Comment on ”A new exactly solvable quantum model in N dimensions” [Phys. Lett. A 375(2011)1431]

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Abstract

We pinpoint that the work about ”a new exactly solvable quantum model in N dimensions” by Ballesteros et al. [Phys. Lett. A **375** (2011) 1431] is not a new exactly solvable quantum model since the flaw of the position-dependent mass Hamiltonian proposed by them makes it less valuable in physics.

Keywords: Position-dependent mass; Arbitrary dimension N; Solvable quantum model

In recent work [1], the authors Ballesteros et al. claimed that they have found a new exactly solvable quantum model in N dimensions given by

$$H = -\frac{\hbar^2}{2(1+\lambda r^2)}\nabla^2 + \frac{\omega^2 r^2}{2(1+\lambda r^2)}, \tag{1}$$

where we prefer to use variable r instead of original one q for convenience.

They found that the spectrum of this model is shown to be hydrogen-like (should be harmonic oscillator-like), and their eigenvalues and eigenfunctions are explicitly obtained by deforming appropriately the symmetry properties of the N-dimensional harmonic oscillator. It should be pointed out that such treatment approach is incorrect since the kinetic energy term should be defined as [2]

$$\nabla_N \frac{1}{m(r)}\nabla_N \psi(\mathbf{r}) = \left(\nabla_N \frac{1}{m(r)}\right)\cdot[\nabla_N \psi(\mathbf{r})] + \frac{1}{m(r)}\nabla_N^2 \psi(\mathbf{r}). \tag{2}$$

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For N-dimensional spherical symmetry, we take the wavefunctions $\psi(\mathbf{r})$ as follows [3]:

$$\psi(\mathbf{r}) = r^{-(N-1)/2} R(r) Y^{l}_{l_{N-2,\ldots,l_1}}(\hat{\mathbf{x}}). \tag{3}$$

Substituting this into the position-dependent effective mass Schrödinger equation

$$\nabla_N \left(\frac{1}{m(r)} \nabla_N \psi(\mathbf{r}) \right) + 2[E - V(r)]\psi(\mathbf{r}) = 0, \tag{4}$$

allows us to obtain the following radial position-dependent mass Schrödinger equation in arbitrary dimensions

$$\left\{ \frac{d^2}{dr^2} + \frac{m'(r)}{m(r)} \left(\frac{N-1}{2r} - \frac{d}{dr} \right) - \frac{\eta^2 - 1/4}{r^2} + 2m(r)[E - V(r)] \right\} R(r) = 0, \tag{5}$$

where $m(r) = (1 + \lambda r^2), m'(r) = dm(r)/dr$ and $\eta = |l - 1 + N/2|$. Since the operator ∇_N does not commutate with the position-dependent mass $m(r)$, then this system does not exist exact solutions at all. This can also be proved unsolvable to Eq.(5) if substituting the position-dependent mass $m(r)$ into it.

On the other hand, the choice of the position-dependent mass $m(r)$ has no physical meaning since the mass $m(r)$ goes to infinity when $r \to \infty$. Moreover, it is shown from Eq.(1) that the position-dependent mass $m(r)$ in kinetic term is equal to $(1 + \lambda r^2)$, but it was taken as $1/(1+\lambda r^2)$ for the harmonic oscillator term. Accordingly, the wrong expression of the Hamiltonian in position-dependent mass Schrödinger equation in arbitrary dimensions N, the flaw of the chosen position-dependent mass $m(r)$ as well as its inconsistence between the kinetic term and the harmonic oscillator term make it less valuable in physics.

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References

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