Production of radially excited charmonium mesons in two-body nonleptonic B_c decays

I. Bediaga[1*] and J. H. Muñoz[1,2†]

[1] *Centro Brasileiro de Pesquisas Fisicas, Rua Xavier Sigaud 150, 22290-180, Rio de Janeiro, RJ, Brazil*

[2] *Departamento de Física, Universidad del Tolima, Apartado Aéreo 546, Ibagué, Colombia*

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Abstract

We have computed branching ratios of two-body nonleptonic $B_c \to X_{c\overline{c}}M$ decays, where $X_{c\overline{c}}$ is the radially excited charmonium $\eta_c(2S)$ or $\psi(2S)$ meson, and M is a pseudoscalar (P) or a vector (V) or an axial-vector ($A(^3P_1)$) meson. We have assumed factorization hypothesis and calculated the form factors in the nonrelativistic ISGW2 quark model. We have compared our predictions with results obtained in other frameworks. We have found that some of these decays have branching ratios of the order of $10^{-3} - 10^{-4}$.

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*bediaga@cbpf.br

†jhmunoz@ut.edu.co

1 Introduction

The heavy B_c meson offers the possibility of studying the two heavy flavors b and c in a meson simultaneously. It can only decay through weak interactions and provides a good scenario to study nonleptonic weak decays of heavy mesons. For B_c processes, the contribution of the c-quark decays with the b-quark being as a spectator is $\approx 70\%$ while the b-quark decays with the c-quark being as a spectator and the weak annihilation decays contribute approximately with 20% and 10%, respectively [1, 2].

The nonleptonic B_c weak decays have been widely studied using different approaches (see the classified bibliography in Ref. [3]). The majority of these studies have considered $l = 0$ and $l = 1$ mesons without radial excitation in final states. In relation to excited charmonium states in B_c decays, some works present a systematic analysis on production of orbitally excited charmonium mesons in exclusive nonleptonic and semileptonic B_c decays using different frameworks (see *e.g.* Refs. [4, 5, 6, 7, 8, 9]). However, nonleptonic B_c decays with radially excited charmonium mesons in final state have received less attention in the literature.

At theoretical level, the observation of a number of new charmoniumlike states above the open charm production threshold [10, 11] has motivated some works on production of excited charmonium states in heavy meson decays. For example, recently, in Ref. [12] was studied the production of radially and orbitally excited $2P$ and $3S$ charmonium states in semileptonic and nonleptonic Bc decays in the framework of the relativistic quark model; in Ref. [13] was computed branching ratios for semileptonic $B_c \to X_{c\overline{c}} l\nu$ decays, where $X_{c\overline{c}}$ is a radially and orbitally excited charmonium meson $2S$, $3S$, $4S$, $1P$, $2P$, $1D$, $2D$, $3D$ in the light-cone QCD sum rules approach; and in Ref. [14] was studied the production of excited charmonium states in nonleptonic B_s decays using generalized factorization together with $SU(3)_F$ symmetry. On the other hand, at experimental level, the high luminosity of the LHC provides the possibility of measuring many decays of the B_c meson [1, 2, 15]. In particular, some of these B_c channels into charmonium states can be measured at the LHCb experiment where it is expected $\mathcal{O}(10^9)$ B_c^+ mesons with a cross section of 1 μb and a luminosity of 1 fb^{-1}.

This article is focused on production of radially excited charmonium $2S$ mesons in two-body nonleptonic weak B_c processes, which arise from the b-quark decay with the c-quark being as a spectator. These decays are produced by the $b \to c\overline{q}_i q_j$ transition, where $q_i = u, c$ and $q_j = d, s$. We have omitted the annihilation contribution because it is expected to be suppressed, and assumed naive factorization, which works reasonably well in two-body nonleptonic B_c decays where the quark-gluon sea is suppressed in the heavy quarkonium [17].

In this work, we have obtained branching ratios of two-body nonleptonic $B_c \to X_{c\overline{c}} M$

decays, where $X_{c\overline{c}}$ is the radially excited charmonium $\eta_c(2S) = \eta_c^{'}$ or $\psi(2S) = \psi^{'}$ meson, and M denotes a pseudoscalar (P) or a vector (V) or an axial-vector $A(^3P_1)$ meson, assuming factorization hypothesis and using the nonrelativistic ISGW2 quark model [18] for evaluating the $B_c \to \eta_c^{'}$ and the $B_c \to \psi^{'}$ transitions. We have compared our results with previous theoretical predictions obtained in other frameworks based on the relativistic quark model, which works with the quasipotential approach in quantum field theory [19], on the QCD relativistic potential model [20], on the relativistic constituent quark model based on the Bethe-Salpeter formalism [21], and on the QCD-motivated nonrelativistic heavy quark potential model [22]. For completeness, we have obtained branching ratios for semileptonic $B_c \to \eta_c^{'}(\psi^{'})l\nu$ decays and compared with other results obtained in the frameworks mentioned above and in the light-cone QCD sum rules [13] and QCD sum rules [23].

This paper is organized as follows. In section II, we discuss the effective weak Hamiltonian and give the form factors for $B_c \to \eta_c^{'}$ and $B_c \to \psi^{'}$ transitions. Numerical results for branching ratios of nonleptonic and semileptonic B_c decays are presented in section III, and conclusions are given in section IV.

2 Hamiltonian and Form Factors

The effective weak Hamiltonian for nonleptonic $B_c \to X_{c\overline{c}}(2S)M$ decays, where $X_{c\overline{c}}(2S)$ denotes a radially excited meson $\eta_c^{'}(2\,^1S_0)$ or $\psi^{'}(2\,^3S_1)$, and M is a pseudoscalar (P) or a vector (V) or an axial-vector (A) meson, is given by

$$\begin{aligned}\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\{&V_{cb}V_{ud}^*[c_1(\mu)(\overline{c}b)(\overline{d}u) + c_2(\mu)(\overline{d}b)(\overline{c}u)]\\ &+ V_{cb}V_{cs}^*[c_1(\mu)(\overline{c}b)(\overline{s}c) + c_2(\mu)(\overline{s}b)(\overline{c}c)]\\ &+ V_{cb}V_{us}^*[c_1(\mu)(\overline{c}b)(\overline{s}u) + c_2(\mu)(\overline{s}b)(\overline{c}u)]\\ &+ V_{cb}V_{cd}^*[c_1(\mu)(\overline{c}b)(\overline{d}c) + c_2(\mu)(\overline{d}b)(\overline{c}c)]\} + h.c.,\end{aligned} \tag{1}$$

where G_F is the Fermi constant, V_{ij} are CKM factors, $(\overline{q}_\alpha q_\beta)$ is a short notation for the $V-A$ current $\overline{q}_\alpha\gamma^\mu(1-\gamma_5)q_\beta$, $C_{1,2}$ are Wilson coefficients. The QCD coefficients are given by $a_{1,2}(\mu) = c_{1,2}(\mu) + \frac{1}{N_c}c_{2,1}(\mu)$. In this work, we have assumed large N_c limit to fix QCD coefficients $a_1 \approx c_1$ and $a_2 \approx c_2$ at $\mu \approx m_b^2$ (there are some works that have assumed this limit such as [20], [22] and [24]).

In order to obtain branching ratios of $B_c \to X_{c\overline{c}}(2S)M$ modes it is necessary to evaluate the hadronic matrix element $\langle X_{c\overline{c}}(2S)M|\mathcal{H}_{eff}|B_c\rangle$. In the framework of factorization approach, this element can be approximated by the product of two matrix elements of single currents:

$$\langle X_{c\overline{c}}(2S)M|\mathcal{H}_{eff}|B_c\rangle \approx \langle M|J^\mu|0\rangle\langle X_{c\overline{c}}(2S)|J_\mu|B_c\rangle + \langle X_{c\overline{c}}(2S)|J^\mu|0\rangle\langle M|J_\mu|B_c\rangle,$$

where J_μ is a bilinear current. In this way, the hadronic matrix element of a four-quark operator can be expressed as the product of a decay constant and form factors.

We have calculated in the ISGW2 model [18] the form factors of the hadronic matrix elements $\langle \eta_c^{'}|J_\mu|B_c\rangle$ and $\langle \psi^{'}|J_\mu|B_c\rangle$. This model is an improved version of the nonrelativistic ISGW model [25].

The parametrization of $B_c \to \eta_c^{'}$ and $B_c \to \psi^{'}$ transitions are given by [25]

$$\begin{aligned}
\langle \eta_c^{'}|J_\mu|B_c\rangle &= f_+^{'}\,(p_{B_c}+p_{\eta_c^{'}})_\mu \;+\; f_-^{'}\,(p_{B_c}-p_{\eta_c^{'}})_\mu, \qquad (2)\\
\langle \psi^{'}|J_\mu|B_c\rangle &= ig^{'}\varepsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}(p_{B_c}+p_{\psi^{'}})^\rho(p_{B_c}-p_{\psi^{'}})^\sigma - f^{'}\epsilon_\mu^*\\
&\quad -(\epsilon^*.p_{B_c})(a_+^{'}(p_{B_c}+p_{\psi^{'}})_\mu + a_-^{'}(p_{B_c}-p_{\psi^{'}})_\mu), \qquad (3)
\end{aligned}$$

where p_{B_c}, $p_{\eta_c^{'}}$ and $p_{\psi^{'}}$ are the 4-momentum of B_c, $\eta_c^{'}$ and $\psi^{'}$ mesons, respectively, ϵ_μ^* is the polarization of the $\psi^{'}$ meson, $f_+^{'}$, $f_-^{'}$, $f^{'}$, $g^{'}$, $a_+^{'}$ and $a_-^{'}$ are form factors.

2.1 Form factors for the $B_c \to \eta_c^{'}$ transition

The form factors $f_+^{'}$ and $f_-^{'}$ for the $B_c \to \eta_c^{'}$ transition are given in the ISGW2 model [18] by

$$f_+^{'} \;+\; f_-^{'} = -\frac{1}{6}\sqrt{\frac{3}{2}}\frac{\beta_{B_c}^2}{\beta_{B_c\eta_c^{'}}^2}\left(1+\frac{m_c}{m_b}\right)\left[7-\frac{\beta_{B_c}^2(5+\tau)}{\beta_{B_c\eta_c^{'}}^2}\right]F_3^{(f_+^{'}\,+\,f_-^{'})}, \qquad (4)$$

$$\begin{aligned}
f_+^{'} \;-\; f_-^{'} = \sqrt{\frac{3}{2}}\frac{\tilde{m}_{B_c}}{m_c}\Bigg\{&\left(\frac{\beta_{B_c}^2-\beta_{\eta_c^{'}}^2}{2\beta_{B_c\eta_c^{'}}^2}+\frac{\tau\beta_{B_c}^2}{3\beta_{B_c\eta_c^{'}}^2}\right)+\\
&\frac{m_c}{6\tilde{m}_{\eta_c^{'}}}\frac{\beta_{B_c}^2}{\beta_{B_c\eta_c^{'}}^2}\left(1+\frac{m_c}{m_b}\right)\left[7-\frac{\beta_{B_c}^2(5+\tau)}{\beta_{B_c\eta_c^{'}}^2}\right]\Bigg\}F_3^{(f_+^{'}\,-\,f_-^{'})}, \qquad (5)
\end{aligned}$$

where

$$F_3^{(f_+^{'}\,\pm\,f_-^{'})} = \left(\frac{\overline{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{\mp\frac{1}{2}}\left(\frac{\overline{m}_{\eta_c^{'}}}{\tilde{m}_{\eta_c^{'}}}\right)^{\pm\frac{1}{2}}\left(\frac{\tilde{m}_{\eta_c^{'}}}{\tilde{m}_{B_c}}\right)^{\frac{1}{2}}\left(\frac{\beta_{B_c}\beta_{\eta_c^{'}}}{\beta_{B_c\eta_c^{'}}^2}\right)^{\frac{3}{2}}\left[1+\frac{r^2(t_m-t)}{24}\right]^{-4}, \qquad (6)$$

$$\beta_{B_c\eta_c^{'}}^2 = \frac{1}{2}(\beta_{B_c}^2 \;+\; \beta_{\eta_c^{'}}^2), \qquad (7)$$

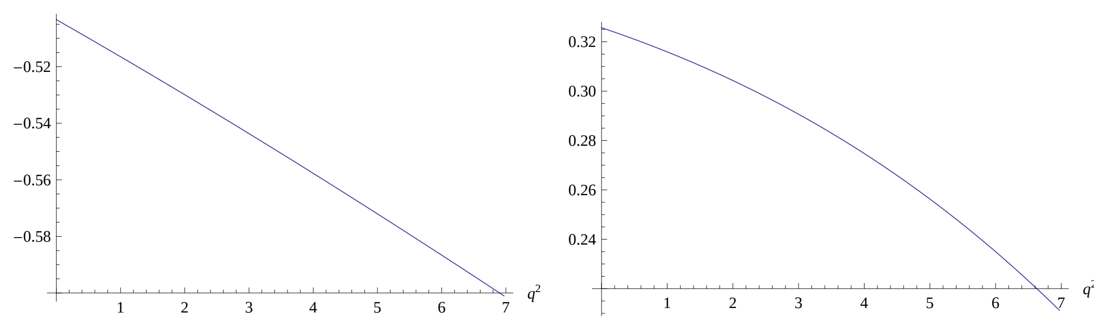
$$\tau = \frac{m_c^2\beta_{\eta_c^{'}}^2(w-1)}{\beta_{B_c}^2\beta_{B_c\eta_c^{'}}^2}, \qquad (8)$$

with

$$r^2 = \frac{3}{4m_b m_c} + \frac{3m_c^2}{2\overline{m}_{B_c}\overline{m}_{\eta_c'}\beta^2_{B_c\eta_c'}} + \frac{16}{27\overline{m}_{B_c}\overline{m}_{\eta_c'}} ln\left[\frac{\alpha_s(\mu_{QM})}{\alpha_s(m_c)}\right], \tag{9}$$

$$w = 1 + \frac{t_m - t}{2\overline{m}_{B_c}\overline{m}_{\eta_c'}}. \tag{10}$$

The values of the β parameters are given in the ISGW2 model. $t_m = (m_{B_c} - m_{\eta_c'})^2$ is the maximum momentum transfer and $t = (p_{B_c} - p_{\eta_c'})^2 = m_M^2$ for two-body nonleptonic $B_c \to \eta_c' M$ decay. $\overline{m}$ is the hyperfine-averaged physical mass, $\tilde{m}_X$ is the sum of the masses of constituent quarks of meson X, $\mu_{QM} \approx 1$ GeV is a quark model scale. In Table I, we show the values of f_+' and f_-' at $q^2 = 0$, t_m in the ISGW2 model. Also, in Fig. 1 we plot these form factors in the kinematical range $0 \leq q^2 \leq (m_{B_c} - m_{\eta_c'})^2$.



(a) (b)

Figure 1: Form factors for the $B_c \to \eta_c'$ transition: (a) $f_-'(q^2)$, (b) $f_+'(q^2)$.

2.2 Form factors for the $B_c \to \psi'$ transition

The form factors f', g' and $a_\pm'$ are given in the ISGW2 model [18] by:

$$f' = (0.899)\sqrt{\frac{3}{2}}\tilde{m}_{B_c}(1+w)\left[\frac{\beta^2_{B_c} - \beta^2_{\psi'}}{2\beta^2_{B_c\psi'}} + \frac{\tau\beta^2_{B_c}}{3\beta^2_{B_c\psi'}}\right] F_3^{(f')}, \tag{11}$$

$$g' = \frac{1}{2}\sqrt{\frac{3}{2}}\left[\left(\frac{1}{m_c} - \frac{m_c\beta^2_{B_c}}{2\mu_-\tilde{m}_{\psi'}\beta^2_{B_c\psi'}}\right)\left(\frac{\beta^2_{B_c} - \beta^2_{\psi'}}{2\beta^2_{B_c\psi'}} + \frac{\tau\beta^2_{B_c}}{3\beta^2_{B_c\psi'}}\right) + \frac{m_c\beta^2_{B_c}\beta^2_{\psi'}}{3\mu_-\tilde{m}_{\psi'}\beta^4_{B_c\psi'}}\right] F_3^{(g')}, \tag{12}$$

$$\begin{aligned} a_+' + a_-' &= -\sqrt{\frac{2}{3}}\frac{\beta^2_{B_c}}{m_c m_b \beta^2_{B_c\psi'}}\left\{\frac{7m_c^2\beta^4_{\psi'}(1+\frac{\tau}{7})}{8\tilde{m}_{B_c}\beta^4_{B_c\psi'}} - \frac{5m_c\beta^2_{\psi'}(1+\frac{\tau}{5})}{4\beta^2_{B_c\psi'}}\right. \\ &\quad \left. - \frac{3m_c^2\beta^4_{\psi'}}{8\tilde{m}_{B_c}\beta^2_{B_c}\beta^2_{B_c\psi'}} + \frac{3m_c\beta^2_{\psi'}}{4\beta^2_{B_c}}\right\} F_3^{(a_+' + a_-')}, \end{aligned} \tag{13}$$

$$a_+^{'} - a_-^{'} = \sqrt{\frac{2}{3}}\frac{3\tilde{m}_{B_c}}{2m_b\tilde{m}_{\psi'}}\left\{1-\frac{\beta^2_{B_c}(1+\frac{\tau}{7})}{\beta^2_{B_c\psi'}}-\frac{m_c\beta^2_{\psi'}}{2\tilde{m}_{B_c}\beta^2_{B_c\psi'}}\left(1-\frac{5\beta^2_{B_c}(1+\frac{\tau}{5})}{3\beta^2_{B_c\psi'}}\right)\right.$$
$$\left.-\frac{7m_c^2\beta^2_{B_c}\beta^2_{\psi'}}{12m_c\tilde{m}_{B_c}\beta^4_{B_c\psi'}}\left(1-\frac{\beta^2_{\psi'}}{\beta^2_{B_c\psi'}}+\frac{\tau\beta^2_{B_c}}{7\beta^2_{B_c\psi'}}\right)\right\}F_3^{(a_+^{'} - a_-^{'})}, \tag{14}$$

where

$$F_3^{(f^{'})} = \left(\frac{\overline{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{\frac{1}{2}}\left(\frac{\overline{m}_{\psi'}}{\tilde{m}_{\psi'}}\right)^{\frac{1}{2}}\left(\frac{\tilde{m}_{\psi'}}{\tilde{m}_{B_c}}\right)^{\frac{1}{2}}\left(\frac{\beta_{B_c}\beta_{\psi'}}{\beta^2_{B_c\psi'}}\right)^{\frac{3}{2}}\left[1+\frac{r^2(t_m-t)}{24}\right]^{-4}, \tag{15}$$

$$F_3^{(g^{'})} = \left(\frac{\overline{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-\frac{1}{2}}\left(\frac{\overline{m}_{\psi'}}{\tilde{m}_{\psi'}}\right)^{-\frac{1}{2}}\left(\frac{\tilde{m}_{\psi'}}{\tilde{m}_{B_c}}\right)^{\frac{1}{2}}\left(\frac{\beta_{B_c}\beta_{\psi'}}{\beta^2_{B_c\psi'}}\right)^{\frac{3}{2}}\left[1+\frac{r^2(t_m-t)}{24}\right]^{-4}, \tag{16}$$

$$F_3^{(a_+^{'} + a_-^{'})} = \left(\frac{\overline{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-\frac{3}{2}}\left(\frac{\overline{m}_{\psi'}}{\tilde{m}_{\psi'}}\right)^{\frac{1}{2}}\left(\frac{\tilde{m}_{\psi'}}{\tilde{m}_{B_c}}\right)^{\frac{1}{2}}\left(\frac{\beta_{B_c}\beta_{\psi'}}{\beta^2_{B_c\psi'}}\right)^{\frac{3}{2}}\left[1+\frac{r^2(t_m-t)}{24}\right]^{-4}, \tag{17}$$

$$F_3^{(a_+^{'} - a_-^{'})} = \left(\frac{\overline{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-\frac{1}{2}}\left(\frac{\overline{m}_{\psi'}}{\tilde{m}_{\psi'}}\right)^{-\frac{1}{2}}\left(\frac{\tilde{m}_{\psi'}}{\tilde{m}_{B_c}}\right)^{\frac{1}{2}}\left(\frac{\beta_{B_c}\beta_{\psi'}}{\beta^2_{B_c\psi'}}\right)^{\frac{3}{2}}\left[1+\frac{r^2(t_m-t)}{24}\right]^{-4}, \tag{18}$$

$$\mu_\pm = \left(\frac{1}{m_c}\pm\frac{1}{m_b}\right)^{-1}. \tag{19}$$

β^2, τ, r^2 and w are given by Eqs. (7), (8), (9) and (10), respectively, substituting $\eta_c^{'}$ by $\psi^{'}$.

In Table I, we show the values of $f^{'}$, $g^{'}$ and $a_\pm^{'}$ at $q^2 = 0,\ t_m$, evaluated in the ISGW2 model. Moreover, in Fig. 2 we display these form factors in the kinematical region $0 \leq q^2 \leq (m_{B_c} - m_{\psi'})^2$.

Table I. Form factors for the $B_c \to \eta_c^{'}$ and the $B_c \to \psi'$ transitions at $q^2 = 0,\ t_m$ in the ISGW2 model.

	$f_+^{'}(q^2)$	$f_-^{'}(q^2)$	$f^{'}(q^2)$	$g^{'}(q^2)$	$a_+^{'}(q^2)$	$a_-^{'}(q^2)$
$q^2 = 0$	0.325	-0.503	3.457	0.073	-0.015	0.059
$q^2 = t_m$	0.211	-0.601	3.552	0.087	-0.017	0.079

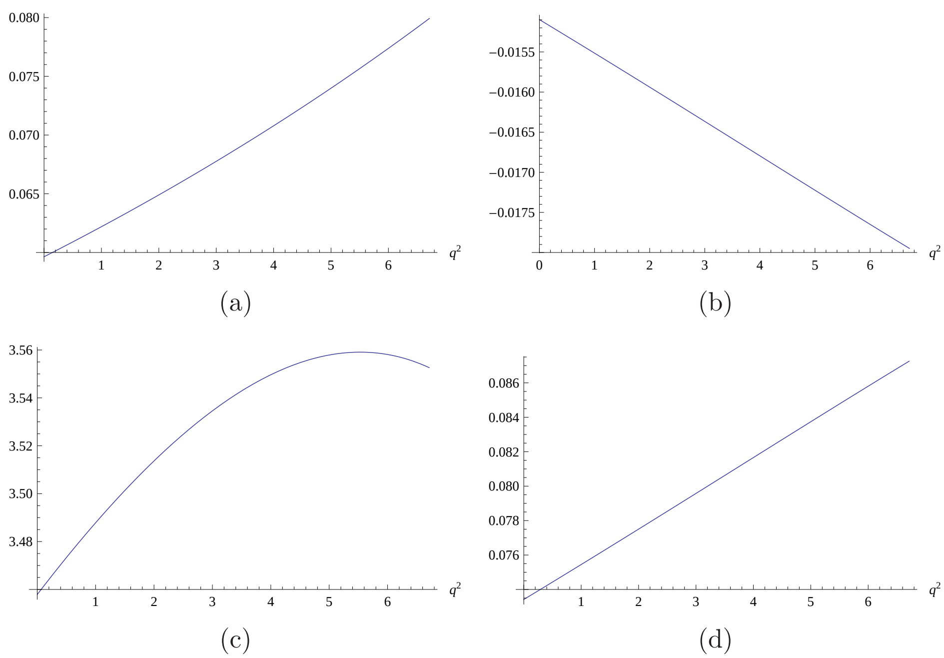


Figure 2: Form factors for the $B_c \to \psi'$ transition: (a) $a'_-(q^2)$, (b) $a'_+(q^2)$, (c) $f'(q^2)$, (d) $g'(q^2)$.

3 Numerical values and discussion

In order to obtain branching ratios of nonleptonic and semileptonic decays with radially excited charmonium mesons in final state, we take the following numerical values:

- For meson masses (in GeV) [26]: $m_{B_c} = 6.277$, $m_{\eta_c'} = 3.637$, $m_{\psi'} = 3.686$, $m_\tau = 1.776$, $m_{\pi^-} = 0.139$, $m_{K^-} = 0.493$, $m_{D^-} = 1.869$, $m_{D_s^-} = 1.968$, $m_{\rho^-} = 0.775$, $m_{K^{*-}} = 0.891$, $m_{D^{*-}} = 2.010$, $m_{D_s^{*-}} = 2.112$, $m_{a_1^-} = 1.230$, $m_{K_1(1270)} = 1.272$, $m_{K_1(1400)} = 1.403$, $m_{B_c^*} = 6.327$.

- For CKM factors [26]: $|V_{cb}| = 40.6 \times 10^{-3}$, $|V_{ud}| = 0.97425$, $|V_{cs}| = 1.023$, $|V_{us}| = 0.2252$, $|V_{cd}| = 0.230$.

- For quark masses (in GeV) [18]: $m_b = 5.2$, $m_c = 1.82$, $m_s = 0.55$, $m_u = m_d = 0.33$.

- For QCD coefficients: $a_1 = 1.14$, $a_2 = -0.2$ (see for example Refs. [5, 20, 22, 24]).

- For decay constants (in GeV): $f_{\pi^-} = 0.131$ [27], $f_{K^-} = 0.160$ [27], $f_{D^-} = 0.227$ [5], $f_{D_s^-} = 0.259$ [28], $f_{\rho^-} = 0.216$ [29], $f_{K^{*-}} = 0.210$ [27], $f_{D^{*-}} = 0.249$ [5], $f_{D_s^{*-}} = 0.266$ [5], $f_{a_1^-} = 0.238$ [29], $f_{K_1(1270)} = -0.170$ [29], $f_{K_1(1400)} = -0.139$ [29], $f_{\eta_c'} = 0.270$ [14], $f_{\psi'} = 0.304$ [13].

- β parameters from the ISGW2 model [18]: $\beta_{B_c} = 0.92$, $\beta_{\eta_c'} = 0.88$, $\beta_{\psi'} = 0.62$, $\beta_D = 0.45$, $\beta_{D_s} = 0.56$.

- $\tau_{B_c} = 0.453 \times 10^{-12}$ s [26].

Expressions for decay widths of two-body nonleptonic $B_c \to X_{c\bar{c}}(2S)M$, where $X_{c\bar{c}}(2S) = \eta_c^{'}(2\ ^1S_0)$, $\psi^{'}(2\ ^3S_1)$ and $M = P,\ V,\ A(^3P_1)$ are well known in the literature (see for example the overview given in Ref. [30]).

In Table II, we show our results for branching ratios of two-body nonleptonic $B_c \to \eta_c^{'}P,\ \eta_c^{'}V,\ \eta_c^{'}A(^3P_1)$ decays and compare with predictions of other approaches based on relativistic quark models [19, 20, 21], and on the QCD-motivated nonrelativistic heavy quark potential model [22]. We have obtained numerical values of branching ratios from these references taking $a_1 = 1.12$ y $a_2 = -0.2$. In general, we can see that branching ratios have close values in all models. Our results agree with predictions of Ref. [22], except for $B_c^- \to \eta_c^{'}D^-(D_s^-)$ decays. In this case, our numerical values are smaller than ones obtained in [22]. On the other hand, results obtained in Ref. [20] are smallest for all channels. For branching ratio of the $B_c^- \to \eta_c^{'}D_s^{*-}$ mode there is a remarkable difference (one order of magnitude) between our numerical value and the one obtained in [20] .

We can see, in Table II, that the CKM favored modes $B_c^- \to \eta_c^{'}\pi^-,\ \eta_c^{'}\rho^-, \eta_c^{'}a_1^-,\ \eta_c^{'}D_s^-, \eta_c^{'}D_s^{*-}$ have branching ratios of the order of $\approx 10^{-4}$. These branching ratios could be measured in the future at the LHCb experiment. We also obtain that

$$\frac{Br(B_c \to \eta_c^{'} V(q_i\overline{q}_j))}{Br(B_c \to \eta_c^{'} P(q_i\overline{q}_j))} \gtrsim (1.4\ -\ 4.8).$$

Let us note that in Refs. [20] and [22] this quotient gives < 1 when $V = D^{*-},\ D_s^{*-}$ and $P = D^-,\ D_s^-$, respectively, .

The $B_c^- \to \eta_c^{'} D_{(s)}^{(*)-}$ decays have two contributions: with W-external emission (proportional to QCD coefficient a_1) and with W-internal emission (proportional to QCD coefficient a_2, which is negative). Branching ratios of these decays are very sensitive to the value of a_2. For the second contribution we need to evaluate the $B_c \to D(D_s)$ and the $B_c \to D^*(D_s^*)$ transitions. We obtained the form factors for these transitions in the ISGW2 model. It is important to note that the interference term in $B_c^- \to \eta_c^{'} D^{*-}$ and $B_c^- \to \eta_c^{'} D_s^{*-}$ modes is positive because a_2 and the form factor $A_0(t = m^2_{\eta_c^{'}})$ (this form factor appears in the parametrization of $B_c \to V$ transition in Ref. [31]) are negative. So, the behavior of the interference term in $B_c \to \eta_c^{'} P$ and $B_c \to \eta_c^{'} V$ decays is different: in $B_c \to \eta_c^{'} D(D_s)$ decays the dominant contribution comes from the W-external emission term while in $B_c \to \eta_c^{'} D^*(D_s^*)$ channels, the contributions that arise from the W-external emission and the interference term are of the same order.

For the $B_c \to \eta_c^{'} A(^3P_1)$ modes, where $A(^3P_1)$ is an axial-vector meson we found that branching of the CKM favored $B_c^- \to \eta_c{'} a_1^-$ decay is of the order of 10^{-4} and is smaller that $Br(B_c \to \eta_c^{'} \rho^-)$. In fact,

$$\frac{Br(B_c \to \eta_c^{'} \rho^-)}{Br(B_c \to \eta_c^{'} a_1^-)} \approx 1.12.$$

On the other hand, when we consider the strange $K_1(1270)$ and $K_1(1400)$ mesons, which are a mixture of K_{1A} and K_{1B} mesons, it is obtained

$$\frac{Br(B_c \to \eta_c^{'} K_1^-(1270))}{Br(B_c \to \eta_c^{'} K_1^-(1400))} \approx 1.73.$$

This quotient can be an additional test for the $K_{1A} - K_{1B}$ mixing angle.

In Table III, we present our predictions for branching ratios of $B_c \to \psi^{'} P,\ \psi^{'} V,\ \psi^{'} A(^3P_1)$ decays and compare our results with those obtained in other approaches based on relativistic [19, 20, 21], and nonrelativistic quark models [22]. We have obtained numerical values of branching ratios from these references taking $a_1 = 1.12$ y $a_2 = -0.2$. Our predictions are the biggest. They are bigger than those obtained in Ref. [20] and in Ref. [22] approximately by a factor of $(1.93 - 12.11)$ and of $(1.18 - 2.28)$, respectively.

We see, in Table III, that the CKM favored $B_c \to \psi^{'} \rho,\ \psi^{'} D_s^*,\ \psi^{'} a_1$ modes, which are decays of the type $B_c \to V(2S)V(1S)$ and $B_c \to V(2S)A(1S)$, respectively, have branching ratios of the order of 10^{-3}. The other CKM favored $B_c \to \psi^{'} \pi(D_s)$ processes, which are

Table II. Branching ratios of $B_c \to \eta_c^{'} M$ decays, where $M = P,\ V,\ A(^3P_1)$.

Decay	This work	[19]	[20]	[21]	[22]
$B_c^- \to \eta_c^{'}\pi^-$	2.4×10^{-4}	1.7×10^{-4}	6.6×10^{-5}	2.2×10^{-4}	2.4×10^{-4}
$B_c^- \to \eta_c^{'}K^-$	1.8×10^{-5}	1.25×10^{-5}	4.9×10^{-6}	1.6×10^{-5}	1.8×10^{-5}
$B_c^- \to \eta_c^{'}D^-$	5.7×10^{-6}		2.2×10^{-6}		2×10^{-5}
$B_c^- \to \eta_c^{'}D_s^-$	8.3×10^{-5}		7.85×10^{-5}		8.7×10^{-4}
$B_c^- \to \eta_c^{'}\rho^-$	5.5×10^{-4}	3.6×10^{-4}	1.4×10^{-4}	5.25×10^{-4}	5.5×10^{-4}
$B_c^- \to \eta_c^{'}K^{*-}$	2.6×10^{-5}	1.9×10^{-5}	7.15×10^{-6}	2.5×10^{-5}	2.8×10^{-5}
$B_c^- \to \eta_c^{'}D^{*-}$	2.1×10^{-5}		7.8×10^{-8}		1.1×10^{-5}
$B_c^- \to \eta_c^{'}D_s^{*-}$	4×10^{-4}		2×10^{-5}		4.4×10^{-4}
$B_c^- \to \eta_c^{'}a_1^-$	4.9×10^{-4}		1.3×10^{-4}		
$B_c^- \to \eta_c^{'}K_1^-(1270)$	1.3×10^{-5}				
$B_c^- \to \eta_c^{'}K_1^-(1400)$	7.5×10^{-6}				

$B_c \to V(2S)P(1S)$ channels, have branching ratios of the order of 10^{-4}. In general, we obtain

$$\frac{Br(B_c \to \psi^{'} V(q_i\overline{q}_j))}{Br(B_c \to \psi^{'} P(q_i\overline{q}_j))} \gtrsim 2.$$

For $B_c \to \psi^{'} A(^3P_1)$ decays, where $A(^3P_1)$ denotes an axial-vector meson, we obtain that branching ratio of $B_c^- \to \psi^{'} a_1^-$ channel is the biggest. A similar result it is obtained in Ref. [20]. In fact,

$$\frac{Br(B_c \to \psi^{'} \rho^-)}{Br(B_c \to \psi^{'} a_1^-)} \approx 0.74.$$

On the other hand, when the axial-vector meson is a strange meson, we obtain

$$\frac{Br(B_c \to \psi^{'} K_1^-(1270))}{Br(B_c \to \psi^{'} K_1^-(1400))} \approx 1.5.$$

This ratio provides an additional test for the $K_{1A} - K_{1B}$ mixing angle.

Let us mention that $B_c \to \psi^{'} D^{(*)}_{(s)}$ decays also have two contributions: one with W-external emission and proportional to QCD coefficient a_1 and another with W-internal emission and proportional to QCD coefficient a_2. Branching ratios of these decays are very sensitive to the value of a_2. For obtaining branching ratios of these processes we need to evaluate the form factors for the $B_c \to D(D_s)$ and the $B_c \to D^*(D^*_s)$ transitions. We computed these form factors in the ISGW2 model. We obtain that in all cases the interference is destructive and it is smaller in $B_c \to \psi^{'} D(D_s)$ decays. We remark that Kiselev [23] found a similar effect in the interference of two-body nonleptonic $B_c \to X_{c\overline{c}}(1S) D^{(*)}_{(s)}$ decays, where $X_{c\overline{c}}(1S)$ is the η_c or the J/ψ meson, i.e., a charmonium meson without radial excitation. In the Table IX of the first paper of Ref. [23] it is showed the value of the interference term in these decays.

From Tables II and III, we obtain that $Br(B_c \to \psi^{'} M) > Br(B_c \to \eta^{'}_c M)$. Specifically, it is found that

$$\frac{Br(B_c \to \psi^{'} P)}{Br(B_c \to \eta^{'}_c P)} \approx (1.6\ -\ 6.3), \quad \frac{Br(B_c \to \psi^{'} V)}{Br(B_c \to \eta^{'}_c V)} \approx (2\ -\ 3), \quad \frac{Br(B_c \to \psi^{'} A)}{Br(B_c \to \eta^{'}_c A)} \approx (3\ -\ 3.6).$$

This ratio is bigger for those decays that have both contributions. On the other hand, for $B_c \to P(2S)V(1S)$ and $B_c \to V(2S)P(1S)$ decays we obtain

$$\frac{Br(B_c \to \eta^{'}_c V(q_i \overline{q}_j))}{Br(B_c \to \psi^{'} P(q_i \overline{q}_j))} \approx 0.8,$$

except for $V = \rho^-$ and $P = \pi^-$. In this case, the ratio is 1.44.

For completeness, we have computed branching ratios for semileptonic $B_c \to \eta^{'}_c(\psi^{'}) l\nu$ decays, with $l = e, \mu, \tau$. In Table IV, we show our results and compare with predictions in other approaches based on QCD sum rules [13, 23], relativistic [19, 20, 21] and nonrelativistic [22] quark models. Our numerical value for $Br(B^-_c \to \eta^{'}_c e^- \overline{\nu}_e)$ is of the order of 10^{-4} and close to the prediction of Ref. [22]. In general, predictions for this branching agree in the different approaches except in the framework of the light-cone QCD sum rules approach [13], where it is obtained the biggest value. For $B^-_c \to \eta^{'}_c \tau^- \overline{\nu}_\tau$ decay, our result is the smallest but close to numerical value of Ref. [23]. The prediction obtained in Ref. [13] is the biggest. It is six times our numerical value.

On the other hand, our prediction for branching ratio of $B^-_c \to \psi^{'} e^- \overline{\nu}_e$ decay is the biggest. It is of the order of 10^{-3}. A similar result is obtained in Refs. [20, 22, 23]. For $B^-_c \to \psi^{'} \tau^- \overline{\nu}_\tau$ channel, we compute the branching ratio using the expression for $d\Gamma(B_c \to V \tau \nu)/dq^2$ displayed in Ref. [32]. Our prediction for the branching of this process is $\sim$ two times the numerical value of Ref. [23]. We obtained for the three kinds of $B_c \to \psi^{'} \tau \overline{\nu}_\tau$ decays that the longitudinal contribution ($= 6.6 \times 10^{-5}$) is comparable with the

Table III. Branching ratios of $B_c \to \psi^{'} M$ decays, where $M = P,\ V,\ A(^3P_1)$.

Decay	This work	[19]	[20]	[21]	[22]
$B_c^- \to \psi^{'} \pi^-$	3.7×10^{-4}	1.1×10^{-4}	2×10^{-4}	6.3×10^{-5}	2.2×10^{-4}
$B_c^- \to \psi^{'} K^-$	2.9×10^{-5}	8×10^{-6}	8.9×10^{-6}	4.45×10^{-6}	1.6×10^{-5}
$B_c^- \to \psi^{'} D^-$	2.4×10^{-5}		7.3×10^{-6}		1.1×10^{-5}
$B_c^- \to \psi^{'} D_s^-$	5.25×10^{-4}		1.2×10^{-4}		4.4×10^{-4}
$B_c^- \to \psi^{'} \rho^-$	1.1×10^{-3}	1.8×10^{-4}	4.8×10^{-4}	1.6×10^{-4}	6.3×10^{-4}
$B_c^- \to \psi^{'} K^{*-}$	5.7×10^{-5}	9.8×10^{-6}	2.7×10^{-5}	8.1×10^{-6}	3.4×10^{-5}
$B_c^- \to \psi^{'} D^{*-}$	6.3×10^{-5}		5.2×10^{-6}		
$B_c^- \to \psi^{'} D_s^{*-}$	1.2×10^{-3}		1.7×10^{-4}		
$B_c^- \to \psi^{'} a_1^-$	1.5×10^{-3}		5.8×10^{-4}		
$B_c^- \to \psi^{'} K_1^-(1270)$	4×10^{-5}				
$B_c^- \to \psi^{'} K_1^-(1400)$	2.7×10^{-5}				

transverse contribution ($= 8.2 \times 10^{-5}$). For this process, the ratio Γ_L/Γ_T is 0.8. A similar result was presented in Ref. [32] for the $B_c \to J/\psi \tau \overline{\nu}_\tau$ mode.

Finally, from our numerical values showed in Table IV, we get the following ratios:

$$\frac{Br(B_c^- \to \eta_c^{'} e^- \overline{\nu}_e)}{Br(B_c^- \to \eta_c^{'} \tau^- \overline{\nu}_\tau)} = 35.4 \quad \text{and} \quad \frac{Br(B_c^- \to \psi^{'} e^- \overline{\nu}_e)}{Br(B_c^- \to \psi^{'} \tau^- \overline{\nu}_\tau)} = 14.$$

The first quotient is too big. In Refs. [13] and [23] it is obtained 13.5 and 12.5, respectively. On the other hand, predictions of Ref. [23] give 11.75 for the second ratio. On the other hand, we can compute the following quotients:

$$\frac{Br(B_c^- \to \psi^{'} e^- \overline{\nu}_e)}{Br(B_c^- \to \eta_c^{'} e^- \overline{\nu}_e)} = 4.6 \quad \text{and} \quad \frac{Br(B_c^- \to \psi^{'} \tau^- \overline{\nu}_\tau)}{Br(B_c^- \to \eta_c^{'} \tau^- \overline{\nu}_\tau)} = 11.5.$$

Results of Ref. [23] give 4.7 and 5, respectively, for these ratios. Our prediction for the second quotient is $\approx$ two times the numerical value obtained from the Ref. [23].

Table IV. Branching ratios of semileptonic $B_c \to \eta_c^{'}(\psi^{'})l\nu$ decays.

Decay	This work	[13]	[23]	[19]	[20]	[21]	[22]
$B_c^- \to \eta_c^{'} e^- \overline{\nu}_e$	4.6×10^{-4}	1.1×10^{-3}	2×10^{-4}	3.2×10^{-4}	2.1×10^{-4}	4.2×10^{-4}	5×10^{-4}
$B_c^- \to \eta_c^{'} \tau^- \overline{\nu}_\tau$	1.3×10^{-5}	8.1×10^{-5}	1.6×10^{-5}				
$B_c^- \to \psi^{'} e^- \overline{\nu}_e$	2.1×10^{-3}		9.4×10^{-4}	3×10^{-4}	1.2×10^{-3}	1.3×10^{-4}	1×10^{-3}
$B_c^- \to \psi^{'} \tau^- \overline{\nu}_\tau$	1.5×10^{-4}		8×10^{-5}				

4 Conclusions

In this work we have studied in a systematic way the production of radially excited charmonium mesons in two-body nonleptonic B_c decays assuming factorization approach and using the nonrelativistic ISGW2 quark model. We have obtained branching ratios for $B_c \to X_{c\overline{c}}(2S)M$ decays, where $X_{c\overline{c}}(2S)$ is the η_c' or the ψ' meson, and M is a pseudoscalar (P) or a vector (V) or an axial-vector ($A(^3P_1)$) meson. We have compared our predictions with previous results obtained in other approaches and given some ratios that could be an additional test for the different frameworks used for calculating these branching ratios. We find that some of these decays have branching ratios of the order of $10^{-3} - 10^{-4}$, which indicates that they could be measured in the future at LHCb experiment. For completeness we compute branching ratios of semileptonic $B_c \to {\eta_c}'(\psi^{'})l\nu$ decays and compare with results obtained in other scenarios.

Our main results are:

- For $B_c \to \eta_c' M$ decays, branching ratios of the CKM favored $B_c^- \to \eta_c'\pi^-(\rho^-)$, $\eta_c' a_1^-$, $\eta_c' D_s(D_s^*)$ modes are of the order of 10^{-4}. We find that the behavior of the interference term in $B_c^- \to \eta_c' D(D_s)$ and $B_c \to \eta_c' D^*(D_s^*)$ decays is different. In the first case, it is negative while in the second case it is positive because the form factor $A_2(t = m^2_{\eta_c'})$ and the QCD coefficient a_2 are negative.
- For $B_c \to \psi^{'} M$ decays, our predictions are the biggest. Branching ratios of the CKM favored $B_c \to \psi^{'}\rho^-$, $\psi^{'} a_1^-$, $\psi^{'} D_s^{*-}$ channels are of the order of 10^{-3}. Branching ratio of the exclusive $B_c \to \psi^{'} a_1^-$ decay is the biggest.
- For the semileptonic $B_c^- \to \psi^{'}\tau\overline{\nu}_\tau$ we obtain that the longitudinal (Γ_L) and transverse (Γ_T) contributions are 8.2×10^{-5} and 6.6×10^{-5}, respectively. So, the ratio Γ_T/Γ_L is 0.8.

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