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Influence of modal filtering on the bandwidth of multimode optical fibers

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The multimode (MM) optical fiber maximum operational range is defined by the fiber bandwidth (related to the intermodal dispersion) rather than by the fiber attenuation. The relationship between the modal bandwidth of the fiber, the launching condition and mode coupling is fairly complicated. There is presented a theoretical study on the modal bandwidth of the multimode fiber. The theory is based on a numerical solution of the coupled mode diffusion equation that allows the bandwidth of the MM optical fiber to be calculated. It is shown that appropriate modal filtering at the receiver side of the fiber link increases the link bandwidth.

Keywords: telecommunications, LAN, multimode optical fiber, light launch, mode coupling, bandwidth.

1. Introduction

The multimode (MM) optical fibers are used nowadays in LANs for high bit rate data transmission (up to 10 Gbit/s) and distances of a few hundred meters. The crucial issue is to determine basic transmission parameters of such a fiber, especially its modal bandwidth as the attenuation is rarely an issue for silica fibers, since the links are not longer than a few hundred meters. Due to the modulation speed, semiconductor lasers have to be used instead of LEDs. The lasers excite fewer modes than LEDs realizing the so called restricted launch (RL). As the modes have different modal group delays and these delays determine the modal bandwidth, it is very important which modes are excited and which are not during the light launch. The situation is even more complicated as the modes propagating in the fiber are coupled and they interchange their energies. A combination of restricted launch and a flaw in the index of refraction profile (such as central dip) leads to a reduction of the MM fiber modal bandwidth [1, 2]. In this paper, we deal with the case of the modal filtering at the fiber output. We shall prove theoretically that appropriate modal filtering at the receiver side of the fiber link increases the link bandwidth. The approach presented combines the numerical solution of the mode diffusion equation with the exact calculation of the launch conditions.

2. Theory

Let us denote by $P(x, z, t)$ the mean value of the power of the mode belonging to the compound mode described by x, at a distance z from the transmitter end of the fiber, and at the time moment t. Here $x = m / m_c$ is the compound mode number (mode group number) normalized to m_c, the latter being the number of maximum guided compound mode. To calculate the frequency response of the optical fiber and its bandwidth it is more suitable to use the Fourier transform of $P(x, z, t)$ [3, 4]

$$F(x, z, f) = \int_{-\infty}^{\infty} P(x, z, t) \exp(-j2\pi f t) \mathrm{d}t \tag{1}$$

Then the equation that describes the behavior of $F(x, z, f)$ (the diffusion equation) may be written as [3, 4]

$$\frac{\partial F}{\partial z} + \left[\alpha(x) - j2\pi\tau(x) f\right] F = \frac{1}{M} \frac{1}{x} \frac{\partial}{\partial x}\left[x d(x) \frac{\partial F}{\partial x}\right] \tag{2}$$

Here, $\tau(x)$ is the mode group delay, $\alpha(x)$ is the mode attenuation, M is the number of all guided modes, and $d(x)$ is the mode-coupling coefficient. In the following, we shall discuss these parameters.

For regular profile of the index of refraction $n(r)$ the mode delay is given by [5, 6]

$$\tau(x) = \frac{N_1}{c}\left[1 + \frac{g - 2 - \varepsilon}{g + 2} \Delta x^{\frac{2g}{g+2}} + \frac{1}{2} \frac{3g - 2 - 2\varepsilon}{g + 2} \Delta^2 x^{\frac{4g}{g+2}}\right] \tag{3}$$

Here N_1 is the group index of refraction at the core center, $\Delta = 0.5\mathrm{NA}^2/n_1^2$ (where NA is the numerical aperture, and $n_1 = n(0)$), ε is the profile dispersion parameter, and g is the parameter that defines the shape of the index profile:

$$n(r) = \begin{cases} n_1\sqrt{1 - 2\Delta\left(\dfrac{r}{a}\right)^g} & 0 \le r \le a \\ n_1\sqrt{1 - 2\Delta} & r \ge a \end{cases} \tag{4}$$

where a is the core radius.

The values of $\tau(x)$ obtained from Eq. (3) were compared with the exact numerical results calculated according to the finite element method presented in [7]. It turned out that except for the highest order modes both methods give very close results. Therefore, we shall use Eq. (3) as a good approximation of the modal delay for regular $n(r)$ profiles.

The modal attenuation $\alpha(x)$ is a function of x. The value of $\alpha(x)$ is determined by effects common to all the modes, such as absorption and Rayleigh scattering as well as phenomena related to the irregularities between core and cladding and the cladding effects. The latter two phenomena bring about a substantial increase of the attenuation of higher modes. For silica fibers this increase is several dB/km [1, 3]. In this paper, we assume that the modal attenuation is given by

$$\alpha(x) = \alpha_0 + \Delta\alpha x^c \tag{5}$$

where α_0 is the common attenuation, $\Delta\alpha$ is the attenuation increase, and c is a parameter describing the shape of function $\alpha(x)$, its value is usually between 4 and 8 [1, 3].

The mode-coupling coefficient $d(x)$ is most often expressed as [6, 7]

$$d(x) = d_0 x^{-2q} \tag{6}$$

where d_0 [1/km] defines the coupling strength between modes belonging to the adjacent compound modes, and the exponential q describes its functional form. Both parameters depend on the fiber type, and its perturbations (core diameter variations, micro- and macrobends, *etc.*). For a gradient index GI (parabolic profile) fiber it is usually assumed that $q = 0$ or $q = -0.5$ [3, 6, 8, 9]. The value of the coupling coefficient d_0 may be as high as 30 1/km [4, 8]. In modern fibers, however, it is usually less, so sometimes one assumes no coupling at all (*i.e.*, $d_0 = 0$, no mode mixing) [10].

In order to solve Eq. (2) it is necessary to define the boundary conditions. These are the following [8, 9, 11]:

$$F(x, z, f)_{x=1} = 0 \tag{7}$$

$$\left[d(x)\frac{\partial F}{\partial x}\right]_{x=0} = 0 \tag{8}$$

$$F(x, 0, f) = p(x) \tag{9}$$

Here, $p(x)$ is the initial power of the mode belonging to the compound mode x:

$$P(x, 0, t) = p(x)\delta(t) \tag{10}$$

and $\delta(t)$ is the delta Dirac function.

The number of modes in a given compound mode x is proportional to x. Besides, two orthogonal polarizations are possible. Thus, the power $P(x)$ of this compound mode

(mode group) is $2xP(x, z, t)$, and the total powers at the input and output of the fiber are respectively

$$P_{\mathrm{in}}(t) = 2\delta(t)\int_0^1 xp(x)\mathrm{d}x \tag{11}$$

$$P_{\mathrm{out}}(t) = 2\int_0^1 xH(x)P(x, z, t)\mathrm{d}x \tag{12}$$

Taking the Fourier transform of (11), (12) we finally get the frequency response of the fiber

$$T(f, z) = \frac{\int_0^1 xH(x)F(x, z, f)\mathrm{d}x}{\int_0^1 xp(x)\mathrm{d}x} \tag{13}$$

Here, $F(x, z, f)$ is the solution of Eq. (2) with the boundary conditions (7)–(9). In this paper, Eqs. (2) and (13) were solved numerically. Writing Eq. (13) we neglected the chromatic dispersion of the fiber. This is justified when the source linewidth is small. For instance, for linewidths below 1 nm and $\lambda = 0.85$ μm this assumption is satisfied for modal bandwidths in excess of 1.8 GHz [12].

In Eqs. (12) and (13), we introduced the possibility of modal filtering at the fiber output. It is described by the function $H(x)$ ($H(x) \leq 1$). In the absence of filtering $H(x) = 1$. Modal filtering may be realized by the following methods:

1. Winding a few turns of the fiber on a reel with a small diameter; such a filter cuts off higher order modes. The smaller the reel diameter, the lower order modes are damped. Due to the possible fiber break the reel diameter must not be very small, so in practice only the modes with $x > 0.5$ can be filtered off [13–15].

2. Using a single mode (SM) patchcord (in general, a small core fiber) as the modal filter at the MM fiber receiver end. The modal transfer function of such a patchcord, $H(x)$, corresponds to the function $p(x)$, see Eq. (9). The principal disadvantage here is high loss (around 10 dB and more) related to the coupling between MM and SM fibers.

3. Spatial filtering of the light beam coming out of the MM fiber end. It may be realized by placing a diaphragm between the fiber end and the photodetector, or simply by moving the photodetector away from the fiber [16]. A more sophisticated approach is also possible: various mode groups are received by different photodetectors and these signals are electronically processed to compensate for different modal delays [17].

Two approaches are possible to calculate the initial distribution of compound mode powers at the fiber input. First, the incoming light beam may be decomposed into the MM fiber modes by calculating the correlations between the input field distribution and distributions of all MM fiber guided modes [1, 2]. These correlations define the coupling coefficients between the input and all the modes. This may be done numerically. The second approach assumes the Gaussian beam input as well as both the weak guiding and infinite parabolic profile of the MM fiber. Then, both the modal fields and coupling coefficients may be expressed analytically, the latter by means of the generalized Laguerre polynomials [18]. Both approaches were tested showing an excellent agreement for a quadratic index of refraction profile. The mean power of a mode belonging to the compound mode m is given by

$$p(m)\Big|_{x = \frac{m}{m_c}} = \left(\frac{1}{k}\right) \sum_{2p + l = m} c_{lp} \tag{14}$$

Here, k is the number of modes in the compound mode m, and c_{lp} is the coupling coefficient between the input field and a linearly polarized mode LP_{lp}.

3. Results

As mentioned before, Eq. (2) in general case may be solved only numerically. To this end, we used finite difference methods [19], namely the explicit method and the DuFort and Frankel method [19]. Each case was solved independently by these two methods until the convergence of the results was obtained. If not stated otherwise the results presented were obtained for a GI MM optical fiber with the following parameters: $g = 1.8$, $\varepsilon = 0.05$, $\lambda = 0.85$ µm, $D = 62.5$ µm, NA = 0.275, $n_1 = 1.48$, $L = 1$ km. The modal bandwidth as defined here corresponds to a 3 dB optical power reduction (optical bandwidth) and it is consistent with a standard definition [12].

We are interested primarily in the influence of the index of refraction flaws on the MM fiber bandwidth. The most often encountered fault, namely the central dip, is modeled here by adding a triangle disturbance to the regular function (3) describing the modal delay:

$$\begin{cases} \tau_1(x) = \tau(x) - \dfrac{N_1}{c} \Delta h \left(1 - \dfrac{x}{x_0}\right) & \text{for} \quad x \le x_0 \\ \tau_1(x) = \tau(x) & \text{for} \quad x > x_0 \end{cases} \tag{15}$$

Here, h is the relative depth of a dip, typically $0 < h < 1$, and x_0 defines the maximum distorted modal group number. The function (15) is somewhat arbitrary, and it is used to test the influence of modal filtering on the fiber bandwidth.

The modal bandwidth for a distorted profile, and the group delay given by (15) ($h = 0.5$, $x_0 = 0.05$) is shown in Fig. 1 for various offsets. The Gaussian beam launch of 9 µm diameter is assumed. It is readily seen that the bandwidth is highly reduced

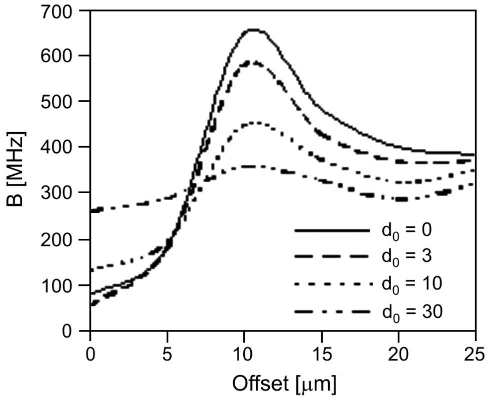

Fig. 1. Modal bandwidth B, versus offset: the exciting beam diameter is 9 μm, $d(x) = d_0$ = const [1/km]. The profile is distorted according to (15): $h = 0.5$ and $x_0 = 0.05$.

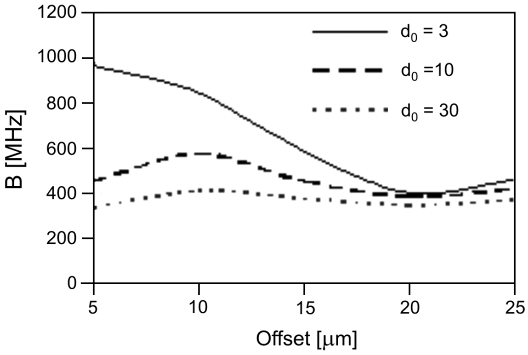

Fig. 2. Modal bandwidth in the presence of modal filtering with the filter (16) and for a distorted profile (15), launch beam diameter is 9 μm, $d(x) = d_0$ = const [1/km].

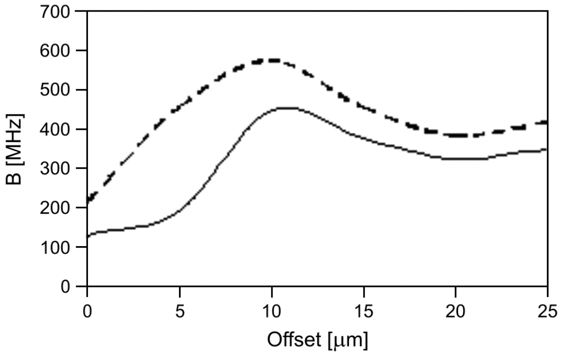

Fig. 3. Modal bandwidth in the absence of modal filtering (solid line), and in its presence (dashed line). Modal filtering is with the filter (16) and for the distorted profile (15), launch beam diameter is 9 μm, $d(x) = d_0 = 10$ [1/km]. Modal filtering loss does not exceed here 6.1 dB.

especially for the central launches because the distortion affects mostly the lower order modes. This bandwidth reduction is less pronounced for intense mode mixing (greater d_0). If the modes are highly coupled the bandwidth only weakly depends on the offset.

It turns out that appropriate modal filtering applied at the fiber output brings about the modal bandwidth increase. In exemplary calculation we used a modal filter described by:

$$H(x) = \begin{cases} 1 & 0.25 \le x \le 0.75 \\ 0 & x > 0.25 \ \text{ or } \ x > 0.75 \end{cases} \tag{16}$$

This is a hypothetical filter that may or may not be realized in practice. Possible approximation is shown in Fig. 5. Such a filter or a similar one eliminates some mode groups that have extremely different modal delays, decreases the impulse spreading,

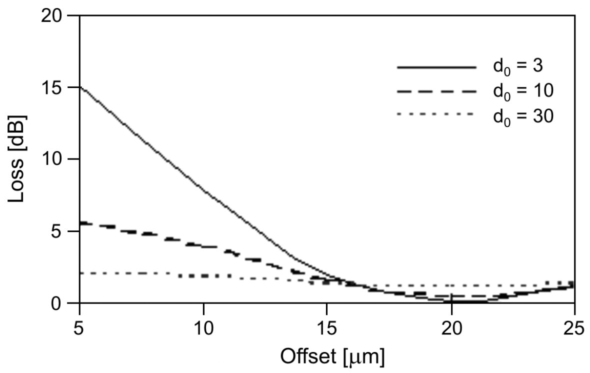

Fig. 4. Modal filtering loss for the data from Fig. 3 for various coupling coefficients $d(x) = d_0$ [1/km].

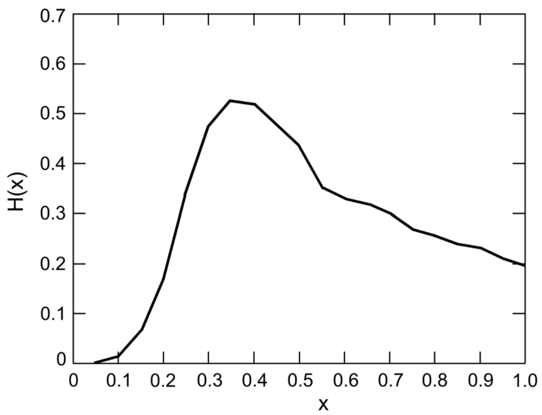

Fig. 5. Transfer function $H(x)$ realized in a MM fiber ($\lambda = 0.78$ μm, $D = 50$ μm, $g = 2$, NA = 0.2, $n_1 = 1.48$) by a circular diaphragm located at the fiber end that is opaque for $r < 10$ μm and for $r > 15$ μm, and transparent elsewhere.

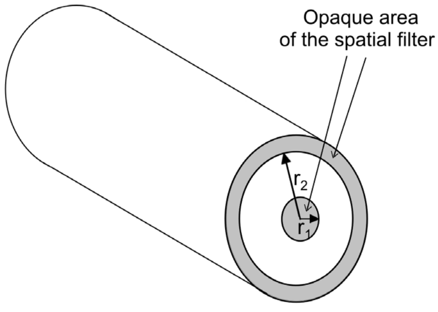

Fig. 6. View of the fiber core end surface. A spatial filter that realizes the transfer function in Fig. 5 is applied. The fiber cladding is not shown.

and increases the fiber modal bandwidth. It is shown in Fig. 2. Figure 3 compares the bandwidths obtained with and without the modal filtering. It is necessary to stress that the modal filtering induces the signal loss. This effect is shown in Fig. 4. Usually, the greater the bandwidth increase, the greater the loss, however, if the filter is appropriately designed, the signal loss is moderate.

The modal filter may be realized, for example, by a circular diaphragm located at the fiber end. The resulting $H(x)$ function is depicted in Fig. 5, and the filter realization in Fig. 6.

4. Conclusions

It has been proved theoretically that it is possible to increase the modal bandwidth of the MM fiber applying the pass-band modal filtering at the fiber output. A twofold bandwidth increase may be obtained with moderate signal loss related to this modal filtering. The mechanism behind the bandwidth improvement is straightforward: the filter simply suppresses the mode groups that have the most different modal delays. Further research is necessary to verify the results obtained. In particular, an experimental confirmation of the bandwidth increase is necessary. Furthermore, optimum filters that may be practically implemented should be found.

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