Scaling laws prediction from a solvable model of turbulent thermal convection

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A solvable turbulent model is used to predict both the structure of the boundary layer and the scaling laws in thermal convection. The transport of heat depends on the interplay between the thermal, viscous and integral scales of turbulence, and thus, on both the Prandtl number and the Reynolds numbers. Depending on their values, a wide variety of possible regimes is found, including the classical 2/7 and 1/3 law, and a new $4/13 = 0.308$ law for the Nusselt power law variation with the Rayleigh number.

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Until the advent of high precision numerical and experimental data, the heat transport in turbulent convection was thought to be governed by the classical 1/3 power law linking the Nusselt number and the Rayleigh number. Deviations from this prediction have now been measured at large Rayleigh numbers, revealing a whole zoology of scaling exponents. They range from 0.2 to 0.25 in low Prandtl number, Mercury experiments [1], to 0.29 ± 0.1 in Helium experiments [2–4], while electro-chemical convection gives the classical 1/3 exponent [5]. A number of theoretical models based on dimensional arguments have been developed to explain these new regimes [1, 2, 6], including the possibility that no real scaling prevails [7]. In this letter, predictions obtained using a solvable model of turbulent convection are presented. This model couples large scale mean sheared velocity and temperature fields $U = (U(z), 0, 0)$ and $\Theta(z)$, with small scale random velocity and temperature fields. This kind of large scale geometry is generally accepted as representative of the Boussinesq convection in the boundary layers, due to the shearing effect of the large convective cells. The model is closed by deriving an equation for the random component from the Boussinesq equation using two simplifying assumptions: i) the non-linear interactions of the small scale scales between themselves is modeled via a turbulent viscosity; ii) the small scale generation via the breaking of large scale structures is modeled by a random small scale forcing with prescribed statistics. This results in a linear stochastic equation for the random small scales, which can be analytically solved in the shear flow geometry by a decomposition of the small waves into localized wave-packets. This model was used to obtain analytical predictions of mean velocity and/or temperature profiles in neutral boundary layer or channel flows [8] or in the Planetary Surface Layer (PSL) [9]. Here, we adapt and generalize these results to determine both the structure of the boundary layers, and various scaling laws relating the Nusselt number Nu and various length scales to the Rayleigh number Ra. Some of our findings are then compared with results from high resolution direct numerical simulations (DNS) at Ra between 10^4 and 10^8, and at Pr between 0.02 and 7, which are described in [10–12].

We consider Boussinesq equations, nondimensionalized by the thermal diffusivity and the cell height:

$$\begin{aligned} \partial_t u_i + \partial_j(u_i u_j) &= -\partial_i p + Ra Pr \theta \delta_{i3} + Pr \Delta u_i + f_i^{(u)}, \\ \partial_t \theta + \partial_j(u_j \theta) &= \Delta\theta + f^{(\theta)}. \end{aligned} \quad (1)$$

Here, Pr is the Prandtl number, and $f^{(u)}$ and $f^{(\theta)}$ are small scale random forces which are introduced to model the seeding of small scales by turbulent plumes detaching from the wall and penetrating the outer region. For simplicity, this forcing is taken as spatially homogeneous and delta correlated in time [13]. These assumptions only influence the magnitude of the Reynolds stresses, not their shape [8] . Averaging (1) over the statistics of the forcing and assuming a shear flow geometry, one obtains the standard equations for the x-component of $\mathbf{U} =< \mathbf{u} >= (U(z), 0, 0)$ and for $\Theta(z) =< \theta >$:

$$\begin{aligned} \partial_t U + \partial_z < u'w' > &= -\partial_x P + Pr \partial_z^2 U, \\ \partial_t \Theta + \partial_z < w'\theta' > &= \partial_z^2 \Theta. \end{aligned} \quad (2)$$

Here, the primes denote fluctuating (small scale) quantities and $<>$ the averaging. In the stationary case, we get from (2) that $\partial_x P$ is a constant, independent of z. In the laminar case where $< u'w' >$ and $< w'\theta' >$ are negligible, we thus obtain the well known parabolic profile for the velocity and the linear profile for the temperature. In the turbulent case, the profiles are linear within the thermal or the viscous layer, while outside this layer, they are given by the condition

$$\partial_z < u'w' >= -\partial_x P, \quad \partial_z < w'\theta' >= 0. \quad (3)$$

To close the system, we need $< u'w' >$ and $< w'\theta' >$. For this, we now derive an equation for the fluctuating quantities, by taking into account the scale separation between the mean field and the random field $l/L = \eta \ll 1$. For this, we decompose the velocity field into localized wave-packets via a Gabor transform (GT):

$$\hat{u}(\mathbf{x}, \mathbf{k}, t) = \int g(\eta^* |\mathbf{x} - \mathbf{x}'|) e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \mathbf{u}(\mathbf{x}', t) d\mathbf{x}', \quad (4)$$

where g is a function which decreases rapidly at infinity and $1 \gg \eta^* \gg \eta$. Because of this, the GT of any large scale field is exponentially small in η^*, and the GT of $U_j \partial_j \mathbf{u'}_i$ can be developed into $U_j \partial_j \hat{u'}_i - \partial_j (U_m k_m) \partial_{k_j} \hat{u'}_i$ (see [8] for details). Using these properties, we obtain after GT of (1) the small scale equation:

$$\begin{aligned} D_t \hat{u}_i &= -ik_i \hat{p} - \hat{w} \partial_z U \delta_{i1} + RaPr \hat{\theta} \delta_{i3} - Pr^t \mathbf{k}^2 \hat{u}_i + \hat{f}_i^{(u)} \\ D_t \hat{\theta} &= -\hat{w} \partial_z \Theta - \mathbf{k}^2 \hat{\theta} + \hat{f}^{(\theta)}. \end{aligned} \tag{5}$$

We have dropped primes on fluctuating quantities for convenient notations and introduced the total derivative $D_t = \partial_t + U\partial_x - \partial_z (U k_x) \partial_{k_z}$. In (5), we have furthermore model the GT of the non-linear terms describing local interactions between fluctuations via the introduction of turbulent viscosity and diffusivity, or equivalently, via a turbulent Prandtl number Pr^t. The linear part of (5) is exact and describes the non-local interactions between the mean and the fluctuating part. We have thus obtained a linear, stochastic equation for the fluctuating quantities to close our system.

To solve the closed system (2) and (5), we introduce two further simplification: i) we restrict ourselves to the 2D case. This is justified because numerical work on 2D thermal convection showed that 2D geometry is sufficient to capture the physical mechanism responsible for the Nu scaling with Ra [12]. Analytical work on neutral shear flow has also proved that 2D geometry is sufficient to capture the correct shape of the equilibrium profile [8]. Note that the vortex stretching, which is theoretically absent in 2D geometry, has been implicitly accounted for via the turbulent viscosity. In real 2D flows, this turbulent viscosity can be ignored [8]. ii) we take $Pr^t = 1$. This is because our model introduces a turbulent Prandtl number whose exact value is unknown, which implies that the correct Pr scaling cannot be captured within our model. General Pr effects will be introduced in another manner, via links between length scales. With these approximations, it was shown in [9] that to leading order in the Reynolds number Re, the solutions of (5) satisfy:

$$\begin{aligned} < w'u' > &= \frac{1}{2\partial_z U (a_+ - a_-)^2} \left(\frac{\partial_z U}{k_*^2} \right)^{-(2+4a_+)/3} \\ & \left(4a_-^2 \lambda_1 + \frac{RaPr}{\partial_z U} \lambda_2 + \left(\frac{RaPr}{\partial_z U} \right)^2 \lambda_3 \right), \\ < w'\theta' > &= \frac{1}{2\partial_z U (a_+ - a_-)^2} \left(\frac{\partial_z U}{k_*^2} \right)^{-(2+4a_+)/3} \\ & \left(-8a_+ a_-^2 \frac{\partial_z U}{RaPr} \mu_1 + 4a_+ a_- \mu_2 - 2a_+ \frac{RaPr}{\partial_z U} \mu_3 \right) \end{aligned} \tag{6}$$

Here, k_* is a characteristic horizontal wavenumber, λ_i and μ_i $(i = 1, ..3)$ are some constant which depend on the forcing correlation functions, and $a_\pm = -(1 \pm \sqrt{1 - 4R_i})/2$, where

$$R_i = RaPr \frac{\partial_z \Theta}{(\partial_z U)^2}, \tag{7}$$

is the Richardson number. In developed convective turbulence, this number is large and negative $R_i \ll -1$. So only the leading order contribution in R_i in the expression (6) will be further considered. Combining (3) with the large R_i expansion of (6), we find that the turbulent profiles are given by the conditions:

$$\begin{aligned} < w'u' > &\approx \frac{u_*^2}{\partial_z U} \propto z, \\ < w'\theta' > &\approx \frac{u_*^2 \sqrt{-R_i}}{RaPr} \propto cte = Nu, \end{aligned} \tag{8}$$

where we have introduced $u_*^2 = (\partial_z U / k_*^2)^{2\sqrt{-R_i}/3}$. Looking for simple solutions where $\partial_z U$ and $\partial_z \Theta$ are power laws of z, the only solution is:

$$\partial_z U \sim \frac{1}{z}, \quad \partial_z \Theta \sim \frac{1}{z^2}, \tag{9}$$

resulting in a constant (with z) Richardson number. This solution corresponds to the standard logarithmic velocity profile, and a temperature profile decreasing like $1/z$. Such a profile had been predicted by Malkus [14] using a maximum principle. It was found to be in good agreement with experiments of large Ra convection [15] and DNS at $Ra = 10^8$ and $Pr = 0.7$ [11, 16]. For the velocity profile, the prediction (9) is difficult to check numerically, because the Reynolds number is too low for the boundary layer to be fully developed [16]. Even in the large Raleigh number experiments of [17], the extent of the turbulent boundary layer is too small (a tenth of the cell) to check this prediction. However, in the PSL, the prediction (9) appears compatible with the measurements [9].

It is also possible to find the leading order behavior in Re and R_i of vertical velocities and temperature fluctuations, using the results of [9]. They are:

$$< w'^2 > \approx R_i \frac{u_*^2}{\partial_z U}, \quad < \theta'^2 > \approx R_i \frac{u_*^2 \partial_z U}{(RaPr)^2}. \tag{10}$$

This give a r.m.s. vertical velocity and temperature varying like $z^{1/2}$ and $z^{-1/2}$, like in the free-fall regime. For the temperature fluctuation, the predicted decrease is more rapid than the classical $z^{-1/3}$ prediction. This feature has been already observed in high Ra convection [17]. In the PSL, the r.m.s. velocity and temperature are compatible with (10) [9].

From (8) and (9), we may also derive interesting exact relations. By matching the turbulent profiles with the viscous or diffusive solutions $Pr \partial_z U = u_\tau^2$ and $\partial_z \Theta = Nu$, where u_τ is the friction velocity and Nu the Nusselt number, we get:

$$\partial_z U = u_\tau^2 \frac{\lambda_V}{z Pr}, \quad \partial_z \Theta = Nu \left(\frac{\lambda_T}{z} \right)^2, \tag{11}$$

Here, we have introduced the thermal length scale $\lambda_T = 1/Nu$ and the viscous length scale $\lambda_V = Pr/u_\tau$. To find the prefactor in (8), we use the law of energy dissipation in a boundary layer geometry which gives $\epsilon \sim< u'w' > \partial_z U$. We then take into account the exact relation $\epsilon = RaNu$ and use (8), to obtain $u_*^2 = RaNu$ and:

$$\frac{< u'w' >}{< w'\theta' >} = \frac{RaPr}{\partial_z U\sqrt{-R_i}} = \frac{Ra}{\partial_z U}. \quad (12)$$

The first equality comes from (8), while the second comes from the link between ϵ and $< u'w >$. Condition (12) shows that R_i is independent of Ra. This condition is actually a necessary condition for stability of the turbulent boundary layer [2]. The definition (7) combined with (11) then implies:

$$Ra = u_\tau^2 NuPr. \quad (13)$$

We may also use (11) and the above estimate to define a characteristic vertical velocity $w_c = \sqrt{< w'^2 > /z}$ and temperature fluctuation $\Delta_c = \sqrt{< \theta'^2 > z}$, which scale as:

$$w_c \sim \sqrt{\frac{RaNu}{u_\tau}}, \quad \Delta_c \sim \sqrt{\frac{u_\tau Nu}{Ra}}. \quad (14)$$

These exact relations derived within our nonlocal model will be the basis of the scaling theory which we now detail.

For this, we follow [11] and introduce a third length scale, noted λ_I, representing the location of the peak of the kinetic horizontal energy spectrum. Its expression can again be found within the nonlocal framework by noting that this length scale coincides with the location of the maximum of the r.m.s. horizontal velocity. In the non-local model, this horizontal velocity is passively advected by the large scale horizontal velocity and obeys $U\partial_x u' = \partial_z^2 u'$. The solution to this equation depends on the structure of the velocity boundary layer: if the Reynolds number is too small, the boundary layer is mainly laminar, $U = u_\tau z/\lambda_V$ and simple dimensional argument show that u' is a function of $z/(x\lambda_V u_\tau^{-1})^{1/3}$. For a fixed aspect ratio Γ, this defines a typical vertical scale of r.m.s. velocity variation $\lambda_I = (\Gamma\lambda_V u_\tau^{-1})^{1/3}$. When the Reynolds number exceeds a critical value of the order of $Re_c \sim 10^5$ [18], the boundary layer turbulence is well developed, and most of the transport of u' is provided by the turbulent logarithmic regime $U \sim u_\tau(lnz + B)$. This regime is hard to obtain at Prandtl numbers of the order one or larger and it is likely that only low Prandtl number system (like Mercury experiments) have the ability to reach this critical Reynolds number [1]. In this regime, u' is a function of $z/(xu_\tau^{-1})^{1/2}$, thereby defining a typical length scale $\lambda_I \sim (\Gamma u_\tau^{-1})^{1/2}$ [19]. The two regimes can be lumped into the single formula $\lambda_I \sim \lambda_V(\Gamma u_\tau Pr^{-2})^\psi$, where $\psi = 1/3$ for $Re < Re_c$ and $\psi = 1/2$ for $Re > Re_c$. Using these results, the logic of our argumentation is now to link u_τ to Nu by relating λ_V and/or λ_I to λ_T, and then use the exact relation (13) to obtain Nu versus Ra. For this, we set $\lambda_T = \lambda_V^\alpha \lambda_I^{1-\alpha}$, where $0 \le \alpha \le 1$ is a parameter which enables to single out three remarkable regimes. In the first one, $\alpha = 1$, $\lambda_T = \lambda_V$: the viscous and thermal layer coincide. This situation might be typical of large aspect ratio cells, or in numerical simulations with stress-free boundary conditions [20]. In our model, it corresponds to $Nu \sim Ra^{1/3}Pr^{-1}$, the classical case. In the second regime, $0 < \alpha < 1$ and the length-scale ordering changes to $\lambda_V < \lambda_T < \lambda_I$. This situation seems to be typical for convection at $Pr \sim 1$ [11]. For illustration purpose, it is interesting to single out two special value of α which are relevant in turbulence: the first one is $\alpha = 2/3$. This corresponds to λ_T being the Taylor micro-scale, given by the square root of the mean energy to the mean enstrophy. The second value is $\alpha = 2/5$. The corresponding scale varies like $\lambda_I(\lambda_I Re)^{-3/10}$, where Re is the Reynolds number based on the cell size and on the velocity at the integral scale U_I. It was shown [21] to correspond to the maximum of the Kolmogorov function and represents the location of the middle of the inertial range. At last, the third regime is obtained with $\alpha = 0$, making $\lambda_T = \lambda_I$. This situation is typical of low Prandtl number convection. In our estimate of α, we have followed standard turbulence phenomenology, and assumed that $\lambda_I = \lambda_V(\lambda_I Re)^{3/4}$. Note that for the heat transport, the exact relation in the two regimes ($\psi = 1/3$ or $1/2$) is:

$$Nu \sim Ra^{\frac{1-\psi[1-\alpha]}{3-\psi[1-\alpha]}}\Gamma^{-\frac{2\psi[1-\alpha]}{3-\psi[1-\alpha]}}Pr^{\frac{5\psi[1-\alpha]-3}{3-\psi[1-\alpha]}}. \quad (15)$$

The Prandtl dependence obtained in our model is stronger than in standard models of turbulent convection. It is not necessarily inconsistent with available data. Such a dependence would for example account for most of the difference between the $Pr = 4$ experiment of [26] and the $Pr = 0.8$ experiment of [22]. However, we do not expect our model to fully capture the general Prandtl dependence, because of our approximation on the turbulent Prandtl number. In the sequel, we thus concentrate on the Rayleigh dependence of the physical quantities. The various scaling exponents predicted by the combination of (13) , (14) and the length scale relation is summarized in Table I. For purpose of comparison with the DNS, we have also included the predictions for $Re < Re_c$ at three special values of α, relevant to the low and order unity Prandtl number.

In the low Prandtl regime, our model predicts a transition between a $Nu = Ra^{1/4}$ regime up to $Nu = Ra^{1/5}$ regime at larger Re or Ra. This is in agreement with the experimental findings of [1]. In the regime of $Pr \sim 1$, the prediction depends on the value of α. At $Re < Re_c$, we find $\beta = 4/13$ ($\alpha = 2/3$) and $\beta = 2/7$ ($\alpha = 2/5$). The two values are close to the 0.29 value usually observed in experiments or in simulations. The 2/7 value is here exactly recovered when the thermal length scale matches the middle of the inertial range, which may help give a novel interpretation of the 2/7 law. The value $\beta = 4/13 = 0.3077$ is in remarkable agreement with the experimental value $\beta = 0.309 \pm 0.0043$ obtained in [4] using an Helium experiment with 0.5 aspect ratio and spanning eleven decades of Rayleigh number. The case where

Name	General case	Value for $Re < Re_c$ ($\psi = 1/3$)		
		$\alpha = 0$	$\alpha = 2/3$	$\alpha = 2/5$
Nu	$\frac{1-\psi(1-\alpha)}{3-\psi(1-\alpha)}$	0.2500	0.3077	0.2857
Lw_c/ν	$\frac{1}{2}\frac{3-2\psi(1-\alpha)}{3-\psi(1-\alpha)}$	0.4375	0.4808	0.4643
Δ_c	$-\frac{1}{2}\frac{1}{3-\psi(1-\alpha)}$	−0.1875	−0.1731	−0.1786
u_τ	$\frac{1}{3-\psi(1-\alpha)}$	0.3750	0.3462	0.3571
λ_V	$-\frac{1}{3-\psi(1-\alpha)}$	−0.3750	−0.3462	0.3571
λ_T	$-\frac{1-\psi(1-\alpha)}{3-\psi(1-\alpha)}$	−0.2500	−0.3077	−0.2857
λ_I	$\frac{\psi-1)}{3-\psi(1-\alpha)}$	−0.2500	−0.2308	−0.2381

TABLE I: Summary of exponents as a function of the Rayleigh number as predicted by the present model. Here, $\psi = 1/3$ when $Re < Re_c$ and $\psi = 1/2$ when $Re > Re_c$. $0 \leq \alpha \leq 1$ is a parameter describing how the thermal length-scale compares with the viscous λ_V or integral scale λ_I, thereby describing different Prandtl regimes, like $\alpha = 0$ for low Prandtl, and $\alpha = 2/3$ or $2/5$ for $Pr \sim 1$. w_c and Δ_c are characteristic r.m.s. vertical velocity and temperature.

$Re > Re_c$ gives slightly different values. They are respectively $\beta = 5/17 = 0.294$ ($\alpha = 2/3$) and $\beta = 7/27 = 0.259$ ($\alpha = 2/5$). The first value is so close to the value obtained before the transition, that it would make any transition difficult to detect experimentally. This may explain why no transition was detected in [4], despite the large Rayleigh values attained. The second value is sufficiently lower than the value before transition, so that it should be detectable in a careful experiment. Finally, in the third regime, we find $\beta = 1/3$ at all Reynolds number, in agreement with the $Pr = 1$ DNS of [12]. Our model seems to rule out the $\beta = 1/2$ which has been predicted by Kraichnan [24] at very large Rayleigh number, and which might have been experimentally detected by Chavanne *et al* [25]. If this regime is confirmed, it might then correspond to a situation where the boundary layer grows unstable, thereby unvalidating a starting assumption of our model. For w_c and Δ_c scaling with Ra, very good agreement between the prediction and the experimental value 0.43 and −0.2 [1] is obtained at low Prandtl number. For $Pr \sim 1$, our estimates of $w_c L/\nu$ coincides for $\alpha = 2/3$ with the value 0.485 ± 0.005 measured in Helium [2], while the exponent for Δ_c is slightly larger than the experimental value −0.147.

Another test of our predictions can also be made by comparison with DNS, where some length scales have been measured. The results are shown in Table II. They are in rather good agreement with the theoretical result, except for the case $Pr = 7$ of [11] where both the non-dimensional vertical velocity and the integral scale exponent deviate substantially from the prediction. It would certainly be interesting to investigate further the meaning of this discrepancy. For instance, it might be due to a Rayleigh dependence of the turbulent Prandtl number. Finally, we note that the aspect ratio is also likely to modify the recirculation pattern within a given cell and change the relative behavior between the length scales, so different scaling may appear at different aspect ratio, as observed in [26].

Name	Verzicco and Camussi		Werne (2D)	Kerr and Herring	
	$Pr = 0.02$	$Pr = 0.7$	$Pr = 7$	$Pr = 0.07$	$Pr = 7$
Lw_c/ν			0.54	0.47	0.56
u_τ			0.39		
λ_T	−0.25	−0.29		−0.26	−0.29
λ_I	−0.18	−0.23		−0.26	−0.11

TABLE II: Summary of exponents as a function of the Rayleigh number as measured in various DNS at aspect ratio 1 for the three first cases, and 4 for the last two.

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[1] S. Cioni, S. Ciliberto and J. Sommeria *J. Fluid Mech.* **335** 111 (1997)

[2] B. Castaing, G. Gunaratne, F. Heslot, L. Kadanoff, A. Libchaber,S. Thomae,X-Z. Wu, S. Zaleski and G. Zanetti *J. Fluid Mech.* **204** 1 (1989).

[3] T. Takeshita, T. Segawa, J.A. Glazier and M. Sano *Phys. Rev. Letter* **76** 1465 (1996).

[4] J.J. Niemela, L. Skrbek, K.R. Sreenivasan and R.J. Donnelly *preprint* (1999).

[5] R.J. Goldstein, H.D. Chiang and D.L. See, *J. Fluid Mech.* **213**, 111 (1990)

[6] B. I. Shraiman and E.D. Siggia *Phys. Rev. A.* **42**, 3650 (1990).

[7] S. Grossmann and D. Lohse *J. Fluid Mech.* **407** 27 (2000).

[8] S. Nazarenko *Phys. Lett. A* **264** 444 (2000); S. Nazarenko, N. Kevlahan and B. Dubrulle *Physica D* **139** 158 (2000); B. Dubrulle, J-P. Laval, S. Nazarenko and N. Kevlahan preprint.

[9] B. Dubrulle, J-P. Laval and P. Sullivan preprint available at http://webast.ast.obs-mip.fr/people/bdubru. (1999).

[10] R. Verzicco and R. Camussi *J. Fluid Mech.* **383** 55 (1999)

[11] R.M. Kerr, *J. Fluid Mech.* **310** 139 (1996); R. Kerr and J. Herring, *submitted to J. Fluid Mech.* 1999.

[12] J. Werne, *Phys. Rev. E* **48** 1020-1035 (1993).

[13] Similar results would be obtained using a shortly correlated forcing.

[14] W.V.R. Malkus *Proc. Roy. Soc. London., A* **225**, 185-195. (1954); W.V.R. Malkus *Proc. Roy. Soc. London., A* **225**, 195-212. (1954)

[15] A.A. Townsend, *J. Fluid Mech.* **5** 209-241 (1959).

[16] R. Kerr, private communication (1999).

[17] A. Tilgner, A. Belmonte and A. Libchaber *Phys. Rev E* **47**, R2253 (1993); A. Belmonte, A. Tilgner and A. Libchaber, *Phys. Rev E* **50** 269 (1994).

[18] H. Schlichting *Boundary layer theory.* Mc Graw Hill.

(1968)
[19] A similar line of argumentation was done by [1] for λ_T, using the transport of Θ instead of u'.
[20] J. Werne, private communication; K. Julien, S. Legg, J. Mc Williams and J. Werne, *J. Fluid Mech.* **322** 243 (1996).
[21] F. Moisy, P. Tabeling and H. Willaime, *Phys. Rev. Letter* **82** 3994 (1999).
[22] S. Ashkenazi and V. Steinberg *Phys. Rev. Letter* **83**, 3641 (1999).
[23] X. Xu, K.M.S. Bajaj and G. Ahlers (2000), preprint. Such a dependence would for example account for most of the difference between the $Pr = 4$ experiment of [26] and the $Pr = 0.8$ experiment of [22].
[24] R. Kraichnan *Phys. Fluids* **5** 1374 (1962)
[25] X. Chavanne, F. Chilla, B. Castaing, B. Hébral, B. Chabaud and J. Chaussy, *Phys. Rev. Letters* **79**, 3648 (1997).
[26] X. Xu, K.M.S. Bajaj and G. Ahlers (2000), preprint.