}(2640)$ and $D_{s J}(2632)^{+}$seem not the orbital excited tensor states or the first radially excited state.

The observation of another three $D_{s}$ mesons: $D_{s 1}^{*}(2700)^{ \pm} \quad[8-10], \quad D_{s J}^{*}(2860)^{ \pm}$[9, 10] and $D_{s J}(3040)^{+}$[10], has evoked much more study of highly excited $Q \bar{q}$ mesons. The masses and the decay widths of $D_{s 1}^{*}(2700)^{ \pm}$and $D_{s J}^{*}(2860)^{ \pm}$were reported by experiments. Furthermore, the ratios of branching fractions, $\frac{\mathcal{B}\left(D_{s 1}^{*}(2700)^{ \pm} \rightarrow D^{*} K\right)}{\mathcal{B}\left(D_{s 1}^{*}(2700)^{ \pm} \rightarrow D K\right)}=0.91 \pm 0.13_{\text {stat }} \pm 0.12_{\text {syst }}$ and $\frac{\mathcal{B}\left(D_{s, J}^{*}(2860)^{+} \rightarrow D^{*} K\right)}{\mathcal{B}\left(D_{s, J}^{*}(2860)^{+} \rightarrow D K\right)}=1.10 \pm 0.15_{\text {stat }} \pm 0.19_{\text {syst }}$, were measured. These states have been explored within some models. $D_{s 1}^{*}(2700)^{ \pm}$was identified with the first radial

[^0]excitation of $D_{s}^{*}(2112)^{ \pm}$[11, 12], or the $D_{s}\left(1^{3} D_{1}\right)$ [13], or the mixture of them 11]. $D_{s J}^{*}(2860)^{ \pm}$was interpreted as the $D_{s}\left(2^{3} P_{0}\right)$ 13, 14] or the $D_{s}\left(1^{3} D_{1}\right)$ 12, 13, 15]. $D_{s J}(3040)^{+}$was identified with the radially excited $D_{s}\left(2 \frac{1}{2}^{+}\right)$12, 16]. However, theoretical predictions of these states are not completely consistent with experiments either on their spectrum or on their decay widths.

Four new charmed states, $D(2550)^{0}, \quad D^{*}(2600)^{0}$, $D(2750)^{0}$ and $D^{*}(2760)^{0}$ (including two isospin partners $D^{*}(2600)^{+}$and $D^{*}(2760)^{+}$) were recently observed by the BaBar collaboration [17]. Some ratios of branching fractions of $D^{*}(2600)^{0}$ and $D(2750)^{0}$ were also measured. In their report, analysis of the masses and helicity-angle distributions indicates that $D(2550)^{0}$ and $D^{*}(2600)$ are possible the first radially excited $S$-wave states $D\left(2^{1} S_{0}\right)$ and $D\left(2^{3} S_{1}\right)$, respectively, while other two charmed candidates are possible the $1 D$ orbitally excited states.

Theoretical analyses indicate that $D(2550)^{0}$ is a good candidate of $2^{1} S_{0}$ though the predicted narrow width of $2^{1} S_{0}$ is inconsistent with the observation [18, 19]. $D^{*}(2600)^{0}$ is interpreted as a mixing state of $2^{3} S_{1}$ and $1^{3} D_{1}$ [18, 19]. The calculation in Ref. [18] indicates that $D^{*}(2760)^{0}$ can be regarded as the orthogonal partner of $D^{*}(2600)^{0}$ (or $\left.1^{3} D_{3}\right)$, but this possibility (or $D^{*}(2760)$ is predominantly the $1^{3} D_{1}$ ) was excluded in Ref. [19], where $D^{*}(2760)$ is identified with the $1^{3} D_{3}$ state. In Ref. [19], the identification of $D(2750)^{0}$ and $D^{*}(2760)$ with the same resonance with $J^{P}=3^{-}$does not favored.

Obviously, these $D$ and $D_{s}$ candidates have not yet been pinned down. In addition to some theoretical deviations from experiments, some theoretical predictions of their strong decays are different in different models. Systematical study of these possible $2 S$ and $1 D$ states in more models is required. In this paper, the method proposed by Eichten et al. 1] is employed to study the strong decay of the heavy-light mesons. We will label them with the notaion $n L\left(J^{P}, j_{q}\right)$ in most cases, where $n$ is the radial quantum number, $L$ is the orbital angular momentum, $J^{P}$ refers to the total angular momentum
and parity, $j_{q}$ is the total angular momentum of the light degrees of freedom.

The paper is organized as follows. In Sec.II, the spectrum of $2 S$ and $1 D D_{s}$ and $D$ will be examined within the Regge trajectory phenomenology. In Sec.III, twobody strong decay of these states will be explored with EHQ's method. Finally, we present our conclusions and discussions in Sec.IV.

## II. MASS SPECTRUM IN REGGE TRAJECTORIES

Linearity of Regge trajectories (RTs) is an important observation in particle physics [22]. In the relativized quark model [20], the RTs for normal mesons are linear. For $Q \bar{q}$ mesons, the approximately linear, parallel and equidistant RTs were obtained both in $\left(J, M^{2}\right)$ and in $\left(n_{r}, M^{2}\right)$ planes in the framework of a QCD-motivated relativistic quark model 23].

However, when RTs are reconstructed with the experimental data, the linearity is always approximate. For orbitally excited states, Tang and Norbury plotted many RTs of mesons and indicated that the RTs are non-linear and intersecting [24]:

$$
\begin{equation*}
M^{2}=a J^{2}+b J+c \tag{1}
\end{equation*}
$$

where the coefficients $a, b, c$ are fixed by the experimental data, and $|a| \ll|b|$ [24]. The coefficients are usually different for different RTs.

For radially excited light $q \bar{q}$ mesons, Anisovich et al. systematically studied the trajectories on the planes ( $n, M^{2}$ ) in the mass region up to $M<2400 \mathrm{MeV}$ [25]. The RTs on $\left(n, M^{2}\right)$ plots behave as

$$
\begin{equation*}
M^{2}=M_{0}^{2}+(n-1) \mu^{2} \tag{2}
\end{equation*}
$$

where $M_{0}$ is the mass of the basic meson, $n$ is the radial quantum number, and $\mu^{2}$ is the slope parameter of the trajectory.

Possible $1 S$ and $2 S D$ and $D_{s}$ states are listed in Table [] , where ${ }^{\dagger} D_{s}^{\prime}(2635)$ is the predicted mass of $2 S\left(1^{-}, \frac{1}{2}\right) D_{s}$ meson. It is easy to notice that these candidates of $1 S$ and $2 S$ meet well with the trajectories on the ( $n, M^{2}$ ) plot according to Eq. (2). The narrow charmed strange state $D_{s J}(2632)^{+}$is located around the mass region of $2 S D_{s}$. However, the exotic relative branching ratio $\Gamma\left(D^{0} K^{+}\right) / \Gamma\left(D_{s}^{+} \eta\right)=0.14 \pm 0.06$ excludes its $2 S\left(1^{-}, \frac{1}{2}\right)$ possibility. Therefore, we denotes the $2 S\left(1^{-}, \frac{1}{2}\right) D_{s}$ meson with ${ }^{\dagger} D_{s}^{\prime}(2635)$. As pointed in Ref. 12], the $2 P$ candidate $D_{s J}(3040)^{+}$meets well with the trajectory on the $\left(n, M^{2}\right)$ plot.

The measured masses of $D(2750)^{0}, D^{*}(2760)$ and $D_{s J}^{*}(2860)^{ \pm}$seem a little lower than most theoretical predictions of the $1 D$ states [20, 21, 23]. In Fig.1, non-linear RTs of $D$ and $D_{s}$ states consisting of $1^{3} S_{1}\left(1^{-}\right), 1^{3} P_{2}\left(2^{+}\right)$ and $1^{3} D_{3}\left(3^{-}\right)$were reconstructed, where the polynomial fits indicate $|a| \ll|b|$. In a relativistic flux tube model,

| States | $\left(0^{-}, \frac{1}{2}\right)$ | $\left(1^{-}, \frac{1}{2}\right)$ | $\left(0^{-}, \frac{1}{2}\right)$ | $\left(1^{-}, \frac{1}{2}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| 2 S | $D(2550)^{0}$ | $D_{1}^{*}(2600)$ | ${ }^{\dagger} D_{s}^{\prime}(2635)$ | $D_{s 1}^{*}(2700)^{ \pm}$ |
| 1 S | $D(1869)^{ \pm}$ | $D^{*}(2007)^{0}$ | $D_{s}(1968)^{ \pm}$ | $D_{s}^{*}(2112)^{ \pm}$ |
| $\mu^{2}\left(\mathrm{GeV}^{2}\right)$ | 2.97 | 2.78 | 3.07 | 2.88 |

TABLE I: $1 S$ and $2 S D$ and $D_{s}$ mesons.
a ratio $b_{h l} / b_{l l}=2$ was obtained at the lowest order [26], where $b_{h l}$ is the coefficient for the heavy-light meson and $b_{l l}$ is the coefficient for the light-light meson in Eq. (1). The $b_{l l}$ (about $0.70 \sim 1.60$ ) has been obtained in Ref. [24]. the fitted $b_{h l}$ of $D$ and $D_{s}$ in Fig. 1 is about 2.74 and 3.03 , respectively. Obviously, the fitted ratio is consistent with the theoretical prediction.


FIG. 1: Non-linear RTs of the $D$ and $D_{s}$ triplet with $N$, $S=1$. The polynomial fits are $M^{2}=-0.23 J^{2}+2.74 J+1.53$ $\left(\mathrm{GeV}^{2}\right)$ and $M^{2}=-0.29 J^{2}+3.03 J+1.72\left(\mathrm{GeV}^{2}\right)$, respectively.

Through the analysis of the spectrum only, $D(2550)^{0}$, $D^{*}(2600)$ and $D_{s 1}^{*}(2700)^{ \pm}$are very possible the first radially excited $D$ and $D_{s}$ states, and $D^{*}(2760)$ and $D_{s J}^{*}(2860)^{ \pm}$are possible the $1^{3} D_{3}$ states.

However, as well known, the RTs can only give a preliminary analysis of the observed states, the investigation of the decay widths and the ratios of branching fractions will be more useful to shed light on the underlying properties of these states.

## III. DECAY WIDTH IN EHQ'S FORMULA

The decay properties of heavy-light mesons have been studied in detail in the heavy quark effective theory (HQET). Here, a concise method proposed by Eichten et al. [1] is employed to study the decays of $D$ and $D_{s}$ mesons. As well known, in the heavy quark symmetry theory, the heavy-light mesons degenerate in $j_{l}^{P}$, i.e., two orbital ground states form a spin doublet $1 S\left(0^{-}, 1^{-}\right)$with $j_{l}^{P}=\frac{1}{2}^{-}$, and the decay amplitude satisfies certain symmetry relations due to the heavy quark symmetry [27].

Therefore, the decays of the two mesons in one doublet are governed by the same transition strength.

In the decay of an excited heavy-light meson $H$, characterized by $n L\left(J^{P}, j_{l}\right)$, to a heavy-light meson $H^{\prime}$ $\left(n^{\prime} L^{\prime}\left(J^{\prime P^{\prime}}, j^{\prime}\right)\right)$ and a light hadron $h$ with spin $s_{h}$ and orbital angular momentum $l$ relative to $H^{\prime}$, the two-body strong decay width (the EHQ's formula) is written as [1]

$$
\begin{equation*}
\Gamma^{H \rightarrow H^{\prime} h}=\zeta\left(\mathcal{C}_{j_{h}, j_{q}, J}^{s_{Q}, j^{\prime}, J^{\prime}}\right)^{2} \mathcal{F}_{j_{h}, l}^{j_{q}, j_{q}^{\prime}}(0) p^{2 l+1} \exp \left(-\frac{p^{2}}{6 \beta^{2}}\right) \tag{3}
\end{equation*}
$$

Where

$$
\mathcal{C}_{j_{h}, J, j_{q}}^{s_{Q}, j_{q}^{\prime}, J^{\prime}}=\sqrt{\left(2 J^{\prime}+1\right)\left(2 j_{q}+1\right)}\left\{\begin{array}{ccc}
s_{Q} & j_{q}^{\prime} & J^{\prime} \\
j_{h} & J & j_{q}
\end{array}\right\}
$$

and $\vec{j}_{h}=\vec{s}_{h}+\vec{l} . \mathcal{F}_{j_{h}, l}^{j_{q}, j_{q}^{\prime}}(0)$ is the transition strength, and $p$ is the momentum of decay products in the rest frame of $H$. The coefficients $\mathcal{C}$ depend only upon the total angular momentum $j_{h}$ of the light hadron, and not separately on its spin $s_{h}$ and the orbital angular momentum $l$ of the decay. The flavor factor $\zeta$ used in this paper for different decay channels can be found in Ref. [21].

For lack of measurements of partial widths in the charmed states, the decay width of $K$ mesons (i.e. $\left.K_{1}(1270) \rightarrow \rho K\right)$ was used to fix the transition strength in Ref. [1]. $c$ and $b$ quarks are much heavier than $u, d$ and $s$ quarks, so the open charmed or bottomed mesons provide better place to test EHQ's formula. Systematical study of $S$ - and $P$-wave heavy-light meons ( $D, B$, $D_{s}$ and $B_{s}$ mesons) by EHQ's formula have been presented in Ref. [28]. In the reference, the EHQ's formula is also obtained by the well-known ${ }^{3} P_{0}$ model [29]. In this way, the transition strength $\mathcal{F}_{j_{h}, l}^{j_{q}, j_{q}^{\prime}}(0)$ obtained in the ${ }^{3} P_{0}$ model includes only two parameters, the dimensionless parameter $\gamma$ and the wave function inverse length scale $\beta(0.35 \sim 0.40 \mathrm{GeV})$ [28].

The relevant transition strengths $\mathcal{F}_{j_{h}, l}^{j_{q}, j_{q}^{\prime}}(0)$ used in this paper are given in Table II. Some expressions in the table are from Ref. [30, 31], and others are obtained in the ${ }^{3} P_{0}$ model in detail in Ref. [28]. For these transition strengths, a constant

$$
\begin{equation*}
\mathcal{G}=\pi^{1 / 2} \gamma^{2} \frac{2^{10}}{3^{4}} \frac{\widetilde{M}_{B} \widetilde{M}_{C}}{\widetilde{M}_{A}} \frac{1}{\beta} \tag{4}
\end{equation*}
$$

was omitted. Here the phase space normalization of Kokoski and Isgur is employed 20, 31].

In the analysis follows, the decay widths of possible $2 S$ and $1 D D$ and $D_{s}$ states are computed in terms of Eq. (3).
(1). $2^{1} S_{0}$ or $\left[2 S\left(0^{-}, \frac{1}{2}\right)\right]$
$D(2550)^{0}$ observed in the decay channel $D^{*+} \pi^{-}$is a good candidate of $2^{1} S_{0}$ charmed meson. Following the procedure in Ref. [28], we take the decay width of $D_{2}^{*}(2460)^{0}$ as an input and obtain the $d$-wave transition strength $\mathcal{F}_{2,2}^{\frac{3}{2}, \frac{1}{2}}(0)=0.964 \mathrm{GeV}^{-4}$ with $\beta=0.38 \mathrm{GeV}^{-1}$,

| $n L\left(j_{l}^{P}\right) \rightarrow n L\left(j_{l}^{P}\right)+\mathcal{P}$ | $\mathcal{F}_{j_{h}, l^{\prime},}^{j_{q}^{\prime}}(0)$ | Polynomial of $p / \beta$ |
| :---: | :---: | :---: |
| $2 S\left(\frac{1}{2}^{-}\right) \rightarrow 1 S\left(\frac{1}{2}^{-}\right)+0^{-}$ | $\mathcal{F}_{1,1}^{\frac{1}{2}, \frac{1}{2}}(0)$ | $\frac{5^{2}}{3^{4}} \frac{1}{\beta^{2}}\left(1-\frac{2}{15} \frac{p^{2}}{\beta^{2}}\right)^{2}$ |
| $2 S\left(\frac{1}{2}^{-}\right) \rightarrow 1 P\left(\frac{1}{2}^{+}\right)+0^{-}$ | $\mathcal{F}_{0,0}^{\frac{1}{2}, \frac{1}{2}}(0)$ | $\frac{1}{2 \cdot 3^{3}}\left(1-\frac{7}{9} \frac{p^{2}}{\beta^{2}}+\frac{2}{27} \frac{p^{4}}{\beta^{4}}\right)^{2}$ |
| $\left.\underline{2 S\left(1^{-}\right.}{ }^{-}\right) \rightarrow 1 P\left(\frac{3}{2}^{+}\right)+0^{-}$ | $\mathcal{F}_{2,2}^{\frac{1}{2}, \frac{3}{2}}(0)$ | $\frac{13^{2}}{3^{7}} \frac{1}{\beta^{4}}\left(1-\frac{2}{39} \frac{p^{2}}{\beta^{2}}\right)^{2}$ |
| $1 D\left(\frac{3}{2}^{-}\right) \rightarrow 1 S\left(\frac{1}{2}^{-}\right)+0^{-}$ | $\mathcal{F}_{1,1}^{\frac{3}{2}, \frac{1}{2}}(0)$ | $\frac{5 \cdot 2}{3^{4}} \frac{1}{\beta^{2}}\left(1-\frac{2}{15} \frac{p^{2}}{\beta^{2}}\right)^{2}$ |
| $1 D\left(\frac{3}{2}^{-}\right) \rightarrow 1 S\left(\frac{1}{2}^{-}\right)+1^{-}$ | $\mathcal{F}_{1,1}^{\frac{3}{2}, \frac{1}{2}}(0)$ | $\frac{2^{2}}{3^{4}} \frac{1}{\beta^{2}}\left(1-\frac{2}{15} \frac{p^{2}}{\beta^{2}}\right)^{2}$ |
| $1 D\left(\frac{3}{2}^{-}\right) \rightarrow 1 P\left(\frac{1}{2}^{+}\right)+0^{-}$ | $\mathcal{F}_{2,2}^{\frac{3}{2}, \frac{1}{2}}(0)$ | $\frac{5}{3^{7}} \frac{1}{\beta^{4}}\left(1+\frac{2}{15} \frac{p^{2}}{\beta^{2}}\right)^{2}$ |
| $1 D\left(\frac{3}{2}^{-}\right) \rightarrow 1 P\left(\frac{3}{2}^{+}\right)+0^{-}$ | $\begin{aligned} & \mathcal{F}_{0,0}^{\frac{3}{2}, \frac{3}{2}}(0) \\ & \mathcal{F}_{2,2}^{\frac{3}{2}, \frac{3}{2}}(0) \\ & \hline \end{aligned}$ | $\begin{gathered} \frac{2^{2} \cdot 5}{3^{3}}\left(1-\frac{5}{18} \frac{p^{2}}{\beta^{2}}+\frac{1}{135} \frac{p^{4}}{\beta^{4}}\right)^{2} \\ \frac{13^{2}}{3^{7.5}} \frac{1}{\beta^{4}}\left(1-\frac{2}{39} \frac{p^{2}}{\beta^{2}}\right)^{2} \\ \hline \end{gathered}$ |
| $1 D\left(\frac{5}{2}^{-}\right) \rightarrow 1 S\left(\frac{1}{2}^{-}\right)+0^{-}$ | $\mathcal{F}_{3,3}^{\frac{5}{2}, \frac{1}{2}}(0)$ | $\frac{2^{3}}{3^{6} \cdot 5} \frac{1}{\beta^{6}}$ |
| $1 D\left(\frac{5}{2}^{-}\right) \rightarrow 1 S\left(\frac{1}{2}^{-}\right)+1^{-}$ | $\mathcal{F}_{3,3}^{\frac{5}{2}, \frac{1}{2}}(0)$ | $\frac{2^{5}}{3^{7} \cdot 5} \frac{1}{\beta^{6}}$ |
|  | $\mathcal{F}_{2,1}^{\frac{5}{2}, \frac{1}{2}}(0)$ | $\frac{2^{4}}{3^{4}} \frac{1}{\beta^{2}}\left(1-\frac{2}{15} \frac{p^{2}}{\beta^{2}}\right)^{2}$ |
| $1 D\left(\frac{5}{2}^{-}\right) \rightarrow 1 P\left(\frac{1}{2}^{+}\right)+0^{-}$ | $\mathcal{F}_{2,2}^{\frac{5}{2}, \frac{1}{2}}(0)$ | $\frac{2^{2} \cdot 5}{3^{7}} \frac{1}{\beta^{4}}\left(1-\frac{1}{15} \frac{p^{2}}{\beta^{2}}\right)^{2}$ |
| $1 D\left(\frac{5}{2}^{-}\right) \rightarrow 1 P\left(\frac{3}{2}^{+}\right)+0^{-}$ | $\mathcal{F}_{2,2}^{\frac{5}{2}, \frac{3}{2}}(0)$ | $\frac{2^{5} \cdot 7}{3^{7} \cdot 5} \frac{1}{\beta^{4}}\left(1-\frac{1}{42} \frac{p^{2}}{\beta^{2}}\right)^{2}$ |
|  | $\mathcal{F}_{4,4}^{\frac{5}{2}, \frac{3}{2}}(0)$ | $\frac{2^{4}}{3^{8.5 \cdot 7}} \frac{1}{\beta^{8}}$ |

TABLE II: The transition strength $\mathcal{F}_{j_{h}, l}^{j_{q}, j_{q}^{\prime}}(0)$, where the sign " $\mathcal{P}$ " denotes a light pseudoscalar meson or a light vector meson.
where

$$
\mathcal{F}_{2,2}^{\frac{3}{2}, \frac{1}{2}}(0)=\mathcal{G} \frac{2^{2}}{3^{4}} \frac{1}{\beta^{4}}
$$

All other transition strength $\mathcal{F}_{j_{h}, l}^{j_{q}, j_{q}^{\prime}}(0)$ in Table $\llbracket$ could be fixed once the mock-meson masses $\widetilde{M}_{i}$ effect has been taken into account. According to our computation [28], the total decay width of $D(2550)^{0} \Gamma=124.1$ MeV . The dominating decay mode is the $D^{*} \pi$ channel with $\Gamma\left(D^{*} \pi\right)=121.0 \mathrm{MeV}$, and the decay width of another allowed $D_{0}^{*}(2400) \pi$ channel is 3.1 MeV (the mass of $D_{0}^{*}(2400)$ is taken as 2318 MeV [2]).

These results agree well with the experiments. It explains the fact that $D(2550)^{0}$ was first observed in $D^{*+} \pi^{-}$[17]. In Fig. 2, the variation of the decay width with $\beta$ is plotted. Obviously, the observed decay width of $D(2550)^{0}$ is well obtained in the reasonable region of $\beta(0.35 \sim 0.40 \mathrm{GeV})$,

In $D_{s}$ states, the mass of the $2^{1} S_{0}$ state is predicted around $2635 \pm 20 \mathrm{MeV}$ (a little smaller than the threshold of $D^{*} \eta$ and $\left.D_{0}^{*}(2400) K\right)$, and $D^{*} K$ is the only two-body strong decay channel. Our result for this decay channel is $\Gamma\left(D^{*} K\right) \approx 82.2 \pm 15.1 \mathrm{MeV}$, so the observed $D_{s J}(2632)^{+}$ is impossible the $2^{1} S_{0} D_{s}$ meson.
(2). Mixing states of $2^{3} S_{1}$ and $1^{3} D_{1}$

The spectrum and the helicity-angle distributions support the suggestion that $D^{*}(2600)$ is the $2^{3} S_{1}$. However, if $D^{*}(2600)$ is a pure $2^{3} S_{1}$ state, its decay width is about $163.7 \mathrm{MeV}\left(\beta=0.38 \mathrm{GeV}^{-1}\right)$. This decay width seems


FIG. 2: The decay width with $\beta$, where one takes $D(2550)^{0}$ as a pure $2^{1} S_{0}$ (green line) state and $D^{*}(2600)^{0}$ as a pure $2^{3} S_{1}$ (red line) state. Dash lines refer to central values of decay width given by experiment.
broader (see Fig. 2) than the experiment ( $93 \pm 6 \pm 13$ $\mathrm{MeV})$. Similarly, the decay width of $D_{s 1}^{*}(2700)^{ \pm}$is 230.5 MeV if it is a pure $2^{3} S_{1}$, which deviates also from the experiment $(125 \pm 30 \mathrm{MeV})$.

In charmonium system, $\psi(2 S)$ and $\psi(3770)$ are two orthogonal partners of mixtures of $2^{3} S_{1}$ and $1^{3} D_{1}$ with $J^{P C}=1^{--} 32$. This mixing scheme has also been employed to explain the decay width and the ratio of branching fractions of $D_{s 1}^{*}(2700)^{ \pm}$and $D_{s J}^{*}(2860)^{ \pm}$[11]. Similarly, we denote two orthogonal partners $\left(J^{P}=1^{-}\right)$ of $D$ and $D_{s}$ as

$$
\begin{align*}
& \left|(S D)_{1}\right\rangle_{L}=\cos \phi\left|2^{3} S_{1}\right\rangle-\sin \phi\left|1^{3} D_{1}\right\rangle  \tag{5}\\
& \left|(S D)_{1}\right\rangle_{R}=\sin \phi\left|2^{3} S_{1}\right\rangle+\cos \phi\left|1^{3} D_{1}\right\rangle
\end{align*}
$$

When $D^{*}(2600)$ and $D_{s 1}^{*}(2700)^{ \pm}$are identified with the $\left|(S D)_{1}\right\rangle_{L}$ of $D$ and $D_{s}$, respectively, their decay widths variation with the mixing angle $\phi$ are calculated and presented in Fig. 3. The experimental decay widths of $D^{*}(2600)$ and $D_{s 1}^{*}(2700)^{ \pm}$are well obtained at $\phi \approx 20^{\circ}$,

The ratios of branching fractions variation with the mixing angle $\phi$ are presented in Fig. 4. When $\phi \approx 20^{\circ}$, the observed ratio of branching fraction $\mathcal{B}\left(D^{*}(2600)^{0} \rightarrow\right.$ $\left.D^{+} \pi^{-}\right) / \mathcal{B}\left(D^{*}(2600)^{0} \rightarrow D^{*+} \pi^{-}\right)=0.32 \pm 0.02 \pm 0.09$ is also obtained. However, theoretical $\mathcal{B}\left(D_{s 1}^{*}(2700) \rightarrow\right.$ $D K) / \mathcal{B}\left(D_{s 1}^{*}(2700) \rightarrow D^{*} K\right)$ seems a little smaller than the experimental data. More experimental measurements of the ratios are required.

The partial widths of all two-body strong decay modes of $D^{*}(2600)^{0}$ and $D_{s 1}^{*}(2700)^{ \pm}$are presented in Table III. Their total decay widths are in accord with experiments. In summary, $D^{*}(2600)^{0}$ and $D_{s 1}^{*}(2700)^{ \pm}$are very possible the $\left|(S D)_{1}\right\rangle_{L}$ of $D$ and $D_{s}$, respectively.

The decay channels $D^{*}(2760)^{0} \rightarrow D^{+} \pi^{-}$and $D_{s J}^{*}(2860)^{+} \rightarrow D^{0} K^{+}$have been observed. However, it is difficult to identify $D^{*}(2760)^{0}$ and $D_{s J}^{*}(2860)^{+}$with the $\left|(S D)_{1}\right\rangle_{R}$, the orthogonal partners of $D^{*}(2600)$ and $D_{s 1}^{*}(2700)^{ \pm}$, respectively. In that case, the decay width


FIG. 3: Decay widths of $D^{*}(2600)^{0}$ (green line) and $D_{s 1}^{*}(2700)^{ \pm}$(red line) with the mixing angle $\phi$. Dash lines refer to central values of decay width given by experiment.


FIG. 4: Ratios of branching fractions with the mixing angle $\phi$ of $D^{*}(2600)^{0}$ (green line) and $D_{s 1}^{*}(2700)^{ \pm}$(red line). Dash lines refer to central values of experiments.
of $D_{s J}^{*}(2860)^{+}$is broader than 200 MeV and the decay width of $D^{*}(2760)^{0}$ ) is broader than 110 MeV , these predictions are much broader than the experimental results.
(3). $1^{3} D_{3}$ or $\left[1 D\left(3^{-}, \frac{5}{2}\right)\right]$
$D^{*}(2760)$ and $D_{s J}^{*}(2860)^{ \pm}$are very possible the $1^{3} D_{3}$ $D$ and $D_{s}$, respectively.

| Modes $^{(1)}$ | $\Gamma_{i}(\mathrm{MeV})$ | Modes $^{(2)}$ | $\Gamma_{i}(\mathrm{MeV})$ | Modes $^{(2)}$ | $\Gamma_{i}(\mathrm{MeV})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $D^{*} K$ | 76.8 | $D^{*} \pi$ | 60.7 | $D_{s} K$ | 3.2 |
| $D K$ | 36.7 | $D \pi$ | 22.5 | $D_{1}^{\prime}(2430) \pi$ | 2.2 |
| $D_{s}^{*} \eta$ | 4.6 | $D^{*} \eta$ | 1.2 | $D_{1}(2420) \pi$ | 0.1 |
| $D_{s} \eta$ | 8.2 | $D \eta$ | 2.0 | $D_{2}^{*}(2460) \pi$ | 0 |
| $\Gamma_{\text {total }}^{(1)}$ | 126.2 |  |  | $\Gamma_{\text {total }}^{(2)}$ | 92.0 |
| Expt. | $125 \pm 30$ |  |  | Expt. | $93 \pm 19$ |

TABLE III: Two-body strong decays of the admixture of $2^{3} S_{1}$ and $1^{3} D_{1}$ with $J^{P}=1^{-}$. Here $\beta=0.38 \mathrm{GeV}^{-1}$ and mixing angle $\phi \approx 19^{0}$. "Modes ${ }^{(1)}$ " refers to decay modes of $D_{s 1}^{*}(2700)$ and "Modes ${ }^{(2)}$ " refers to decay modes of $D^{*}(2600)$.

| Modes $^{(a)}$ | $\Gamma_{i}(\mathrm{MeV})$ | Modes $^{(b)}$ | $\Gamma_{i}(\mathrm{MeV})$ | Modes $^{(b)}$ | $\Gamma_{i}(\mathrm{MeV})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $D^{*} K$ | 12.3 | $D^{*} \pi$ | 12.4 | $D_{s} K$ | 0.9 |
| $D K$ | 28.4 | $D \pi$ | 22.0 | $D_{s}^{*} K$ | 0.1 |
| $D_{s}^{*} \eta$ | 0.6 | $D^{*} \eta$ | 0.2 | $D_{1}^{\prime}(2430) \pi$ | 1.1 |
| $D_{s} \eta$ | 3.0 | $D \eta$ | 0.8 | $D_{1}(2420) \pi$ | 0.4 |
| $D K^{*}$ | 0.5 | $D \rho$ | 0.1 | $D_{2}^{*}(2460) \pi$ | 1.3 |
| $D_{s} \omega$ | 0.2 | $D \omega$ | 0 | - | - |
| $\Gamma_{\text {total }}^{(a)}$ | 44.9 |  |  | $\Gamma_{\text {total }}^{(b)}$ | 39.3 |
| Expt. | $48 \pm 7$ |  |  | Expt. | $60.9 \pm 8.7$ |

TABLE IV: Two-body strong decays of the states $1^{3} D_{3}$. "Modes ${ }^{(a)}$ " refers to decay modes of $D_{s J}^{*}(2860)$ and "Mode ${ }^{(b)}$ " refers to those of $D^{*}(2760)$.
$D^{*}(2760)^{0}$ was observed in the decay channel $D^{+} \pi^{-}$ and was suggested to be a $D$-wave charmed meson 17. If $D^{*}(2760)^{0}$ has the same $J^{P}$ with the $1^{3} D_{1}$, it would have a broad width through the mixing scheme mentioned above.

Under the assumption that both $D^{*}(2760)$ and $D_{s J}^{*}(2860)^{ \pm}$are the $1^{3} D_{3}$ states, their partial widths and total decay widths are given in Table IV. The predicted decay widths of them are in accord with experimental results.
$D(2750)^{0}$ has mass close to that of $D^{*}(2760)$, if these two states are the same state of $1^{3} D_{3}$, the predicted ratio $\Gamma\left(D^{*}(2760) \rightarrow D \pi\right) / \Gamma\left(D^{*}(2760) \rightarrow D^{*} \pi\right)=$ 1.78 (see Table IV) is much larger than the observed $\mathcal{B}\left(D^{*}(2760)^{0} \rightarrow D^{+} \pi^{-}\right) / \mathcal{B}\left(D(2750)^{0} \rightarrow D^{*+} \pi^{-}\right)=$ $0.42 \pm 0.05 \pm 0.11$. This fact supports the suggestion that $D(2750)^{0}$ and $D^{*}(2760)$ are two different charmed states 17, 19].

For $D_{s J}^{*}(2860)^{ \pm}$, the predicted $\Gamma\left(D_{s J}^{*}(2860)^{ \pm} \rightarrow\right.$ $\left.D^{*} K\right) / \Gamma\left(D_{s J}^{*}(2860)^{ \pm} \rightarrow D K\right)=0.43$ is much smaller than the experimental $\frac{\mathcal{B}\left(D_{s, J}^{*}(2860)^{+} \rightarrow D^{*} K\right)}{\mathcal{B}\left(D_{s, J}^{*}(2860)^{+} \rightarrow D K\right)}=1.10 \pm$ $0.15_{\text {stat }} \pm 0.19_{\text {syst }}$.

It is noticed that the mass gaps of the corresponding ground state between $D$ and $D_{s}$ are about 100 MeV [2]. The mass gap between $D_{s 1}^{*}(2700)^{ \pm}$and $D^{*}(2600)$, and the mass gap between $D_{s J}^{*}(2860)^{ \pm}$and $D^{*}(2760)$ are also about 100 MeV . The mass gap supports also the suggestion that $D_{s 1}^{*}(2700)^{ \pm}$is a similar state as $D^{*}(2600)$ with the same $J^{P}$. Therefore, there should exist a charmedstrange $D_{s J}(2850)^{ \pm}$which has the same $\left(J^{P}, j_{q}\right)$ of $D(2750)^{0}$ with mass close to $D_{s J}^{*}(2860)^{ \pm}$.
(4). $1 D\left(2^{-}, \frac{3}{2}\right)$ and $1 D\left(2^{-}, \frac{5}{2}\right)$
$D(2750)^{0}$ was observed in $D^{*+} \pi^{-}$and is possible a $1 D\left(2^{-}, \frac{3}{2}\right)$ or $1 D\left(2^{-}, \frac{5}{2}\right)$, there exists similar assignment for the suggested $D_{s J}(2850)^{ \pm}$. The partial widths of some two-body decay modes of $D(2750)^{0}$ and $D_{s J}(2850)^{ \pm}$in the two possible assignments have been computed and presented in Table V.

If $D_{s J}(2850)^{ \pm}$is the $1 D\left(2^{-}, \frac{3}{2}\right)$, the predicted ratio of branching fraction $\mathcal{B}\left(D_{s J}(2850) \rightarrow\right.$ $\left.D^{*} K\right) / \mathcal{B}\left(D_{s J}(2860) \rightarrow D K\right)$ is about 2.42. Theoretical

| Modes $^{\dagger}$ | $\left(2^{-}, \frac{3}{2}\right)$ | $\left(2^{-}, \frac{5}{2}\right)$ | Modes $^{\ddagger}$ | $\left(2^{-}, \frac{3}{2}\right)$ | $\left(2^{-}, \frac{5}{2}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $D^{*} \pi$ | 58.9 | 20.0 | $D^{*} K$ | 96.2 | 19.2 |
| $D^{*} \eta$ | 5.4 | 0.2 | $D_{s}^{*} \eta$ | 21.7 | 0.9 |
| $D_{s}^{*} K$ | 8.6 | 0.2 | $D K^{*}$ | 4.3 | 18.0 |
| $D \rho$ | 1.9 | 9.2 | - | - | - |
| $D \omega$ | 0.7 | 3.3 | $D_{s} \omega$ | 2.7 | 13.3 |
| $D_{0}^{*}(2400) \pi$ | 0.6 | 10.9 | $D_{0}^{*}(2400) K$ | 0.2 | 0.2 |
| $D_{1}^{\prime}(2430) \pi$ | 0.2 | 1.4 | - | - | - |
| $D_{1}(2420) \pi$ | 0.5 | 1.4 | - | - | - |
| $D_{2}^{*}(2460) \pi$ | 1.2 | 0.3 | - | - | - |
| $\Gamma_{\text {total }}^{(+)}(\mathrm{MeV})$ | 77.9 | 47.9 | $\Gamma_{\text {total }}^{(\ddagger)}(\mathrm{MeV})$ | 125.1 | 51.6 |
| Expt. | - | $71 \pm 17$ | Expt. | - | - |

TABLE V: Two-body strong decays of the states $\left(2^{-}, \frac{3}{2}\right)$ and $\left(2^{-}, \frac{5}{2}\right)$. "Modes ${ }^{\dagger}$ " and "Modes ${ }^{\ddagger}$ " refer to decay modes of $D(2750)^{0}$ and $D_{s J}(2850)$, respectively.
predictions of the decay width and the ratio of branching fraction $\mathcal{B}\left(D^{*}(2760)^{0} \rightarrow D^{+} \pi^{-}\right) / \mathcal{B}\left(D(2750)^{0} \rightarrow\right.$ $D^{\star+} \pi^{-}=0.52$ of $D(2750)^{0}$ are in accord with experiment.

If $D(2750)^{0}$ and $D_{s J}(2850)^{ \pm}$are the $1 D\left(2^{-}, \frac{5}{2}\right)$, $D(2750)^{0}, D^{*}(2760)^{0}$ and $D_{s J}(2850)^{ \pm}, D_{s J}^{*}(2860)^{ \pm}$form the $1 D\left(2^{-}, 3^{-}\right)$doublet of $D$ and $D_{s}$, respectively. For charmed mesons $D(2750)^{0}$ and $D^{*}(2760)^{0}$, we obtained $\mathcal{B}\left(D^{0}\left[\frac{5}{2}^{-}\right] \rightarrow D^{+} \pi^{-}\right) / \mathcal{B}\left(D^{0}\left[\frac{5}{2}^{-}\right] \rightarrow D^{\star+} \pi^{-}\right) \approx 0.44$, which is in accord with the observed $\mathcal{B}\left(D^{*}(2760)^{0} \rightarrow\right.$ $\left.D^{+} \pi^{-}\right) / \mathcal{B}\left(D(2750)^{0} \rightarrow D^{*+} \pi^{-}\right)=0.42 \pm 0.05 \pm 0.11$. We obtained $\mathcal{B}\left(D_{s J}^{+}\left[\frac{5}{2}^{-}\right] \rightarrow D^{*} K\right) / \mathcal{B}\left(D_{s J}^{+}\left[\frac{5}{2}^{-}\right] \rightarrow D K\right) \approx$ 1.71 for the charmed-strange mesons $D_{s J}(2850)^{ \pm}$and $D_{s J}^{*}(2860)^{ \pm}$, and the observed $\frac{\mathcal{B}\left(D_{s J}^{*}(2860)^{+} \rightarrow D^{*} K\right)}{\mathcal{B}\left(D_{s, J}^{*}(286)^{+} \rightarrow D K\right)}=$ $1.10 \pm 0.15_{\text {stat }} \pm 0.19_{\text {syst }}$. Theoretical predictions are in accord with experiments within the uncertainties of the ${ }^{3} P_{0}$ model.

In our computation, a simple device known as the Shmushkevich factory [33] is employed. In this device, when a sample of a doublet $1 D\left(2^{-}, 3^{-}\right)$with random polarizations of the $c$ quark are considered, $5 / 12$ of this sample is $1 D\left(2^{-}, \frac{5}{2}\right)$, and $7 / 12$ is $1 D\left(3^{-}, \frac{5}{2}\right)$. In the transition with one final ground ( $s$-wave) charmed state, the charmed state has also randomly polarized $c$ quark, which means that $1 / 4$ of the charmed state is $D$ and $3 / 4$ of them is $D^{*}$. Therefore, in the decays of $1 D\left(2^{-}, 3^{-}\right)$into ground charmed states, $3 / 7$ of $1 D\left(3^{-}, \frac{5}{2}\right)$ decay into $D$ and $4 / 7$ of $1 D\left(3^{-}, \frac{5}{2}\right)$ decay into $D^{*}$. In this case, the partial width of $D^{*} K$ observed by experiment is the total one of both $D(2750)^{0}$ and $D^{*}(2760)^{0}$.

Of course, the two states in the doublet $1 D\left(2^{-}, 3^{-}\right)$ ( $1 D\left(2^{-}, \frac{5}{2}\right)$ and $1 D\left(3^{-}, \frac{5}{2}\right)$ ) have masses close to each other while their mass splitting is comparable to the uncertainty of their masses, it will be difficult to distinguish these two states through the channel of $D \pi$ and $D^{*} \pi$. However, the state $1 D\left(2^{-}, \frac{5}{2}\right)$ decays through the $P$-wave and the $F$-wave while the state $1 D\left(3^{-}, \frac{5}{2}\right)$ can
only decay through the $F$-wave. Therefore, the widths of decay channels $D \rho$ and $D \omega$ of $D(2750)^{0}$ would much broader than those of $D^{*}(2760)^{0}$. The observation of the channels $D \rho$ and $D \omega$ in forthcoming experiments will be useful to pin down these states.

## IV. CONCLUSIONS AND DISCUSSIONS

In this work, we study the possible $2 S$ and $1 D D$ and $D_{s}$ states, especially the four new $D$ candidates observed by the BaBar Collaboration. Both the mass and the decay width indicate that $D(2550)^{0}$ is a good candidate of the $2^{1} S_{0}$ charmed state. The observed $D_{s J}(2632)^{+}$ is impossible the $2^{1} S_{0} D s$ meson, which is predicted to have mass about $2635 \pm 20 \mathrm{MeV}$ and decay width about $82.2 \pm 15.1 \mathrm{MeV}$.
$D^{*}(2600)$ and $D_{s 1}^{*}(2700)^{ \pm}$are very possible the admixtures of $2^{3} S_{1}$ and $1^{3} D_{1}$ with $J^{P}=1^{-}$and a mixing angle $\phi \approx 19^{0}$. Our analysis does not support the possibility that $D^{*}(2760)$ and $D_{s J}^{*}(2860)^{ \pm}$are the orthogonal partners of $D^{*}(2600)$ and $D_{s 1}^{*}(2700)^{ \pm}$, respectively.

An unobserved meson, corresponding to $D(2750)^{0}$, $D_{s J}(2850)^{ \pm}$, may exist, more measurement of $D_{s J}^{*}(2860)^{ \pm}$is required. $D^{*}(2760)$ and $D_{s J}^{*}(2860)^{ \pm}$ could be identified with the $1^{3} D_{3} D$ and $D_{s}$ states, respectively. $D(2750)^{0}$ and $D^{*}(2760)$ favor to form the doublet $1 D\left(2^{-}, 3^{-}\right)$. The possibility that $D(2750)^{0}$ is the $1 D\left(2^{-}, \frac{3}{2}\right)$ state has not been excluded, so the observation of the channels $D \rho$ and $D \omega$ in experiment would be important for the identification of $D(2750)^{0}$ and $D^{*}(2760)^{0}$. Based on the simple device known as the Shmushkevich factory, some ratios of branching fractions given by experiments are well understood.

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