Chiral symmetry breaking and the spin content of the ρ and ρ' mesons

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Abstract

Using interpolators with different $SU(2)_L \times SU(2)_R$ transformation properties we study the chiral symmetry and spin contents of the ρ- and ρ'-mesons in lattice simulations with dynamical quarks. A ratio of couplings of the $\overline{q}\gamma^i\tau q$ and $\overline{q}\sigma^{0i}\tau q$ interpolators to a given meson state at different resolution scales tells one about the degree of chiral symmetry breaking in the meson wave function at these scales. Using a Gaussian gauge invariant smearing of the quark fields in the interpolators, we are able to extract the chiral content of mesons up to the infrared resolution of ~ 1 fm. In the ground state ρ meson the chiral symmetry is strongly broken with comparable contributions of both the $(0, 1) + (1, 0)$ and $(1/2, 1/2)_b$ chiral representations with the former being the leading contribution. In contrast, in the ρ' meson the degree of chiral symmetry breaking is manifestly smaller and the leading representation is $(1/2, 1/2)_b$. Using a unitary transformation from the chiral basis to the $^{2S+1}L_J$ basis, we are able to define and measure the angular momentum content of mesons in the rest frame. This definition is different from the traditional one which uses parton distributions in the infinite momentum frame. The ρ meson is practically a $^3S_1$ state with no obvious trace of a "spin crisis". The ρ' meson has a sizeable contribution of the $^3D_1$ wave, which implies that the ρ' meson cannot be considered as a pure radial excitation of the ρ meson.

1 Introduction

The structure of hadrons in the infrared is a challenging topic. At low resolution scales (i..e., large distances $\mathcal{O}(1$ fm$)$) both, confinement and chiral symmetry breaking, are crucial phenomena. They influence mass and angular momentum generation of hadrons. These phenomena are of primary interest both theoretically and experimentally. To understand physics at these deeply nonperturbative scales one needs direct information about the chiral and the angular momentum content of hadrons.

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Such information can be obtained from dynamical lattice simulations. Using a set of interpolators that form a complete basis with respect to the $SU(2)_R \times SU(2)_L$ transformations, one is able to define in a gauge invariant manner and measure the chiral content of mesons at different resolution scales [1]. One basically measures a ratio of couplings of different interpolators to a given hadron. Such a ratio tells us something about chiral symmetry breaking in a hadron wave function. If chiral symmetry were unbroken in a hadron, then only interpolators with definite chiral transformation properties would couple to this hadron. Chiral symmetry breaking in a hadron would imply that interpolators with different chiral transformation properties would create this hadron from the vacuum.

The output of the two-flavor dynamical simulations with $m_{AWI} \sim 15 - 30$ MeV [1] was that at the resolution scales $0.15 - 0.6$ fm the ρ meson is approximately a 55% - 45% mixture of the two possible chiral representations $(0,1) + (1,0)$ and $(1/2, 1/2)_b$. Given a unitary transformation from the quark-antiquark chiral basis to the $^{2S+1}L_J$ basis in the rest frame [2], we were able to extract the angular momentum content of the ρ meson in the rest frame [1]. The result was that the ρ meson at the scales $0.15 - 0.6$ fm is approximately a $^3S_1$ state with a tiny contribution of a $^3D_1$ wave. In this definition of total angular momentum (in the rest frame) there is no "spin crisis", at least for the ρ meson. This definition of the spin content of a hadron is very different from the traditional one. The latter relies on the parton distributions in the infinite momentum frame extracted from the deep inelastic scattering with polarization [3]. Accordingly only about 30% spin of the nucleon is carried by the spins of valence quarks [4], which gave rise to the term "spin crisis". Similar results within this same definition are obtained on the lattice both for the nucleon and mesons, for a review and references see [5]. Then a natural question is which of these two definitions does reflect the spin content of a hadron?

In [6] we have also measured the chiral and angular momentum content of the first excitation of the ρ meson, $\rho' \equiv \rho(1450)$. As compared to the ground state ρ, we have observed a very different dependence of the chiral and angular momentum content on the resolution scale. In particular, we have found weaker chiral symmetry breaking in the ρ' state with the $(1/2, 1/2)_b$ representation being the leading one in the infrared. We have also found a significant contribution of the $^3D_1$ wave in the ρ' wave function. This means that the ρ' cannot be considered as a pure radial excitation of the ρ meson.

In these studies only two different resolution scales were used. This did not allow to reliably extrapolate the results up to the resolution scale of the excited hadron size, ~ 1 fm. In the present paper we extend our correlation matrix from 4×4 to 6×6 by providing three different Gaussian smearings of the quark fields in the vector and tensor interpolators. This allows us to discuss the chiral symmetry and angular momentum content of both ρ and ρ' mesons in the infrared region of 1 fm, where mass is generated.

2   Theoretical foundations of the method

In this paper we study the chiral and angular contents of the ρ-mesons, consequently we restrict our discussions specifically to the $I = 1, J^{PC} = 1^{--}$ states. But the formalism is generic and can be used for mesons with other quantum

numbers as well.

A chiral classification of some interpolators is performed in Ref. [7] and a full classification of the quark-antiquark states as well as of the corresponding interpolators is done in Ref. [8]. In the case of the $I = 1, J^{PC} = 1^{--}$ states there are two allowed chiral representations, $(0,1) + (1,0)$ and $(1/2,1/2)_b$. The state that transforms as $(0,1) + (1,0)$ can be created from the vacuum by the vector current,

$$O_\rho^V(x) = \overline{q}(x)\,\gamma^i\vec{\tau}\,q(x) \;, \qquad (1)$$

and the state that belongs to the $(1/2,1/2)_b$ representation can be created by the pseudotensor operator,

$$O_\rho^T(x) = \overline{q}(x)\,\sigma^{0i}\vec{\tau}\,q(x) \;. \qquad (2)$$

The chiral partner of the first operator is the axial vector current,

$$O_{a_1}(x) = \overline{q}(x)\,\gamma^i\gamma^5\vec{\tau}\,q(x) \;, \qquad (3)$$

that creates from the vacuum the a_1 states, $I = 1, J^{PC} = 1^{++}$. The chiral partner of the second operator is the operator

$$O_{h_1}(x) = \varepsilon^{ijk}\overline{q}(x)\,\sigma^{jk}\,q(x) \;, \qquad (4)$$

that couples to the $I = 0, J^{PC} = 1^{+-}$ h_1 mesons.

It is well established in quenched [9] and dynamical [1, 6, 10] lattice simulations that the ground and excited states of the ρ meson can be created from the vacuum by both the vector and pseudotensor operators. This fact by itself means that chiral symmetry is broken in the vacuum and in the physical states, and these states are mixtures of these two representations [1, 6].

The chiral basis in the quark-antiquark system is a complete one and can be connected to the complete angular momentum basis in the rest frame via the unitary transformation [2]

$$\begin{pmatrix} |(0,1) \oplus (1,0); 1\; 1^{--}\rangle \\ |(1/2,1/2)_b; 1\; 1^{--}\rangle \end{pmatrix} = U \cdot \begin{pmatrix} |1;{}^3S_1\rangle \\ |1;{}^3D_1\rangle \end{pmatrix} \qquad (5)$$

with

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} \end{pmatrix} \;. \qquad (6)$$

Consequently, if we know the mixture of the two allowed chiral representations in a physical state, we are also able to obtain the angular momentum content of this state in the rest frame.

In particular, we can answer a question whether or not a spin of a meson is carried by spins of its valence quarks in the rest frame. This definition of the spin content is different from the traditional one that relies on the parton distributions in the infinite momentum frame. According to the latter only about 30% of the nucleon spin is carried by the valence quarks, which has been referred to as "spin crisis". Hence we can compare a spin content of a meson obtained according to our definition with the spin content extracted from the parton distributions in the infinite momentum frame.

3   Reconstruction of the coupling constants with the variational method

In order to resolve a few subsequent physical states with the same quantum numbers one has to choose a convenient set of operators O_i with the same quantum numbers and a significant overlap with the physical states, and calculate a cross-correlation matrix at zero spatial momentum (i.e., in the rest frame) [11],

$$C(t)_{ij} = \langle O_i(t) O_j^\dagger(0) \rangle = \sum_{n=1}^{\infty} a_i^{(n)} a_j^{(n)*} \mathrm{e}^{-E^{(n)}t} \;, \tag{7}$$

with the coefficients giving the overlap of the operators with the physical state,

$$a_i^{(n)} = \langle 0|O_i|n\rangle \;. \tag{8}$$

With a set of operators spanning a complete and orthogonal basis with respect to some symmetry group, these overlaps (coupling constants) give the complete information about symmetry breaking. The interpolating composite operators O_i are not normalized on the lattice and consequently the absolute values of the coupling constants $a_i^{(n)}$ cannot be obtained. However, a ratio of the couplings is a well defined quantity and can be computed as [1]

$$\frac{a_i^{(n)}}{a_k^{(n)}} = \frac{\widehat{C}(t)_{ij} u_j^{(n)}}{\widehat{C}(t)_{kj} u_j^{(n)}} \;. \tag{9}$$

Here $\widehat{C}$ is the cross-correlation matrix from (7), a sum is implied for the index j on the right-hand side and $u_j^{(n)}$ are the eigenvectors obtained from the generalized eigenvalue problem,

$$\widehat{C}(t)_{ij} u_j^{(n)} = \lambda^{(n)}(t, t_0) \widehat{C}(t_0)_{ij} u_j^{(n)} \;, \tag{10}$$

with t_0 being some normalization point in Euclidean time.

In our calculation a set of interpolators O_i complete with respect to $SU(2)_L \times SU(2)_R$ and the angular momentum basis consists of the vector (1) and pseudotensor (2) operators. However, there is an infinite amount of nonlocal operators with the same chiral and angular momentum structure like (1) and (2) but with different radial spatial form. We want to construct these nonlocal operators in such a way that each of them would probe the hadron structure at a given physical resolution scale.

In the continuum the corresponding amplitudes are given as

$$\langle 0|\overline{q}(0)\gamma^\mu q(0)|V(p;\lambda)\rangle \;\; = \;\; m_\rho f_\rho^V e_\lambda^\mu \;, \tag{11}$$

$$\langle 0| \left(\overline{q}(0)\sigma^{\alpha\beta} q(0)\right)(\mu)|V(p;\lambda)\rangle \;\; = \;\; \mathrm{i} f_\rho^T(\mu) e_\lambda^\mu (e_\lambda^\alpha p^\beta - e_\lambda^\beta p^\alpha) \;, \tag{12}$$

where $V(p;\lambda)$ is the vector meson state with the mass m_ρ, momentum p and polarization λ. The vector current is conserved, consequently the vector coupling constant f_ρ^V is scale-independent. The pseudotensor "current" is not conserved and is subject to a nonzero anomalous dimension. Consequently the pseudotensor coupling $f_\rho^T(\mu)$ manifestly depends on the scale μ. In the rest frame the ratio

$$\frac{f_\rho^V}{f_\rho^T(\mu)} = \frac{\langle 0|\overline{q}(0)\gamma^i q(0)|V(\lambda)\rangle}{\langle 0| \left(\overline{q}(0)\sigma^{0i} q(0)\right)(\mu)|V(\lambda)\rangle} \tag{13}$$

Set	β_{LW}	$a\,m_0$	#conf	a [fm]	m_π [MeV]	m_ρ [MeV]	$m_{\rho'}$ [MeV]
A	4.70	-0.050	200	0.1507(17)	526(7)	911(11)	1964(182)
B1	4.65	-0.060	300	0.1500(12)	469(5)	870(10)	1676(106)
B2	4.65	-0.070	200	0.1406(11)	296(6)	819(18)	1600(181)
C	4.58	-0.077	300	0.1440(12)	323(5)	795(15)	1580(159)

Table 1: Specification of the data used here; for the gauge coupling only the leading value β_{LW} is given, m_0 denotes the bare mass parameter of the CI action. Further details on the action, the simulation and the determination of the lattice spacing and the π- and ρ-masses are found in [16, 17].

Set	R_n [fm]	R_w [fm]	R_{uw} [fm]
A	0.36	0.67	–
B1	0.34	0.69	0.81
B2	0.34	0.66	0.85
C	0.33	0.66	–

Table 2: Specification of the smearing radii R.

coincides with the ratio of matrix elements (9) with $i \equiv V$; $k \equiv T$.

4   Physical resolution scale

We want to probe the hadron structure at infrared scales, where mass is generated. The hadron interpolators that create and annihilate the hadrons are built from quark fields. With the local lattice interpolators of type $\overline{q}(x)\Gamma q(x)$ we study the hadron structure at the scale given by the lattice spacing a. Given a reasonably small value of a we can fix a smaller resolution (larger size) of our probe by a gauge invariant smearing of the quark fields in the interpolator. Namely, we smear the quark fields in the source and sink in spatial directions with a Gaussian profile of the size R . Technically this Gaussian type of smearing is achieved by Jacobi smearing [12]. For examples of the resulting quark profiles see, e.g., [13].

The eigenvectors of the cross correlation matrix then give us information on the contribution of the various interpolators (with different Dirac structure and built from differently smeared quark sources) to the physical state.

Then, even in the continuum limit $a \to 0$ we may probe the hadron structure at a scale fixed by R. Such a definition of the resolution is similar to the experimental one, where an external probe is sensitive only to the quark fields (it is blind to gluonic fields) at a resolution that is determined by the typical momentum transfer in spatial directions.
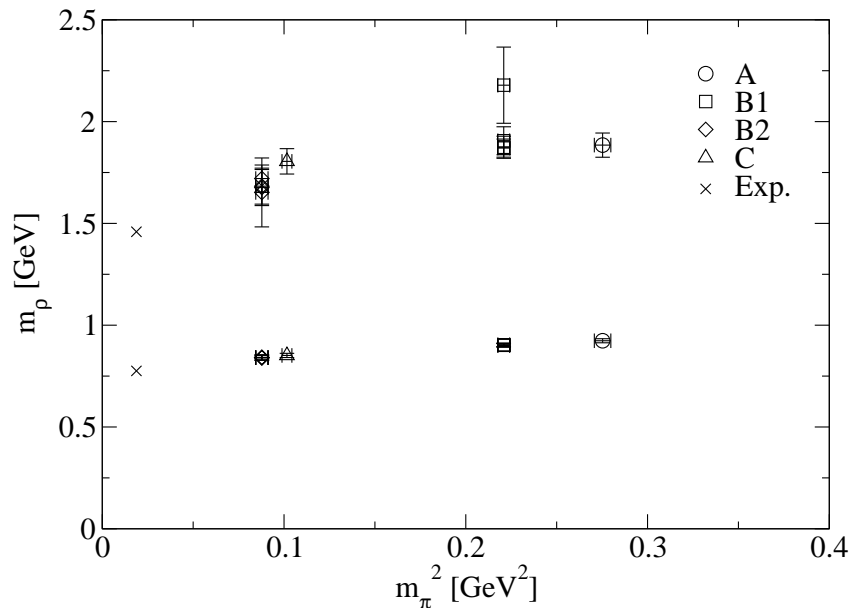
Figure 1: The masses of both ρ and ρ' states extracted from different 4×4 and 6×6 correlation matrices. The crosses indicate the mass values from experiments.

5 Lattice details and choices of correlation matrix

In our study we use Chirally Improved fermions [14] and the Lüscher-Weisz gauge action [15]. The lattice size is $16^3 \times 32$. We use dynamical gauge configurations with two mass-degenerate light quarks. With the lattice spacing ≈ 0.15 fm the spatial volume of the lattice is $\approx 2.4^3$ fm^3. For the ground states and some their first excitations such a volume turns out to be sufficient to get approximately correct masses of hadrons in the physical limit [17], though it is certainly too small to consider higher excitations. In our present study we limit ourselves to the ρ and ρ' states. For details on the simulation we refer the reader to the Table 1 and to [16, 17] .

We use three different smearing radii R for the quark fields in the source and sink, see Table 2. The "narrow" smearing width (index n) varies between 0.33 and 0.36 fm, depending on the set of configurations. The "wide" smearing radius (index w) lies between 0.66 and 0.69 fm and the "ultrawide" one is 0.81 – 0.85 fm (index uw). Hence we can study the hadron structure at resolutions 0.33 fm – 0.85 fm and will be able to extrapolate the results up a resolution of $\mathcal{O}(1$ fm$)$.

We have the following set of operators:

$$O_n^V = \overline{u}_n \gamma^i d_n \ , \quad O_w^V = \overline{u}_w \gamma^i d_w \ , \quad O_{uw}^V = \overline{u}_{uw} \gamma^i d_{uw} \ ,$$
$$O_n^T = \overline{u}_n \gamma^t \gamma^i d_n \ , \quad O_w^T = \overline{u}_w \gamma^t \gamma^i d_w \ , \quad O_{uw}^T = \overline{u}_{uw} \gamma^t \gamma^i d_{uw} \ , \qquad (14)$$

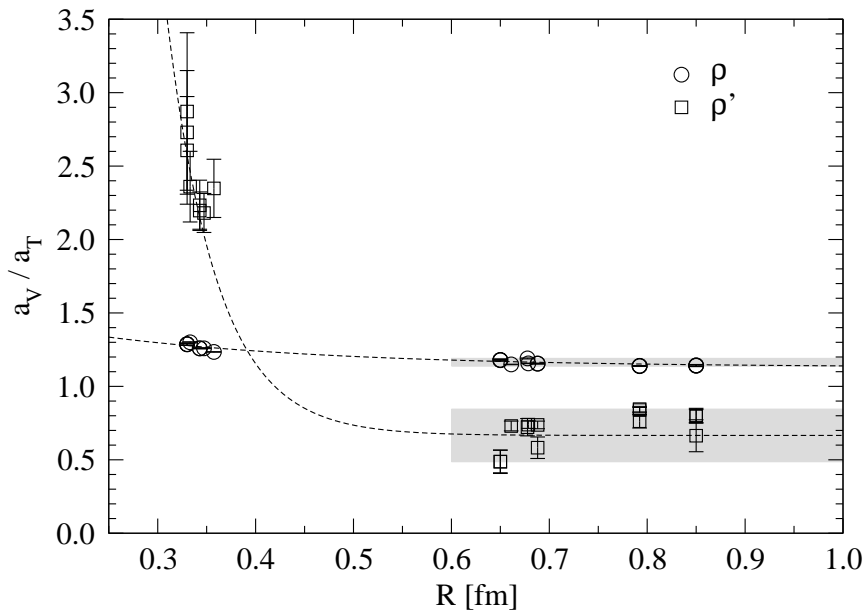where γ^i is one of the spatial Dirac matrices and γ_t is the γ-matrix in (Eu-

Figure 2: A ratio of the vector to the pseudotensor couplings versus a resolution scale R, as extracted from all 4×4 and 6×6 correlation matrices. Broken lines are drawn only to guide the eye.

clidean) time direction. For the sets A and C we construct 4×4 correlation matrices (i.e., with both vector and pseudotensor interpolators using narrow and wide smearing radii), while for the sets B1 and B2 we study the 6×6 correlation matrix (with narrow, wide and ultrawide smearings for both vector and pseudotensor operators) as well as different possible 4×4 sub-matrices.

For the parameters of the simulation the ρ mass is below the p wave decay energy. As has been discussed in [17] no coupling to the $\pi\pi$ channel is observed, which may be due to the fact that no meson-meson interpolator has been explicitly included in the set. We thus may identify the second lowest observed energy level with the ρ' (see also [18]). The masses of both ρ and ρ' states, extracted from different sets and correlation matrices, are shown in Fig. 1.

6 Chiral symmetry breaking and the angular momentum content of ρ and ρ' mesons

A measure of chiral symmetry breaking in ρ and ρ' states at some resolution R is given by the ratio a_V/a_T, which is the same as the ratio (13), obtained at a given resolution scale. At $\mu \to \infty$ (i.e., at $a \to 0$, $R \to 0$) this ratio is divergent, because the pseudotensor operator decouples from the physical state in the asymptotic freedom regime. This can be understood in two ways. In the asymptotic freedom regime chiral symmetry is not broken, hence only one of the two interpolators (which have different chiral transformation properties) can couple to the state. The vector current is conserved and the constants

f^V are scale-independent. The pseudotensor "current" is not conserved and in the asymptotic freedom regime its coupling approaches zero [9]. Indeed, in our previous study [1] we did observe this behavior for the ρ meson towards the ultraviolet regime. However, the way how this ratio approaches the ultraviolet regime for different physical states is a priori unknown and explicitly depends on the hadron wave functions.

The results for this ratio for both the ρ and ρ' states obtained from different 4×4 and 6×6 correlation matrices are consistent with each other. Fig. 2 shows the results for ρ and ρ' from both sets of correlation matrices. A complete set of operators would include all possible smearing radii R and the correlation matrix would be of infinite dimension. Given the consistency of the results fror 4×4 and 6×6 we can deduce reliable physical information already from the present results.

In particular, we clearly see that the ratio for the ρ and ρ' mesons approaches the ultraviolet regime in a very different manner. For the ρ' state the pseudotensor operator decouples much faster towards the ultraviolet than for the ρ meson. This demonstrates that the wave functions of these two states are significantly different. Since the ratio reflects a degree of chiral symmetry breaking at different scales, this symmetry breaking is very different for both states.

The ratio a_V/a_T, which is a ratio of two possible chiral representations in a hadron wave function, also defines an angular momentum decomposition of a state via the unitary transformation (5), (6). In particular, in the asymptotic freedom regime, where only the $(0,1) + (1,0)$ representation couples, the angular momentum content of ρ mesons is fixed to be $\sqrt{2/3}\,|^3S_1\rangle + \sqrt{1/3}\,|^3D_1\rangle$. Deviation from this superposition of the S- and D-waves in a state towards the infrared regime is due to chiral symmetry breaking in this state. From Fig. 2 we clearly see that the ratio is very different for both states at all possible scales. Hence chiral symmetry breaking as well as the angular momentum generation are also very different for both states. This difference shows that the ρ' state cannot be considered as simply a radial excitation of the ρ meson, since the latter would require that their angular momentum content is the same at different resolutions.

Existing data at the resolutions $R \sim 0.65 - 0.85$ fm allows us to extrapolate results up to a resolution of 1 fm. A tentative extrapolation with uncertainties is represented by a shadowed area on Fig. 2. Given that the physical size of the ρ state is of the order $0.7 - 0.8$ fm, as could be deduced from the experimental charge radii of the nucleon and ρ-meson, and for the excited ρ' meson it should be of the order of 1 fm, we can deduce chiral symmetry breaking and the angular momentum content of both states at the infrared scales of their size.

For the ground state ρ meson this ratio is within $a_V^\rho/a_T^\rho = 1.14 - 1.19$ while for its first excitation this ratio $a_V^{\rho'}/a_T^{\rho'} = 0.48 - 0.84$. Chiral symmetry is almost maximally broken (i.e., close to 1) in the ground state ρ, while a degree of chiral symmetry breaking in the ρ' state is essentially smaller, which is consistent with effective chiral restoration in highly excited hadrons [8]. In the ground state ρ the slightly leading representation is $(0,1) + (1,0)$, while in the excited ρ' state a leading chiral representation is $(1/2, 1/2)_b$.

Given these ratios, we can obtain the angular momentum contents of both mesons from (5), (6). For the ρ meson it is approximately $0.99\,|^3S_1\rangle - 0.1\,|^3D_1\rangle$.

Hence the ground state in the infrared is practically a pure $^3S_1$ state with a tiny admixture of the $^3D_1$ wave.

In contrast, in the excited ρ meson there is a sizeable contribution of the $^3D_1$ wave. In the latter case the angular momentum content is between the following two lower and upper bound values. For the lower bound it is $0.88\left|^3S_1\right\rangle - 0.48\left|^3D_1\right\rangle$ and for the upper bound it is $0.97\left|^3S_1\right\rangle - 0.25\left|^3D_1\right\rangle$. This once again demonstrates that the first excitation of the ρ meson cannot be considered as a pure radial excitation of the ground state ρ. Obviously, both radial and orbital degrees of freedom are excited which reflects yet unknown dynamics of confinement and chiral symmetry breaking.

The fact that the angular momentum content of the ρ meson is given by the $^3S_1$ state means that according to a definition used in our study there is no "spin crisis". The spin of the ρ meson in the rest frame is carried by spins of its valence quarks dressed by gluons. The gluonic field is important for the angular momentum generation, because it is this field that provides chiral symmetry breaking and is responsible for most of the hadron mass. However, it is not clear to us whether it is possible to separate contributions of quarks and gluons in the highly non perturbative, confining regime.

7 Conclusions

We summarize the most important implications of our study. It is possible to define and measure a degree of chiral symmetry breaking in mesons at different resolution scales. We are able to define and measure in a gauge invariant manner the angular momentum content of mesons in the rest frame. The angular momentum content of hadrons is deeply connected with chiral symmetry breaking in hadrons. Chiral symmetry is strongly broken in the ρ meson and its wave function (Bethe-Salpeter amplitude) is approximately a 55% - 45% mixture of the chiral representations $(0,1)+(1,0)$ and $(1/2,1/2)_b$. The angular momentum content of the ρ meson is almost completely represented by the $^3S_1$ partial wave at resolution scales of $0.15 - 1$ fm. According to our definition of the spin content, the spin of the ρ meson is carried by its valence quarks. Chiral symmetry breaking in the excited ρ' meson is weaker and the leading chiral representation in this case is $(1/2,1/2)_b$. There is a significant contribution of the $^3D_1$ wave in the ρ' wave function and consequently the ρ' meson cannot be considered as a radial excitation of the ρ.

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