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Optimum location of sensors used for mould parameters estimation

E. Majchrzak*, J. Mendakiewicz
Department for Strength of Materials and Computational Mechanics
Silesian University of Technology, Konarskiego 18a, 44-100 Gliwice, Poland
* Corresponding author. E-mail address: ewa.majchrzak@polsl.pl

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Abstract

Heat transfer processes proceeding in the system casting-mould-environment are considered. In particular, the inverse problem connected with the estimation of thermal conductivity and volumetric specific heat of mould material is presented. To estimate the parameters, the additional information concerning the temperature history at the points selected from domain considered is necessary. The essential problem is a proper choice of sensors localization. The application of sensitivity analysis assures the increase of identification efficiency and this problem is here presented. In the final part of the paper the examples of computations are shown.

Keywords: Application of information technology to the foundry industry, Solidification process, Numerical techniques, Inverse problems, Identification methods

1. Introduction

The system casting-mould-environment is considered. The aim of investigations is to assure the best input data to solve the inverse problem which consists in the determination of thermal conductivity and volumetric specific heat of mould material on a basis of temperature measurements. A fundamental problem is the selection of sensors location. The additional problem is connected with determination of sufficient sensors number. It is said that the number of sensors should be greater or equal to the number of identified parameters. One of the methods warranting the proper localization of sensors (thermocouples) bases on the sensitivity analysis and the numerical solution of sensitivity models gives the essential information concerning the best position of sensors. This approach will be here presented.

In particular the 2D problem is considered (cast iron solidifying is the typical sand mix mould) and the sensitivity models are constructed using the direct approach [1, 2, 3, 4, 5, 6, 7, 8]. On a stage of numerical computations both the basic problem and sensitivity ones are solved using the explicit scheme of the finite difference method (FDM) for non-linear parabolic equations [9].

2. Formulation of problem

The energy equation describing the casting solidification has the following form [9, 10, 11]

$$x \in \Omega: \quad C(T)\frac{\partial T(x,t)}{\partial t} = \lambda \nabla^2 T(x,t) \tag{1}$$

where $C(T)$ is the substitute thermal capacity of alloy, λ is the thermal conductivity, T, x, t denote the temperature, geometrical co-ordinates and time.

The equation (1) is supplemented by the equation concerning a mould sub-domain

$$x \in \Omega_m: \quad c_m \frac{\partial T_m(x,t)}{\partial t} = \lambda_m \nabla^2 T_m(x,t) \tag{2}$$

where c_m is the mould volumetric specific heat, λ_m is the mould thermal conductivity.

In the case of typical sand moulds on the contact surface between casting and mould the continuity condition in the form

$$x \in \Gamma_c : \begin{cases} -\lambda(T)\ \mathbf{n} \cdot \nabla T(x, t) = -\lambda_m\ \mathbf{n} \cdot \nabla T_m(x, t) \\ T(x, t) = T_m(x, t) \end{cases} \tag{3}$$

can be accepted.

On the external surface of the system the Robin condition

$$x \in \Gamma_0 : \quad -\lambda_m\ \mathbf{n} \cdot \nabla T_m(x, t) = \alpha\left[T_m(x, t) - T_a\right] \tag{4}$$

is given (α is the heat transfer coefficient, T_a is the ambient temperature).
For time $t = 0$ the initial condition

$$t = 0 : \quad T(x, 0) = T_0(x)\ , \quad T_m(x, 0) = T_{m\,0}(x) \tag{5}$$

is also known.

In the case of cast iron solidification the following approximation of substitute thermal capacity can be taken into account [2]

$$C(T) = \begin{cases} c_L\ , & T > T_L \\ \dfrac{c_L + c_S}{2} + \dfrac{Q_{aus}}{T_L - T_E}\ , & T_E < T \le T_L \\ \dfrac{c_L + c_S}{2} + \dfrac{Q_{eu}}{T_E - T_S}\ , & T_S < T \le T_E \\ c_S\ , & T \le T_S \end{cases} \tag{6}$$

where T_L, T_S are the liquidus and solidus temperatures, respectively, T_E is the temperature corresponding to the beginning of eutectic crystallization, Q_{aus}, Q_{eu} are the latent heats connected with the austenite and eutectic phases evolution, c_L, c_S are constant volumetric specific heats of molten metal and solid one, respectively. It is assumed that the thermal conductivity and volumetric specific heat of mould are unknown, while the remaining parameters appearing in governing equations are known. To identify these parameters the additional information connected with the course of the process analyzed is necessary. So, it is assumed that the temperature history at the points (sensors) selected from domain considered is known, namely

$$T_{d\,i}^f = T_d\left(x_i, t^f\right), \quad i = 1, 2, \ldots, M, \quad f = 1, 2, \ldots, F \tag{7}$$

where M is the number of sensors.

A fundamental problem is the selection of sensors position. The proper localization of sensors (thermocouples) can be obtained using the sensitivity analysis.

3. Sensitivity analysis

To determine the sensitivity functions the governing equations (1)-(5) are differentiated with respect to thermal conductivity λ_m and volumetric specific heat c_m, respectively. Differentiation of equations (1)-(5) with respect to λ_m leads to the following additional boundary-initial problem

$$x \in \Omega : \ C(T)\frac{\partial}{\partial \lambda_m}\left(\frac{\partial T(x, t)}{\partial t}\right) = \lambda \frac{\partial}{\partial \lambda_m}\left[\nabla^2 T(x, t)\right]$$

$$x \in \Omega_m : c_m \frac{\partial}{\partial \lambda_m}\left(\frac{\partial T_m(x, t)}{\partial t}\right) = \nabla^2 T_m(x, t) + \lambda_m \frac{\partial}{\partial \lambda_m}\left[\nabla^2 T_m(x, t)\right]$$

$$x \in \Gamma_c : \begin{cases} -\lambda\,\mathbf{n} \cdot \dfrac{\partial}{\partial \lambda_m}\left[\nabla T(x, t)\right] = -\mathbf{n} \cdot \nabla T(x, t) - \\ \lambda_m\,\mathbf{n} \cdot \dfrac{\partial}{\partial \lambda_m}\left[\nabla T_m(x, t)\right] \\ \dfrac{\partial T(x, t)}{\partial \lambda_m} = \dfrac{\partial T_m(x, t)}{\partial \lambda_m} \end{cases} \tag{8}$$

$$x \in \Gamma_0 : \ -\mathbf{n} \cdot \nabla T(x, t) - \lambda_m\,\mathbf{n} \cdot \frac{\partial}{\partial \lambda_m}\left[\nabla T_m(x, t)\right] = \alpha \frac{\partial T_m(x, t)}{\partial \lambda_m}$$

$$t = 0 : \quad \frac{\partial T(x, 0)}{\partial \lambda_m} = 0\ , \quad \frac{\partial T_m(x, 0)}{\partial \lambda_m} = 0$$

The following notation is introduced

$$Z_1(x, t) = \frac{\partial T(x, t)}{\partial \lambda_m}\ , \quad Z_{m1}(x, t) = \frac{\partial T_m(x, t)}{\partial \lambda_m} \tag{9}$$

and then the equations (8) can be written in the form

$$x \in \Omega : \ C(T)\frac{\partial Z_1(x, t)}{\partial t} = \lambda \nabla^2 Z_1(x, t)$$

$$x \in \Omega_m : c_m \frac{\partial Z_{m1}(x, t)}{\partial t} = \lambda_m \nabla^2 Z_{m1}(x, t) + \frac{c_m}{\lambda_m}\frac{\partial T_m(x, t)}{\partial t}$$

$$x \in \Gamma_c : \begin{cases} -\lambda\,\mathbf{n} \cdot \nabla Z_1(x, t) = -\lambda_m\,\mathbf{n} \cdot \nabla Z_{m1}(x, t) - \\ \mathbf{n} \cdot T_m(x, t) \\ Z_1(x, t) = Z_{m1}(x, t) \end{cases} \tag{10}$$

$$x \in \Gamma_0 : \ -\mathbf{n} \cdot \nabla T_m(x, t) - \lambda_m\,\mathbf{n} \cdot \nabla Z_{m1}(x, t) = \alpha Z_{m1}(x, t)$$

$$t = 0 : \quad Z_1(x, 0) = 0\ , \quad Z_{m1}(x, 0) = 0$$

In similar way the equations (1)-(5) are differentiated with respect to c_m and then

$$x \in \Omega: \; C(T)\frac{\partial}{\partial c_m}\left(\frac{\partial T(x,t)}{\partial t}\right) = \lambda \frac{\partial}{\partial c_m}\left[\nabla^2 T(x,t)\right]$$

$$x \in \Omega_m: \frac{\partial T_m(x,t)}{\partial t} + c_m \frac{\partial}{\partial c_m}\left(\frac{\partial T_m(x,t)}{\partial t}\right) =$$

$$\lambda_m \frac{\partial}{\partial c_m}\left[\nabla^2 T_m(x,t)\right]$$

$$x \in \Gamma_c: \begin{cases} -\lambda \mathbf{n} \cdot \dfrac{\partial}{\partial c_m}\left[\nabla T(x,t)\right] = -\lambda_m \mathbf{n} \cdot \dfrac{\partial}{\partial c_m}\left[\nabla T_m(x,t)\right] \\ \dfrac{\partial T(x,t)}{\partial c_m} = \dfrac{\partial T_m(x,t)}{\partial c_m} \end{cases} \tag{11}$$

$$x \in \Gamma_0: -\lambda_m \mathbf{n} \cdot \frac{\partial}{\partial c_m}\left[\nabla T_m(x,t)\right] = \alpha \frac{\partial T_m(x,t)}{\partial c_m}$$

$$t = 0: \quad \frac{\partial T(x,0)}{\partial c_m} = 0 \; , \; \frac{\partial T_m(x,0)}{\partial c_m} = 0$$

Or

$$x \in \Omega: \; C(T)\frac{\partial Z_2(x,t)}{\partial t} = \lambda \nabla^2 Z_2(x,t)$$

$$x \in \Omega_m: c_m \frac{\partial Z_{m2}(x,t)}{\partial t} = \lambda_m \nabla^2 Z_{m2}(x,t) - \frac{\partial T_m(x,t)}{\partial t}$$

$$x \in \Gamma_c: \begin{cases} -\lambda \mathbf{n} \cdot \nabla Z_2(x,t) = -\lambda_m \mathbf{n} \cdot \nabla Z_{m2}(x,t) \\ Z_2(x,t) = Z_{m2}(x,t) \end{cases} \tag{12}$$

$$x \in \Gamma_0: -\lambda_m \mathbf{n} \cdot \nabla Z_{m2}(x,t) = \alpha Z_{m2}(x,t)$$

$$t = 0: \quad Z_2(x,0) = 0 \; , \; Z_{m2}(x,0) = 0$$

where

$$Z_2(x,t) = \frac{\partial T(x,t)}{\partial c_m}, \quad Z_{m2}(x,t) = \frac{\partial T_m(x,t)}{\partial c_m} \tag{13}$$

4. Optimum location of sensors

The 2D problem is considered as shown in Figure 1. To solve the system of equations (1)-(5) and additional problems (10), (12) connected with the sensitivity functions, the explicit scheme of the finite difference method (FDM) for non-linear parabolic equations [9] is applied. The following input data have been introduced λ = 30 [W/(mK)], c_L = 5.88 [MJ/(m^3 K)], c_S = 5.4 [MJ/(m^3 K)], Q_{aus}= 923 [MJ/m^3], Q_{eu} = 994 [MJ/m^3], pouring temperature T_0 = 1300 ^{o} C, liquidus temperature T_L = 1250 ^{o} C, border temperature T_E =1160 ^{o} C, solidus temperature T_S = 1110 ^{o}C, initial mould temperature T_{m0} = 20 ^{o} C. The problems have been solved with initial estimate of parameters λ_m^0 = 0.5 [W/(mK)] and c_m^0= 1 [MJ/(m^3 K)].

The regular mesh created by 25×15 nodes with constant step h = 0.002 [m] (Figure 2) has been introduced, time step Δt = 0.1 [s]. It is assumed that only two sensors will be taken into account (it corresponds to the number of estimated parameters) and the optimal location of these sensors should be found. Additionally, the possible co-ordinates of sensors correspond to the co-ordinates of FDM nodes, because the values of sensitivities for this set of points are directly known. The thermocouples should be located at the points for which the local and temporary values of Z_{m1} and Z_{m2} achieve the maximum. In a general case the problem can be complex because the sensitivity fields are time-dependent and position of maximum values for different times can be different. Fortunately, it turned out that in the case considered the geometrical co-ordinates of points corresponding to the maximums practically do not change with time (see Figures 3, 4 and 5). Duration of this feature of sensitivity fields is about 90 s and this interval has been taken into account on the stage of inverse problem solution. For the further times the sensitivity of temperature field with respect to identified parameters becomes close to a constant function. In Figure 5 the domain corresponding to maximum of functions Z_{m1} and Z_{m2} is shown. On the basis of this information the thermocouples can be located at the points 1 and 2 marked on this Figure.

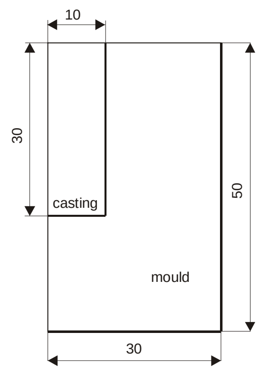

Fig. 1. Casting-mould system

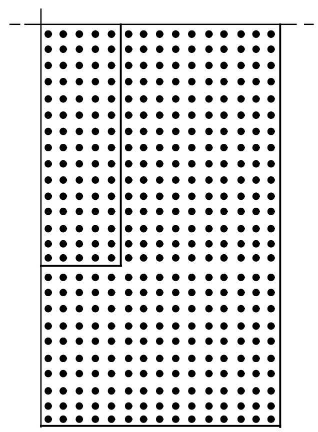

Fig. 2. Discretization

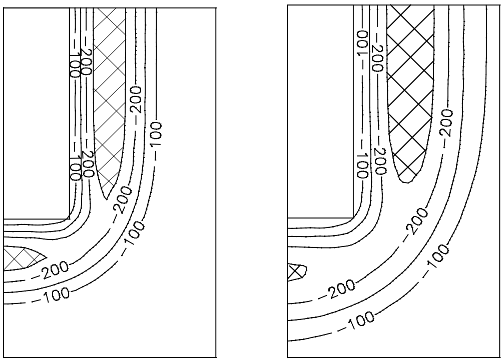

Fig. 3. Distribution of function $\partial T/\lambda_m$ for 30 and 90 s

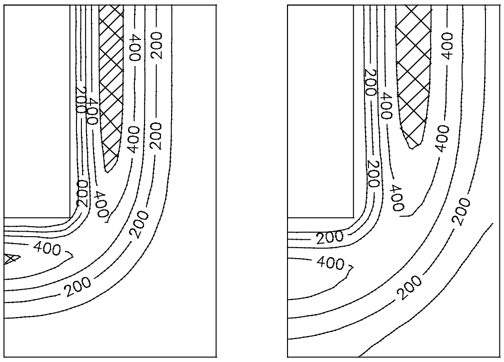

Fig. 4. Distribution of function $\partial T/c_m$ for 30 and 90 s

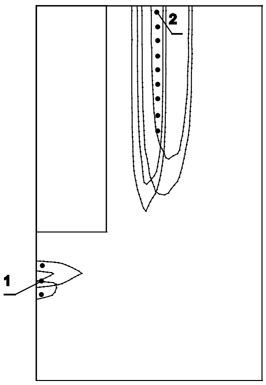

Fig. 5. Optimum sensors localization

5. Conclusions

The problem of optimum localization of sensors is analyzed. The identification of two parameters (thermal conductivity and volumetric specific heat of mould material) is considered and for this case the proper localization of sensors (thermocouples) basing on the sensitivity analysis is presented.

Acknowledgements

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