Socio-Rational Secret Sharing as a New Direction in Both Rational Cryptography and Game Theory

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Abstract. This article is a journey starting at solution concepts in Game Theory, passing through reputation systems in Artificial Intelligence, and ending at a primary primitive in Cryptography. We introduce new concepts like *a rational foresighted player, social game*, and *social equilibrium*. We therefore propose a novel scheme, named *socio-rational secret sharing*, in which rational players have long-term interactions in a social context. In this society, players run secret sharing protocols while founding and sustaining a trust network among themselves. We combine *rational secret sharing*, proposed by Halpern and Teague [7], with *social secret sharing*, introduced by Nojoumian et. al. [18], in order to provide a new solution concept. To motivate our approach, consider a repeated secret sharing game such as sealed-bid auctions, where each auctioneer is supposed to receive shares of secure bids belonging to independent auctions. If we assume each party has a reputation value, we can then penalize (or reward) players who are selfish (or unselfish) from game to game in a long-term interaction. This social reinforcement rationally stimulates players to be cooperative. Despite of all the existing protocols, ours is independent of the security assumption and the communication model of the secret sharing scheme that is being used, and has a single reconstruction round.

Keywords: social secret sharing, rational secret sharing

1 Introduction

The classical (t, n)-*secret sharing scheme* was proposed by [22, 3] where a *dealer* distributes shares of a secret α among n players $P_1, \ldots, P_n$ such that any group of t or more players can reconstruct the secret in the absence of the dealer whereas any group of size less than t cannot gain any information about the secret.

In the Shamir's threshold secret sharing scheme, the dealer first generates a random polynomial $f(x) \in \mathbb{Z}_q$ such that $f(0) = \xi$, and then sends shares $f(i)$ to player P_i for $1 \leq i \leq n$. As a result, any set of t players can simply

reconstruct ξ by Lagrange interpolation while no set of fewer than t players has any information about the secret. The standard assumption in the traditional secret sharing scheme is that each player is either *honest* or *malicious* where (1) At least t honest parties cooperate in the reconstruction phase in order to recover the secret, and (2) Total number of malicious players is less than t.

As started by Halpern and Teague [7], a new research direction was initiated in the area of secret sharing and multiparty computation in a game-theoretic sense. In this new construction, players are *rational* rather than being honest or malicious meaning that they only act in their own self-interest. As we illustrate later, the classical secret sharing completely fails in this setting (see [4, 9] for an overview in this direction). Recently, the authors in [18, 19] introduced the notion of social secret sharing by constructing a publicly known trust network among players. In this construction, weights of players, i.e., the number of shares each can hold, are periodically updated such that players who cooperate end up with more shares than those who defect, that is, non-cooperative participants.

1.1 Our Construction in Nutshell

The general idea is that each *rational foresighted player* is selfish and also has concerns about the future gain or loss, and the game (secret sharing) is going to be repeated over time for an unknown number of times.

Each player has a reputation value which is adjusted each time the game is played. The initial value of the reputation is zero and its computation is public. If a player cooperates (e.g., reveal his share) his trust value is enhanced, otherwise, it is decreased. Then, the utility that each player gains would be based on the combination of the following factors:

1. Estimation of the future gain or loss due to the trust adjustment (virtual)
2. Learning the secret at the current time (real)
3. The number of other players learning the secret at the moment (real)

To estimate the future impact, we consider the following scenario: whenever a player cooperates, we assume he also gains some extra units of utility (that is, he has a chance to learn more secrets in the next games), and whenever he defects, we assume he also loses some extra units of utility (that is, he loses a chance to learn more secrets in the next games). This gain or loss is virtual at the current time but will be realized in the future. As an example, consider a scenario in which a retail store says if you buy something today (cooperate), you will receive a discount (utility) on your next purchase (future).

In other words, if the reputation of P_i is decreased, he will have less chance to be invited to the future secret sharing. Otherwise, P_i is going to be invited to more secret sharing. To realize this scenario, in each new secret sharing or game, the dealer selects players based on their reputation values, e.g., %50 from reputable players, %30 from newcomers, and %20 from non-reputable parties.

1.2 Motivation

Our motivation therefore is to melt in the idea of the social secret sharing scheme into its rational counterpart in order to provide a new solution concept. In this new construction, players sustain the stated trust network overtime to improve (or purify) their society. In fact, players enter into long-term interactions for handling an unknown number of independent secret sharing schemes.

We intend to provide a new solution independent of the security assumption and the communication model of the applied secret sharing with only a single round of reconstruction. Moreover, we would like to consider the players' behaviors even in the intermediate computations between sharing and reconstruction phases, such as, proactive share update, joint random number generation, etc. All these intermediate computations can be as critical as the reconstruction phase in real-world applications. To further motivate our socio-rational secret sharing, consider the following two scenarios in a sealed-bid auction:

Selfish Behavior at the Reconstruction Phase: each auctioneer is supposed to sequentially receive shares of secure bids belonging to an unknown number of independent auctions. An auctioneer may deviate in the winner determination phase of an auction with this hope that he is the only one who learns the outcome, similar to rational secret sharing. (However, as we stated earlier, dealers can reduce the utility of selfish players by not inviting them to the next games.)

Selfish Behavior Between Sharing and Reconstruction Phases: suppose an auctioneer has been involved in many independent societies each of which is running a sequence of secure auctions. If this player simultaneously receives many requests for intermediate computations (e.g., jointly generate a random number as a mask) from those societies, due to the time and resource limitations, he must decide to which one he should contribute based on the utility he gains.

1.3 Contribution

As our main contribution, we propose a *socio-rational secret sharing scheme* where players have incentive to cooperate not only in the reconstruction phase but also in the intermediate computations throughout a secret's lifetime. This incentive is sustained from one scheme to another one by a motivation of the personal reputation's enhancement. In fact, players avoid any selfish behavior due to the social reinforcement of the trust network. Having a *social trust network* not only affects the rational cryptography but also can significantly impress game-theoretic constructions.

Some of the existing solutions are not really fair since the protocol is aborted if a selfish player deviates. This means that a selfish player can force the protocol to be executed for an exponential number of iterations (an unfair punishment), which may cause the cryptographic primitives used in the scheme to be broken. But our proposed protocol has an *everlasting punishment* or *reward* approach while providing *opportunities* for newcomers. It also worth mentioning that any secret sharing scheme can be used in our proposed dealer-free construction.

Our contribution is totally different from the *punishment strategy* used in the *repeated prisoners' dilemma* [21] where players penalize potential deviants. As the authors have mentioned, the major point behind the repeated games is the fact that if each participant believes any deviation terminates the mutual cooperation (resulting in a subsequent loss that outweighs the short-term gain), he then prefers to cooperate. For instance, consider the prisoners' dilemma with $\mathcal{C}ooperation$ and $\mathcal{D}efection$ actions. Both players cooperate until one of them deviates. Then, the other player chooses $\mathcal{D}$ for a specific number of times as a punishment. Meanwhile, the deviant rewards the punisher by selecting $\mathcal{C}$ as a compensation. Finally, the game returns to the mutual cooperation. Indeed, our approach has the following advantages over the punishment strategy:

- In our model, a player is not just an abstract entity who selects actions. It also has a social characteristic reflected in his reputation that shows his trustworthiness. This attribute is solely built by the player himself.
- The punishment strategy is performed by selecting actions that are harmful for deviants whereas the punishment or reward in our model is independent of the action's selection, i.e., losing or gaining reputation and utility.
- Our approach avoids penalizing innocent players or the punisher himself. It also avoids being involved, to some extend, in a game with seriously selfish players who are not reputable at the first place.
- The punishment strategy does not consider that a game may have various importance and utility weights whenever it is repeatedly played. For instance, whether it is a secret sharing for a missile launch or for a safety box.
- The punishment strategy has a discrete penalizing approach whereas our construction has a continuous impact on deviants. For example, it may take a long time to regain the lost reputation.
- Our proposed approach not only consider the punishment or reward but also defines six different scenarios in order to fairly deal with various types of players including good player, bad player, and newcomers.

Our contribution is also different from the constructions forming histories and beliefs such as *subgame perfect equilibrium* or *Bayesian equilibrium* [21]. In the former, players reassess their decisions based on the past *history*, i.e., a sequence of previous actions. In the latter, the game is designed to deal with the situations in which parties are not certain about the characteristics of each others. Therefore, they form *beliefs*, i.e., a probability distributions over actions, to anticipate any future behaviors.

Let P_i be a specific player and P_j for $1 \leq j \neq i \leq n$ denotes all the other players except P_i. More specifically, our trust calculation method and social setting differs from these kinds of solution concepts in the following aspects:

- In forming a belief about P_i's intentions both parties contribute. That is, P_i is indirectly involved by his behavior, i.e., action selections, and P_j-s are directly involved by the methodology that they use in order to form the probability distribution over actions. A belief may or may not be a common knowledge meaning that various players may have different judgments and

beliefs about P_i. Whereas, the reputation of P_i in a trust network is solely constructed by his behavior through a trust function, which is a commonly known function for the reputation measurement. That is, the reputation is a direct reflection of P_i's attitude (no misunderstanding), and he very well knows the impact of his decision on other players' mind considering his current characteristic, whether he is known as a good player, a bad player, or a newcomer. He can also estimate how much extra utility he may gain or lose after his reputation's adjustment, which is a strong enforcement.

- Histories and beliefs are more general compared to the reputation system in a trust network. This means a belief as a probability distribution can be defined over any set of actions for any types of players. Whereas, the reputation is built over a specific set of actions, such as $\mathcal{C}$ooperation and $\mathcal{D}$efection ($\mathcal{X}$: corruption as a malicious behavior might be also considered in a mixed model), for a specific types of players, such as good or bad players, and newcomers. As a result, the reputation system is simpler to be analyzed in a solution concept and is more suitable for cryptographic constructions.
- In the history and belief systems all measurements are inside of the game-theoretic model whereas our reputation system isolates these computations from the game. For instance, two separate probability distributions can be defined over players' types and actions by considering the past behaviors [1]. But our publicly known trust function combines these two measurements in a single reputation value outside of the game-theoretic model (although these values might be interpreted similar to types and beliefs). In other words, the punishment or reward is embedded inside of our reputation system which continuously affects players' utilities in the game-theoretic model, i.e., losing utility due to the reputation's decline or losing reputation and not being selected for the future sharing.

2 Preliminaries

In this section, required background regarding some fundamental game-theoretic concepts along with social and rational secret sharing schemes are presented.

2.1 Game-Theoretic Concepts

Definition 1. *Let $\mathcal{A} \stackrel{\text{def}}{=} \mathcal{A}_1 \times \cdots \times \mathcal{A}_n$ be an action profile for n players. A game $\Gamma = (\mathcal{A}_i, u_i)$ for $1 \leq i \leq n$, presented in normal form, is a set of possible actions $\mathcal{A}_i$ and a utility function $u_i : \mathcal{A} \mapsto \mathbb{R}$ for each player P_i. We refer to a vector of actions $\boldsymbol{a} = (a_1, \ldots, a_n) \in \mathcal{A}$ as an outcome of the game.*

Definition 2. *The utility function u_i illustrates the preferences of player P_i over different outcomes. We say P_i prefers outcome $\boldsymbol{a}$ to $\boldsymbol{a}'$ iff $u_i(\boldsymbol{a}) > u_i(\boldsymbol{a}')$, and he weakly prefers outcome $\boldsymbol{a}$ to $\boldsymbol{a}'$ if $u_i(\boldsymbol{a}) \geq u_i(\boldsymbol{a}')$.*

[1] P_i is good or bad with 0.7 or 0.3 probabilities respectively. Based on that, a P_j may belief that P_i reveals or not reveals his share with 0.9 or 0.1 probabilities accordingly.

In order to allow players to follow randomized strategies (i.e., strategy is the way of choosing actions), we define σ_i as a probability distribution over $\mathcal{A}_i$ for a player P_i meaning that he samples $a_i \in \mathcal{A}_i$ according to σ_i. A strategy is said to be *pure-strategy* if each σ_i assigns probability 1 to an action, otherwise, it is said to be *mixed-strategy*. Let $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_n)$ be the vector of players' strategies, and let $(\sigma_i', \boldsymbol{\sigma}_{-i}) \stackrel{\text{def}}{=} (\sigma_1, \ldots, \sigma_{i-1}, \sigma_i', \sigma_{i+1}, \ldots, \sigma_n)$, i.e., only P_i changes σ_i into σ_i' and other players' strategies remain the same in the vector. Therefore, $u_i(\boldsymbol{\sigma})$ denotes the expected utility of P_i under the strategy vector $\boldsymbol{\sigma}$. A rational player's goal is to maximize this utility. In all the following definitions, one can substitute an action $a_i \in \mathcal{A}_i$ with its probability distribution $\sigma_i \in \mathcal{S}_i$ or vice versa.

Definition 3. *A vector of strategies $\boldsymbol{\sigma}$ is a Nash equilibrium if for all i and any $\sigma_i' \neq \sigma_i$ it holds that $u_i(\sigma_i', \boldsymbol{\sigma}_{-i}) \leq u_i(\boldsymbol{\sigma})$. This means no one gains any advantage by deviating from the protocol as long as other parties follow the protocol.*

Definition 4. *Let $\mathcal{S}_{-i} \stackrel{\text{def}}{=} \mathcal{S}_1 \times \cdots \times \mathcal{S}_{i-1} \times \mathcal{S}_{i+1} \times \cdots \times \mathcal{S}_n$. A strategy $\sigma_i \in \mathcal{S}_i$ (or an action) is weakly dominated by a strategy $\sigma_i' \in \mathcal{S}_i$ (or another action) with respect to $\mathcal{S}_{-i}$ if:*

1. *For all $\boldsymbol{\sigma}_{-i} \in \mathcal{S}_{-i}$, it holds that $u_i(\sigma_i, \boldsymbol{\sigma}_{-i}) \leq u_i(\sigma_i', \boldsymbol{\sigma}_{-i})$.*
2. *There exists a $\boldsymbol{\sigma}_{-i} \in \mathcal{S}_{-i}$ such that $u_i(\sigma_i, \boldsymbol{\sigma}_{-i}) < u_i(\sigma_i', \boldsymbol{\sigma}_{-i})$.*

Meaning that P_i can never improve its utility by playing σ_i, and can sometimes improve it by not playing σ_i. [2]

Definition 5. *Given $\Gamma = (\mathcal{A}_i, u_i)$ for $1 \leq i \leq n$ and a strategy profile $\mathcal{S}$, let $\mathcal{DOM}_i(\mathcal{S})$ denotes the set of strategies in $\mathcal{S}_i$ that are weakly dominated by other strategies with respect to $\mathcal{S}_{-i}$. Let $\mathcal{S}_i^0$ denotes the initial set of P_i's strategies.*

1. *For $k \geq 1$, define $\mathcal{S}_i^k \stackrel{\text{def}}{=} \mathcal{S}_i^{k-1} \setminus \mathcal{DOM}_i(\mathcal{S}^{k-1})$.*
2. *Let $\mathcal{S}_i^\infty \stackrel{\text{def}}{=} \bigcap_k \mathcal{S}_i^k$, i.e., strategies that are survived in all elimination rounds.*

We say σ_i survives iterated deletion of weakly dominated strategies if $\sigma_i \in \mathcal{S}_i^\infty$.

Nash equilibrium surviving iterated deletion of weakly dominated strategies are used in [7, 6, 14, 1].

2.2 Rational Secret Sharing Scheme

In this section, we quickly review *rational secret sharing*, initiated by Halpern and Teague [7]. This construction was later improved by Gordon and Katz [6]. The scheme consists of a *dealer* who holds a secret and n players $P_1, \ldots, P_n$.

The protocol proceeds in a sequence of iterations of which only one iteration is the real reconstruction phase (i.e., the last iteration) and the rest are just fake iterations for trapping selfish players. At the end of each iteration, the

[2] A strategy $\sigma_i \in \mathcal{S}_i$ is strictly dominated if player P_i can always improve its utility by not playing σ_i.

protocol either terminates due to the selfish behavior's observation/cooperative share reconstruction, or proceeds to the next iteration otherwise. Indeed, in any given round, players do not know whether the current iteration is the real reconstruction phase in which a player may gain more utility by being silent and not sending his share to others, or just a test round in which a player must cooperate, otherwise, the other players abort the protocol.

To make this more clear, consider the following scenario for a player P_j. If players P_i for $1 \leq i < t-1$ or $t-1 < i \leq n$ reveal their shares, nothing changes whether P_j reveals his share or not. In the former case, no one learns the secret. In the latter case, everyone learns the secret. On the other hand, if players P_i for $i = t-1$ reveal their shares, then P_j can not only learn the secret with his own private share (i.e., t shares are sufficient to use Lagrange interpolation) but also can prevent others to learn it by not revealing his share, i.e., the preference of a self-interested player in rational secret sharing. In other words, for each P_i, revealing the share is *weakly dominated* by not revealing the share. As a result, no one reveals his share and the secret is never reconstructed.

Let $u_i(\boldsymbol{a})$ denotes the utility of player P_i for a specific outcome $\boldsymbol{a}$ of the protocol. Suppose $l_i(\boldsymbol{a})$ is a bit defining whether P_i has learned the secret or not in a specific outcome. We therefore consider $\delta(\boldsymbol{a}) = \sum_i l_i(\boldsymbol{a})$ which denotes the number of players who have learned the secret. As proposed in [7], the following assumptions regarding players' utility functions are made:

- $l_i(\boldsymbol{a}) > l_i(\boldsymbol{a}') \Rightarrow u_i(\boldsymbol{a}) > u_i(\boldsymbol{a}')$.
- If $l_i(\boldsymbol{a}) = l_i(\boldsymbol{a}')$ and $\delta(\boldsymbol{a}) < \delta(\boldsymbol{a}') \Rightarrow u_i(\boldsymbol{a}) > u_i(\boldsymbol{a}')$.

The first assumption means P_i prefers the outcome in which he learns the secret, that is, since $l_i(\boldsymbol{a}) = 1$ and $l_i(\boldsymbol{a}') = 0$, therefore, he prefers $\boldsymbol{a}$. The second one means P_i prefers the outcome in which the fewest number of other players learn the secret. As illustrated in [7], the Nash equilibrium is too weak for rational secret sharing. As a result, they suggested to design a protocol that applies a Nash equilibrium surviving iterated deletion of weakly dominated strategies.

2.3 Social Secret Sharing Scheme

In this section, we review *social secret sharing*, introduced by Nojoumian et. al. [18, 19], where shares are allocated based on a player's reputation and the way he interacts with other parties. In this scheme, weights of players are adjusted such that participants who cooperate receive more shares compared to non-cooperative parties. This is similar to human social life where people share more secrets with whom they really trust and vice versa.

In other words, each player initially receives a constant number of shares. Consequently, players are assigned weights based on their behaviors. As a result, each player receives a number of shares according to his trust value which is the representation of a player's reputation over time. The authors apply the trust management approach proposed in [17]. We review this technique in order to use it in our social network. The design of this function is out of the scope of our construction meaning that one can apply an arbitrary trust function.

Definition 6. *Let $\mathcal{T}_i^j(p)$ be the trust value assigned by P_j to P_i in period p. Let $\mathcal{T}_i : \mathbb{N} \mapsto \mathbb{R}$ be the trust function representing the reputation of P_i.*

$$\mathcal{T}_i(p) = \frac{1}{n-1} \sum_{j \neq i} \mathcal{T}_i^j(p) \text{ where } -1 \leq \mathcal{T}_i(p) \leq +1 \text{ and } \mathcal{T}_i(0) = 0$$

$\mathcal{T}_i(p) = \mathcal{T}_i^j(p)$ if P_j for $1 \leq j \neq i \leq n$ have equal trust values for P_i, i.e., trust values (personal quantity) are equal to the reputation value (social quantity).

The general idea in [17] is to support good players, discredit bad ones, and create opportunities for newcomers whom we do not know much about their behaviors. The authors demonstrate and resolve the problem of a highly cited construction in the literature [23]. As shown in Table 1, six possible actions and three sets $\mathcal{B}, \mathcal{N}, \mathcal{G}$ are defined for bad, new, and good players respectively, where α, β define boundaries on the trust values for different sets of players.

Trust Value	**Cooperation:** $P_i(\mathcal{C})$	**Defection:** $P_i(\mathcal{D})$
$P_i \in \mathcal{B} \Rightarrow \mathcal{T}_i(p) \in [-1, \beta)$	Encourage	Penalize
$P_i \in \mathcal{N} \Rightarrow \mathcal{T}_i(p) \in [\beta, \alpha]$	Give a Chance	Take a Chance
$P_i \in \mathcal{G} \Rightarrow \mathcal{T}_i(p) \in (\alpha, +1]$	Reward	Discourage

Table 1. Six Possible Actions for the Trust Management [17]

This construction applies monotonically increasing and decreasing functions $\mu(x)$ and $\mu'(x)$, in the case of cooperation $\mathcal{C}$ and defection $\mathcal{D}$, to publicly update the trust (or reputation) value of each player P_i. For instance, by assigning $\eta = 0.01 < \theta = 0.05 < \kappa = 0.09$, we can simply define various points and construct an appropriate trust function via regression.

$$P_i(\mathcal{C}) \Rightarrow \mathcal{T}_i(p) = \mathcal{T}_i(p-1) + \mu(x) \qquad P_i(\mathcal{D}) \Rightarrow \mathcal{T}_i(p) = \mathcal{T}_i(p-1) - \mu'(x)$$

$$\mu(x) \in \begin{cases} [\eta, \theta) & P_i \in \mathcal{B} \\ \theta & P_i \in \mathcal{N} \\ (\theta, \kappa] & P_i \in \mathcal{G} \end{cases} \qquad \mu'(x) \in \begin{cases} (\theta, \kappa] & P_i \in \mathcal{B} \\ \theta & P_i \in \mathcal{N} \\ [\eta, \theta) & P_i \in \mathcal{G} \end{cases}$$

It worth mentioning that the authors also define λ as the *transaction cost* to fairly deal with *cheap* cooperations and *expensive* defections.

3 Literature Review

As we mentioned, the notion of *rational secret sharing* is introduced by Halpern and Teague [7], which is later improved by Gordon and Katz [6]. Assuming the same game-theoretic model, Lysyanskaya and Triandopoulos [14] provide

a solutions in a *mixed-behavior* setting in which players are either rational or malicious, and Abraham et. al. [1] define a notion of resistance to coalitions and present a *coalition-resistant* protocol. All these constructions use simultaneous channels (either a broadcast channel or secure private channels) which means each player must decide on the value he wants to broadcast before observing the values broadcasted by others, i.e., a strategic game.

The proposed constructions in [12, 13, 8] rely on *physical assumptions* such as secure envelopes and ballot boxes, which might be impossible or hard to be implemented for distant players. With the same assumptions, Micali and shelat [16] provide a purely rational secret sharing using a verifiable trusted channel. They show that all the existing solutions not only rely on the players' rationality, but also on their beliefs. As a result, they cannot guarantee that all rational players learn the secret. For instance, if P_i believes that equilibrium (a, b) will be played whereas P_j believes equilibrium (a', b') is going to be played, then the game ends up with (a, b') which may not be an equilibrium at all.

Kol and Naor [11] introduce an equilibrium notion of *strict Nash equilibrium* in an information-theoretic secure setting. In Nash equilibrium, no deviations are advantageous (i.e., no incentive to deviate). In its strict counterpart, all deviations are disadvantageous (i.e., an incentive not to deviate). They first consider both simultaneous and non-simultaneous broadcast channels and then provide a new solution to avoid the simultaneous channel at the cost of increasing the round complexity by using the synchronous broadcast channel.

They later [10] show that all the existing computational-based cryptographic protocols are susceptible to backward induction because of the cryptographic primitives used in the beginning of those protocols, that is, they can surely be broken after an exponential number of rounds. The authors then illustrate a new cryptographic coalition-resilient approach that is *immune to backward induction* by considering simultaneous as well as non-simultaneous broadcast channels.

The *computational strict Nash equilibrium* is introduced in [5] which is a stable solution concept with respect to trembles. This construction is dealer-free and can tolerate a coalition of size $t-1$ without using simultaneous channels. In can even be run over asynchronous point-to-point networks. Finally, it is efficient in terms of computation, share size, and round complexity.

Maleka et. al. [15] present *repeated rational secret sharing*, with the exact same approach proposed in [21], by considering two punishment strategies. In the former, each player reveals his share as long as other players cooperate. As soon as the first defection is observed, players do not reveal their shares in every subsequent game. In the latter, players do not send their shares to the deviant for k subsequent games after observing the first defection. These constructions are severely problematic. In the first one, each player not only punishes the deviant but also other players including himself. In the second method, a player may deviate in an *expensive* reconstruction without having any concern for k subsequent *cheap* reconstructions. Indeed, the nature of a punishment strategy must depend on how much future outcomes are worth for each player. Finally, they only consider a fixed number of m players without considering newcomers.

Other constructions are recently proposed in the literature. For instance, Ong et. al. [20] illustrate a protocol which is fair when the reconstruction phase is executed with many rational players together with a minority of honest parties. Asharov and Lindell [2] show that in all the existing protocols, the designer needs to know the actual utility values of players. They then show that it is possible to achieve utility independence through the relaxation of assumptions.

4 Socio-Rational Secret Sharing ($\mathcal{SRS}$)

We first provide the formal definitions of *social game* which is repeated for an unknown number of times, *social equilibrium*, and *socio-rational secret sharing*. In our model, we assume that trust values are equal to the reputation value of each P_i, that is, $\mathcal{T}_i(p) = \mathcal{T}_i^j(p)$ for $1 \leq j \neq i \leq n$ where $\mathcal{T}_i(0) = 0$. The construction of this function is independent of the proposed protocol, therefore, we apply the existing function presented in [17]. Since the significance of each secret sharing is different from game to game, we assume that the *transaction cost* is considered for each sharing outside of the game, i.e., it is embedded inside of the trust calculation scheme and has its impacts on players' utilities.

Definition 7. *Let $\mathcal{A} \stackrel{\text{def}}{=} \mathcal{A}_1 \times \cdots \times \mathcal{A}_n$ and $\mathcal{T} \stackrel{\text{def}}{=} \mathcal{T}_1 \times \cdots \times \mathcal{T}_n$ be the action and reputation profiles respectively. In a society S of size $|S| = N$, a **social game** $\Gamma = (\mathcal{A}_i, \mathcal{T}_i, u_i)$, for $1 \leq i \leq n$ and $n \leq N$, is repeatedly played and contains a set of possible actions $\mathcal{A}_i$, a trust value $\mathcal{T}_i$, and a utility function $u_i : \mathcal{A} \times \mathcal{T}_i \mapsto \mathbb{R}$ for each player P_i. The value $\mathcal{T}_i(p)$ in period p is computed by a trust function $\mathcal{T} : \mathbb{N} \mapsto \mathbb{R}$, and $\boldsymbol{a} = (a_1, \ldots, a_n) \in \mathcal{A}$ is said to be the game's outcome.*

Definition 8. *A vector of strategies $\boldsymbol{\sigma}$ is said to be a **social equilibrium** in a social game if for all i and any $\sigma_i' \neq \sigma_i$ it holds that $u_i(\sigma_i', \boldsymbol{\sigma}_{-i}) \leq u_i(\boldsymbol{\sigma})$, and consequently, it is said to be **strict social equilibrium** if $u_i(\sigma_i', \boldsymbol{\sigma}_{-i}) < u_i(\boldsymbol{\sigma})$. This is due to $u_i : \mathcal{A} \times \mathcal{T}_i \mapsto \mathbb{R}$, i.e., a player with $\mathcal{T}_i$ cannot gain any benefit in the society by deviating from the protocol as long as others follow the protocol.*

The utility function is a central part in every game since each player makes decision based on his expected utility. The *utility assumption* refers to players' preferences over the game's outcome whereas the *utility computation* illustrate the method of computing the utility of each player, i.e., utility function.

4.1 Utility Assumption

Similar to Section 2.2, let $u_i(\boldsymbol{a})$ denotes P_i's utility of $\boldsymbol{a}$, let $l_i(\boldsymbol{a})$ denotes if P_i has learned the secret, and define $\delta(\boldsymbol{a}) = \sum_i l_i(\boldsymbol{a})$. Finally, assume $\mathcal{T}_i^{\boldsymbol{a}}(p)$ shows the reputation of P_i after $\boldsymbol{a}$ which has happened in $p-1$.

A. If $l_i(\boldsymbol{a}) = l_i(\boldsymbol{a}')$ and $\mathcal{T}_i^{\boldsymbol{a}}(p) > \mathcal{T}_i^{\boldsymbol{a}'}(p) \Rightarrow u_i(\boldsymbol{a}) > u_i(\boldsymbol{a}')$.
B. $l_i(\boldsymbol{a}) > l_i(\boldsymbol{a}') \Rightarrow u_i(\boldsymbol{a}) > u_i(\boldsymbol{a}')$.
C. If $l_i(\boldsymbol{a}) = l_i(\boldsymbol{a}')$ and $\delta(\boldsymbol{a}) < \delta(\boldsymbol{a}') \Rightarrow u_i(\boldsymbol{a}) > u_i(\boldsymbol{a}')$.

The preference A illustrates that whether P_i learns the secret or not he prefers to stay reputable. B and C are the same assumptions of rational secret sharing.

Definition 9. *In a social game, a* ***rational foresighted player*** *has prioritized preferences: $A^{\rho_1} \succ B^{\rho_2} \succeq C^{\rho_3}$: A (greediness) is strictly preferred to B by the multiplicative factor ρ_1, B (selfishness) is at least as good as C by a significance factor ρ_2, and C (selfishness) with the impact factor ρ_3, where $\rho_1 \gg \rho_2 \geq \rho_3 \geq 1$.*

This means that a rational foresighted player has a long-term vision and first prefers to achieve the highest level of trustworthiness. Only in that case, he would be involved in the future games and consequently gains more profits; this can be interpreted as greediness. He secondly prefers the outcome in which he learns the secret. Finally, he desires the fewest number of other players learn the secret. We next construct a new utility function which satisfies all three preferences.

4.2 Utility Computation

Our proposed function $u_i : \mathcal{A} \times \mathcal{T}_i \mapsto \mathbb{R}$ shows the utility that each player P_i can gain in a specific outcome, considering his reputation.

I. Sample Function. Let $\omega_i(\boldsymbol{a}) = 3/(2 - \mathcal{T}_i^{\boldsymbol{a}}(p))$ where $\mathcal{T}_i^{\boldsymbol{a}}(p) \in [-1, +1]$, and let $\tau_i(\boldsymbol{a}) = \mathcal{T}_i^{\boldsymbol{a}}(p) - \mathcal{T}_i^{\boldsymbol{a}}(p-1)$. We also consider $\Omega > 0$ to be a unit of utility and then compute the utility $u_i(\boldsymbol{a})$ that each player is supposed to receive. To satisfy all the stated assumptions, consider the following mathematical terms:

$$A : \frac{|\tau_i(\boldsymbol{a})|}{\tau_i(\boldsymbol{a})} \times \omega_i(\boldsymbol{a}) \times \Omega, \text{ i.e., future loss or gain} \tag{1}$$

$$B : l_i(\boldsymbol{a}) \times \Omega \text{ where } l_i(\boldsymbol{a}) \in \{0, 1\} \tag{2}$$

$$C : \frac{\Omega}{\delta(\boldsymbol{a}) + 1} \text{ where } \delta(\boldsymbol{a}) = \sum_i l_i(\boldsymbol{a}) \tag{3}$$

(1) The first equation would be $+\omega_i(\boldsymbol{a})\Omega$ if a player cooperates and it would be $-\omega_i(\boldsymbol{a})\Omega$ otherwise. This means a player gains or loses at least 1 and at most 3 (depending on his reputation value reflected in ω_i) units of utility in the future sharing due to his current behavior; although the gain or loss might be more than our estimation. (2) The second equation illustrate that a player gains one unit of utility if he learns the secret at the moment and he loses this opportunity otherwise. (3) The final equation expresses that one unit of utility is divided among all players that have learned the secret. We therefore combine these equations with their corresponding coefficients.

$$u_i(\boldsymbol{a}) = \rho_1 \left(\frac{|\tau_i(\boldsymbol{a})|}{\tau_i(\boldsymbol{a})} \times \omega_i(\boldsymbol{a}) \times \Omega \right) + \rho_2 \left(l_i(\boldsymbol{a}) \times \Omega \right) + \rho_3 \left(\frac{\Omega}{\delta(\boldsymbol{a}) + 1} \right) \tag{4}$$

$$= \Omega \times \left(\rho_1 \left(\frac{|\tau_i(\boldsymbol{a})|}{\tau_i(\boldsymbol{a})} \times \omega_i(\boldsymbol{a}) \right) + \rho_2 \left(l_i(\boldsymbol{a}) \right) + \rho_3 \left(\frac{1}{\delta(\boldsymbol{a}) + 1} \right) \right) \tag{5}$$

The proposed function shows that if a player defects (or cooperates) with a selfish (or unselfish) motivation, he may gain (or lose) a unit of utility Ω at the moment but he will definitely lose (or gain), at least, a unit of utility Ω in the future. Although this may not be the case in reality since the cost (or gain) of a defection (or cooperation) is mush more than that due to the reduction (or amplification) of the reputation, as an everlasting characteristic which remains with the player for his entire life. In addition, we later show that the dealer gives less (or more) chance of contribution to non-reputable (or reputable) players in the future games.

II. General Function. It worth mentioning that one can design any arbitrary function as long as it satisfies our utility assumptions. For instance, instead of using $\delta(\boldsymbol{a})$, we can define a function $f(\delta(\boldsymbol{a}))$ where $f : \{0, \ldots, n\} \mapsto \mathbb{R}$ in order to consider the number of players learning the secret, and so forth. A perfect design of this function is out of the scope of this paper. Our intention is to show the impact of the first property on the rational secret sharing scheme when rational foresighted players with future concerns are considered.

Proposition 1. *A utility function F_i with the following linear combination of preference factors $\rho_1 \gg \rho_2 \geq \rho_3 \geq 1$ and functions $f_1(\mathcal{T}_i), f_2(l_i), f_3(\delta)$ satisfies the preference of a rational foresighted player, that is, $A^{\rho_1} \succ B^{\rho_2} \succeq C^{\rho_3}$, where $|f_1|$ is a monotonically increasing function, f_3 is a monotonically decreasing function (excluding zero), and $|f_1(\mathcal{T}_i)| \geq f_2(l_i) \geq f_3(\delta)$ except that $f_2(0) < f_3(0)$.*

$$F_i(\boldsymbol{a}) = \Omega\Big(\rho_1 f_1(\mathcal{T}_i) + \rho_2 f_2(l_i) + \rho_3 f_3(\delta)\Big) \tag{6}$$

$$f_1 : \begin{cases} \mathbb{R}_{>0} & \tau_i > 0 \\ \mathbb{R}_{<0} & \textit{otherwise} \end{cases} \qquad f_2 : \begin{cases} 0 & l_i = 0 \\ \mathbb{R}_{>0} & l_i = 1 \end{cases} \qquad f_3 : \begin{cases} 1 & \delta = 0 \\ \mathbb{R}_{>0} & \delta \in \{1 \ldots n\} \end{cases}$$

Proof. Due to the lack of space we leave the proof but only analyze the situation in which $f_2(0) < f_3(0)$, i.e., $l_i = 0$ and $\delta = 0$ (no one has learned the secret). This means that at most $t-1$ players, where t is the threshold, have cooperated. As a result, P_i is either among the cooperative players who have revealed their shares or among the non-cooperative parties. However, the first property states that P_i potentially gains more utilities if he enhances his reputation, no matter if he learns the secret or not. Therefore, in the case of the cooperation we have:

$$P_i(\mathcal{C}) : (\tau_i > 0, f_2 = 0, f_3 = 1) \text{ then } u_i^{P_i(\mathcal{C})}(\boldsymbol{a}) = \Omega\Big(\rho_1 f_1 + \rho_3\Big)$$

We can simply compute the utility of P_i in the case of the defection as follows:

$$P_i(\mathcal{D}) : (\tau_i < 0, f_2 = 0, f_3 = 1) \text{ then } u_i^{P_i(\mathcal{D})}(\boldsymbol{a'}) = \Omega\Big(-\rho_1 f_1 + \rho_3\Big)$$

Consequently we get $u_i^{P_i(\mathcal{C})}(\boldsymbol{a}) > u_i^{P_i(\mathcal{D})}(\boldsymbol{a'})$ which confirms the statement. □

4.3 Proposed Protocol

Before illustrating any details, we first define socio-rational secret sharing. For the sake of simplicity, let assume all players consider the pure-strategy.

Definition 10. *A **socio-rational secret sharing** $\Gamma \in \Delta$ is a social game with (1) an action set $\mathcal{A}_i = \{\mathcal{C}, \mathcal{D}\}$, (2) a trust function $\mathcal{T}_i$ for rational foresighted players with types $\mathcal{G}, \mathcal{N}, \mathcal{B}$ who are involved in various societies, as defined in Section 2.3, and (3) a utility function $u_i : \mathcal{A} \times \mathcal{T}_i \mapsto \mathbb{R}$, as defined in Section 4.2.*

I. Sharing. The sharing phase is similar to that of the regular secret sharing. The only difference is the way that the dealer selects players for secret sharing in the society. In fact, he gives more chance to reputable players compared to unreliable parties. This method of selection realizes the term $\rho_1 f_1(\mathcal{T}_i)$ in our proposed utility function. Suppose ϕ is the probability distribution over types $\mathcal{B}, \mathcal{N}, \mathcal{G}$ meaning that the dealer selects n out of N, where $n \leq N$, players from the society based on this non-uniform probability distribution.

$$\phi = \sum_{j \in \{\mathcal{B}, \mathcal{N}, \mathcal{G}\}} \phi_j = 1 \text{ where } \phi_{\mathcal{B}} \ll \phi_{\mathcal{N}} < \phi_{\mathcal{G}} \tag{7}$$

This means, although it is the best approach to mostly invite the reputable players for any secret sharing in the society, it is not fair if the dealer does not provide any opportunity for newcomers or if he completely ignores bad players. Once in a while he should give a chance even to bad players to compensate for their past behaviors. This is a realistic approach even in human society and can be seen as the forgiveness factor of the dealer.

II. Reconstruction. The secret recovery phase is also similar to that of the standard secret sharing schemes meaning that any construction can be selected independent of its security assumption and the communication model. We only attach our reputation system a long with our game-theoretic model to the selected scheme. Finally, since players' reputation and the trust function are public information, therefore, all computations associated to the reputation system is done by any authority or a committee of players on a public board.

It worth mentioning that it is not required to consider unknown number of iterations in the reconstruction of a secret (which is the case in all the existing rational secret sharing) since we consider a socio-rational secret sharing scheme as a long-term game. In other words, those iterations for a single reconstruction are conceptually stretched over time on multiple reconstructions of different secrets in a social setting.

We initially analyze the 2-out-of-2 case by considering our sample function in Equation (5), and then use the general form of our utility function.

Theorem 1. *In our $(2, 2)$-socio-rational secret sharing, $\mathcal{C}$ strictly dominates $\mathcal{D}$, considering our sample utility function. In other words, $\mathcal{D}$ is strictly dominated by $\mathcal{C}$. As a result, $(\mathcal{C}, \mathcal{C})$ is a strict social equilibrium which is a unique solution.*

Proof. We compute the utility of each outcome for P_i. For the sake of simplicity we assume $\omega_i(\boldsymbol{a}) = 1$; although the proof is valid for all values of $\omega_i(\boldsymbol{a}) \in [1, 3]$:

1. $(\mathcal{C}, \mathcal{C}) : (\tau_i > 0, l_i = 1, \delta = 2)$ then $u_i^{(\mathcal{C},\mathcal{C})}(\boldsymbol{a}) = \Omega\Big(\rho_1 + \rho_2 + \frac{\rho_3}{3}\Big)$
2. $(\mathcal{C}, \mathcal{D}) : (\tau_i > 0, l_i = 0, \delta = 1)$ then $u_i^{(\mathcal{C},\mathcal{D})}(\boldsymbol{a}) = \Omega\Big(\rho_1 + \frac{\rho_3}{2}\Big)$
3. $(\mathcal{D}, \mathcal{C}) : (\tau_i < 0, l_i = 1, \delta = 1)$ then $u_i^{(\mathcal{D},\mathcal{C})}(\boldsymbol{a}) = \Omega\Big(-\rho_1 + \rho_2 + \frac{\rho_3}{2}\Big)$
4. $(\mathcal{D}, \mathcal{D}) : (\tau_i < 0, l_i = 0, \delta = 0)$ then $u_i^{(\mathcal{D},\mathcal{D})}(\boldsymbol{a}) = \Omega\Big(-\rho_1 + \rho_3\Big)$

We simply ignore the common term Ω. Since $\rho_1 \gg \rho_2 \geq \rho_3 \geq 1$, we have:

$$\begin{aligned}
u_i^{(\mathcal{C},\mathcal{D})}(\boldsymbol{a}) &= \rho_1 + \frac{\rho_3}{2} \\
&\leq \rho_1 + \frac{\rho_2}{2} \\
&< \rho_1 + \rho_2 \\
&< \rho_1 + \rho_2 + \frac{\rho_3}{3} = u_i^{(\mathcal{C},\mathcal{C})}(\boldsymbol{a}) \\
u_i^{(\mathcal{D},\mathcal{C})}(\boldsymbol{a}) &= -\rho_1 + \rho_2 + \frac{\rho_3}{2} \\
&< \rho_2 + \frac{\rho_3}{2} \\
&< \rho_1 + \frac{\rho_3}{2} = u_i^{(\mathcal{C},\mathcal{D})}(\boldsymbol{a}) \\
u_i^{(\mathcal{D},\mathcal{D})}(\boldsymbol{a}) &= -\rho_1 + \rho_3 \\
&\leq -\rho_1 + \rho_2 \\
&< -\rho_1 + \rho_2 + \frac{\rho_3}{2} = u_i^{(\mathcal{D},\mathcal{C})}(\boldsymbol{a})
\end{aligned}$$

Consequently, we gain the following payoff inequality which proofs the theorem:

$$\overbrace{u_i^{(\mathcal{C},\mathcal{C})}(\boldsymbol{a}) > u_i^{(\mathcal{C},\mathcal{D})}(\boldsymbol{a})}^{P_i \text{ cooperates}} > \overbrace{u_i^{(\mathcal{D},\mathcal{C})}(\boldsymbol{a}) > u_i^{(\mathcal{D},\mathcal{D})}(\boldsymbol{a})}^{P_i \text{ defects}} \tag{8}$$

The interesting observation is the difference between two consecutive utilities $u_i^{(\mathcal{C},\mathcal{D})}(\boldsymbol{a})$ and $u_i^{(\mathcal{D},\mathcal{C})}(\boldsymbol{a})$. This means, it is better for P_i to cooperate even if he knows he will not learn the secret whereas the other party will learn it. On the other hand, even if P_i learns the secret by deviating at the moment and using the share of the other party, he will gain less utility. This is due to the potential future gain or loss and the significance of the reputation in a society. □

As we mentioned earlier, a more realistic approach by common sense is to consider a suitable multiplicative factor ρ_1 for the estimation of the future loss (or gain). In that case, the enforcement for the cooperation would be even more and the following inequality is going to be hold:

$$u_i^{(\mathcal{C},\mathcal{C})}(\boldsymbol{a}) > u_i^{(\mathcal{C},\mathcal{D})}(\boldsymbol{a}) \overset{\rho_1}{\gg} u_i^{(\mathcal{D},\mathcal{C})}(\boldsymbol{a}) > u_i^{(\mathcal{D},\mathcal{D})}(\boldsymbol{a}) \tag{9}$$

In all the existing solutions, the payoff matrix shown in Table 2 has been considered. This matrix satisfies the properties B and C without defining any impact factors, where $\mathcal{U}^+ > \mathcal{U} > \mathcal{U}^- > \mathcal{U}^{--}$. The payoff matrix associated with socio-rational secret sharing is illustrated in Table 3, satisfying all three preference assumptions.

P_1 \ P_2	$\mathcal{C}$ooperation	$\mathcal{D}$efection
$\mathcal{C}$ooperation	$\mathcal{U}$,$\mathcal{U}$	$\mathcal{U}^{--}$,$\mathcal{U}^{+}$
$\mathcal{D}$efection	$\mathcal{U}^{+}$,$\mathcal{U}^{--}$	$\mathcal{U}^{-}$,$\mathcal{U}^{-}$

Table 2. $(2, 2)$-Rational SS [2]

P_1 \ P_2	$\mathcal{C}$ooperation	$\mathcal{D}$efection
$\mathcal{C}$ooperation	$\mathcal{U}^{+}$,$\mathcal{U}^{+}$	$\mathcal{U}$,$\mathcal{U}^{-}$
$\mathcal{D}$efection	$\mathcal{U}^{-}$,$\mathcal{U}$	$\mathcal{U}^{--}$,$\mathcal{U}^{--}$

Table 3. $(2, 2)$-Socio-Rational SS

First of all, we should stress that our socio-rational game is a non-cooperative game. In fact, the cooperation is self-enforcing due to the reputation and future concerns of rational foresighted players who decide individually. In a cooperative game, this enforcement is provided by a third party and players do not really compete. Second, this payoff matrix does not mean that players never deviate. As an example, consider a scenario that a player is involved in different societies. If he is required to cooperate for secret reconstructions of various schemes at the same time, he will select the one in which he can gain more utility, of course, by considering his reputation.

Theorem 2. *In a socio-rational secret sharing scheme with n parties, $\mathcal{C}$ strictly dominates $\mathcal{D}$ for all players P_i, assuming the preferences of rational foresighted parties. Consequently, the vector $\boldsymbol{a}^{\mathcal{C}} = (a_1^{\mathcal{C}}, \ldots, a_n^{\mathcal{C}})$ (or $\boldsymbol{\sigma}^{\mathcal{C}} = (\sigma_1^{\mathcal{C}}, \ldots, \sigma_n^{\mathcal{C}})$ in the case of the mixed-strategy) is a strict social equilibrium as a unique solution.*

Proof. We first compute the utility of each outcome based on Equation (6) for the least possible threshold $t = 2$ where $n > 2$, i.e., two shares are enough to learn the secret. $\mathcal{C}_i$ (or $\mathcal{D}_i$) means P_i cooperates (or defects), $\mathcal{C}_{-i}$ (or $\mathcal{D}_{-i}$) means, excluding P_i, all the other players cooperate (or defect), and $\mathcal{M}_{-i}$ means, excluding P_i, some players cooperate **and** some of them defect; we have both $\mathcal{C}$ooperation and $\mathcal{D}$efection.

1. $(\mathcal{C}_i, \mathcal{C}_{-i}) : (\tau_i > 0, l_i = 1, \delta = n)$

$$u_i^{(\mathcal{C}_i,\mathcal{C}_{-i})}(\boldsymbol{a}) = \Omega\Big(\rho_1 f_1 + \rho_2 f_2 + \rho_3 f_3(n)\Big)$$

2. $(\mathcal{C}_i, \mathcal{M}_{-i}) : (\tau_i > 0, l_i = 1, \delta = n)$

$$u_i^{(\mathcal{C}_i,\mathcal{M}_{-i})}(\boldsymbol{a}) = \Omega\Big(\rho_1 f_1 + \rho_2 f_2 + \rho_3 f_3(n)\Big)$$

3. $(\mathcal{C}_i, \mathcal{D}_{-i}) : (\tau_i > 0, l_i = 0, \delta = n-1)$

$$u_i^{(\mathcal{C}_i,\mathcal{D}_{-i})}(\boldsymbol{a}) = \Omega\Big(\rho_1 f_1 + \rho_3 f_3(n-1)\Big)$$

4. $(\mathcal{D}_i, \mathcal{C}_{-i}) : (\tau_i < 0, l_i = 1, \delta = n)$

$$u_i^{(\mathcal{D}_i,\mathcal{C}_{-i})}(\boldsymbol{a}) = \Omega\Big(-\rho_1 f_1 + \rho_2 f_2 + \rho_3 f_3(n)\Big)$$

5. $(\mathcal{D}_i, \mathcal{M}_{-i}) : (\tau_i < 0, l_i = 1, \delta \in \{n-1, n\})$

$$u_i^{(\mathcal{D}_i,\mathcal{M}_{-i})}(\boldsymbol{a}) = \Omega\Big(-\rho_1 f_1 + \rho_2 f_2 + \rho_3 f_3(\delta)\Big)$$

6. $(\mathcal{D}_i, \mathcal{D}_{-i}) : (\tau_i < 0, l_i = 0, \delta = 0)$

$$u_i^{(\mathcal{D}_i,\mathcal{D}_{-i})}(\boldsymbol{a}) = \Omega\Big(-\rho_1 f_1 + \rho_3 f_3(0)\Big)$$

As before, we ignore the common term Ω. Since the utilities of items 1 and 2 as well as 4 and 5 are almost equal, we ignore the items 2 and 5. Based on our proposed utility function, we have:

$$\begin{aligned}
u_i^{(\mathcal{C}_i,\mathcal{D}_{-i})}(\boldsymbol{a}) &= \rho_1 f_1 + \rho_3 f_3(n-1) \\
&\leq \rho_1 f_1 + \rho_3 f_2 \\
&\leq \rho_1 f_1 + \rho_2 f_2 \\
&< \rho_1 f_1 + \rho_2 f_2 + \rho_3 f_3(n) = u_i^{(\mathcal{C}_i,\mathcal{C}_{-i})}(\boldsymbol{a}) \\
u_i^{(\mathcal{D}_i,\mathcal{C}_{-i})}(\boldsymbol{a}) &= -\rho_1 f_1 + \rho_2 f_2 + \rho_3 f_3(n) \\
&\leq -\rho_1 f_1 + \rho_2 f_2 + \rho_3 f_2 \\
&\leq -\rho_1 f_1 + \rho_2 f_2 + \rho_2 f_2 \\
&< -\rho_1 f_1 + \rho_1 f_2 + \rho_1 f_2 = \rho_1(2f_2 - f_1) \\
&\leq \rho_1 f_1 \text{ since } f_2 \leq f_1 \\
&< \rho_1 f_1 + \rho_3 f_3(n-1) = u_i^{(\mathcal{C}_i,\mathcal{D}_{-i})}(\boldsymbol{a}) \\
u_i^{(\mathcal{D}_i,\mathcal{D}_{-i})}(\boldsymbol{a}) &= -\rho_1 f_1 + \rho_3 f_3(0) \\
&\leq -\rho_1 f_1 + \rho_3 f_2 \\
&\leq -\rho_1 f_1 + \rho_2 f_2 \\
&< -\rho_1 f_1 + \rho_2 f_2 + \rho_3 f_3(n) = u_i^{(\mathcal{D}_i,\mathcal{C}_{-i})}(\boldsymbol{a})
\end{aligned}$$

Consequently, we gain the following payoff inequality:

$$\overbrace{u_i^{(\mathcal{C}_i,\mathcal{C}_{-i})}(\boldsymbol{a}) > u_i^{(\mathcal{C}_i,\mathcal{D}_{-i})}(\boldsymbol{a})}^{P_i \text{ cooperates}} > \overbrace{u_i^{(\mathcal{D}_i,\mathcal{C}_{-i})}(\boldsymbol{a}) > u_i^{(\mathcal{D}_i,\mathcal{D}_{-i})}(\boldsymbol{a})}^{P_i \text{ defects}} \tag{10}$$

The same result can be gained for any values of t, which proofs the theorem. □

Expected Utility. In this section, we show how each player can compute his expected utility. This is specifically useful if he would like to decide on different requests received from various societies, i.e., $\mathcal{EU}_i^{\mathcal{C}}(\boldsymbol{a}) \overset{?}{>} \mathcal{EU}_i^{\mathcal{C}}(\boldsymbol{a}')$ where $\boldsymbol{a}, \boldsymbol{a}'$ are for different games.

We stress that the utility value represents the relation between actions and their corresponding consequences for a player whereas the *expected utility* of P_i is an estimation of gain or loss when he plays with a player P_j. We therefore compute the expected utility with a linear combination of utility values and probability of P_j's cooperation. Let $\epsilon_j = (\mathcal{T}_j^{\boldsymbol{a}}(p) + 1)/2$ where $\epsilon_j \in [0, 1]$ shows how probably the opponent P_j may cooperate.

$$\mathcal{EU}_i^{\mathcal{C}}(\boldsymbol{a}) = \epsilon_j \, \mathcal{U}^{+} + (1 - \epsilon_j) \, \mathcal{U} \tag{11}$$
$$\mathcal{EU}_i^{\mathcal{D}}(\boldsymbol{a}) = \epsilon_j \, \mathcal{U}^{-} + (1 - \epsilon_j) \, \mathcal{U}^{--} \tag{12}$$

Corollary 1. *In our socio-rational secret sharing,* $\mathcal{EU}_i^{\mathcal{C}}(\boldsymbol{a}) \overset{always}{>} \mathcal{EU}_i^{\mathcal{D}}(\boldsymbol{a})$.

Proof.

$$\begin{aligned}
\mathcal{EU}_i^{\mathcal{C}}(\boldsymbol{a}) &> \mathcal{EU}_i^{\mathcal{D}}(\boldsymbol{a}) \\
\epsilon_j \, \mathcal{U}^{+} + (1 - \epsilon_j) \, \mathcal{U} &> \epsilon_j \, \mathcal{U}^{-} + (1 - \epsilon_j) \, \mathcal{U}^{--} \\
\epsilon_j \, (\mathcal{U}^{+} - \mathcal{U}^{-}) &> (1 - \epsilon_j) \, (\mathcal{U}^{--} - \mathcal{U}) \\
\frac{\mathcal{U}^{+} - \mathcal{U}^{-}}{\mathcal{U}^{--} - \mathcal{U}} &< \frac{1 - \epsilon_j}{\epsilon_j} \quad \text{since } (\mathcal{U}^{--} - \mathcal{U}) < 0
\end{aligned}$$

Since $\epsilon_j \in [0, 1]$ and the LH side is negative, the inequality is always hold. □

Corollary 2. *In our social setting, players have more motivation to cooperate with trustworthy parties or contribute in a society with more reliable participants.*

Proof. Suppose P_i is involved in two games with P_j and P_k who have different reputation values, for instance, let $\epsilon_j > \epsilon_k$. Assume P_i receives the same unit of utility in both games, and let $\boldsymbol{a}_{ij}, \boldsymbol{a}_{ik}$ be the outcomes of two games accordingly. We therefore consider the following inequality:

$$\begin{aligned}
\mathcal{EU}_i^{\mathcal{C}}(\boldsymbol{a}_{ij}) &\overset{?}{>} \mathcal{EU}_i^{\mathcal{C}}(\boldsymbol{a}_{ik}) \\
\epsilon_j \, \mathcal{U}^{+} + (1 - \epsilon_j) \, \mathcal{U} &\overset{?}{>} \epsilon_k \, \mathcal{U}^{+} + (1 - \epsilon_k) \, \mathcal{U} \\
\epsilon_j \, \mathcal{U}^{+} - \epsilon_k \, \mathcal{U}^{+} &\overset{?}{>} (1 - \epsilon_k) \, \mathcal{U} - (1 - \epsilon_j) \, \mathcal{U} \\
(\epsilon_j - \epsilon_k) \, \mathcal{U}^{+} &\overset{?}{>} (\epsilon_j - \epsilon_k) \, \mathcal{U} \quad \text{since } \epsilon_j > \epsilon_k \\
\mathcal{U}^{+} &> \mathcal{U} \quad \text{which is true}
\end{aligned}$$

□

5 Conclusion and Future Direction

This paper provides a multidisciplinary research connecting three major areas of Computer Science in order to propose a novel solution for one of the most fundamental cryptographic primitives.

As we illustrated, the social network with reputation consideration is a strong self-enforcement for players to cooperate, for instance, a player may change his non-cooperative approach after analyzing his reputation, or after estimating his future loss. In our social setting, players can compensate for their past behavior. On the other hand, reputable players can gain more profits as long as they act properly, and newcomers can fairly start their social interactions. Finally, we should stress that having a trust network by considering long-term interactions can be seen as a new direction in game theory itself, specifically, the theoretical models used in social sciences such as economics and political science because elements in those frameworks are more close to human social behavior.

As our future work, we are interested to consider other complicated models. For instance, using *referral chain* in which two players who are interacting for the first time, can gain some information with respect to each other's reputation through other parties or common friends. We also would like to scrutinize the impact of a situation in which a player is involved in *various societies* while he is holding different reputation values associated to each one. Finally, it would be interesting to build a *hybrid model* where both reputation and belief are considered. In that case, by considering all the other parameters of the game, the reputation can be viewed as an estimation of the past behaviors whereas the belief can be considered as an anticipation of the future activities.

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