Incoherently Coupled Grey Photovoltaic Soliton Families due to Two-photon Photorefractive Effect

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Abstract：To study the grey photovoltaic soliton families in two-photon photovoltaic photorefractive materials under open-circuit conditions，the dynamical evolution equation and the numerical solution of the grey photovoltaic soliton families were established. The existence curves of the grey photovoltaic soliton families were simulated by the numerical method. The results show that the soliton families are derived under the assumption that the multiple carrier beams share the same polarization，wavelength，and are mutually incoherent. Such grey photovoltaic soliton families reduce to grey-grey photovoltaic soliton pairs when they contain only two components. Furthermore，such grey soliton families will reduce to dark soliton families or soliton pairs when the greyness of soltion takes zero. Relevant examples were presented where the photovoltaic photorefractive material was $LiNbO_3$.

Key words：Nonlinear optics；Two-photon photorefractive effect；Grey photovoltaic soliton families

0　Introduction

Photorefractive spatial solitons (PRSS) were first observed experimentally by Duree et al.[1] just a year after their theoretical prediction by Segev et al.[2] in 1992 and have been the subject of active research both theoretically and experimentally since then[3-17]. At present，several different types of PRSS have been investigated，such as the quasi-steady-state solitons[1-2]，the screening solitons[3-4]，the photovoltaic (PV) solitons[5-6] and the screening photovoltaic (SP) solitons[7-8]. All of above mentioned solitons result from the single-photon PR effect. Very recently，Castro-Camus et al. provided a new concise model of the two-photon PR effect [9]. Later，the screening solitons [10]，PV solitons[11] and SP solitons[12] due to two-photon PR effect have been predicted. On the other hand，incoherently coupled bright-bright，dark-dark，bright–dark and grey-grey soliton pairs have been predicted for screening solitons or PV solitons[13-16] that result from the two-photon PR effect. In this paper，we will show that the incoherently coupled grey PV soliton families can also be supported in two-photon PV-PR materials. In the steady state and under open-circuit conditions，the grey PV solitons families' solutions are obtained. Moreover，such soliton families will deduce to dark soliton families when the greyness of soliton takes zero. Relevant examples are presented where the photovoltaic photorefractive crystal is $LiNbO_3$ to illustrate our results.

1　Theoretical model

To study the incoherently coupled grey PV soliton families in two-photon PV-PR materials under open-circuit conditions，we consider N optical beams that propagate collinearly in a two-photon PV-PR material along the z axis and are permitted to diffract only along the x direction. The incident beams have the same polarization，wavelength，and are mutually incoherent. The PV-PR material is taken here to be $LiNbO_3$，with its optical c axis oriented along the x axis. Moreover，let us assume that the polarizations of the incident optical beams are all parallel to c axis and the crystal is illuminated by the gating beam. As usual，the optical fields are expressed in terms of slowly varying envelopes φ_j，i. e. ，$\boldsymbol{E}_j = \hat{x} \varphi_j(x,z)$ exp (ikz)，$j=1,2,\cdots,N$，where $k = k_0 n_e = (2\pi/\lambda_0) n_e$，$\lambda_0$ is the free space wavelength，and n_e is the unperturbed extraordinary index of refraction.

Under these conditions，the N optical beams satisfy the following envelope evolution equations[17]

$$\mathrm{i}\frac{\partial \varphi_j}{\partial z}+\frac{1}{2k}\frac{\partial^2 \varphi_j}{\partial x^2}-\frac{k_0 n_e r_{33} E_{sc}}{2}\varphi_j=0$$
$$(j=1,2,\cdots,N) \qquad (1)$$

where r_{33} is the electro-optic coefficient，and E_{sc} is the induced space-charge field. In the case that the diffusion effect is neglected，the space-charge field can be approximately given by[11]

$$E_{sc}=-E_p\frac{s_2 I_2(I_2+I_{2d}+\gamma_1 N_A/s_2)}{(s_1 I_1+\beta_1)(I_2+I_{2d})} \qquad (2)$$

where $E_p=\kappa\gamma N_A/e\mu$ is the PV field，κ is the PV constant，N_A is the acceptor or trap density，μ and e are the electron mobility and the charge，respectively；γ is the recombination factor of the conduction band (CB) to valence band (VB) transition，γ_1 is the recombination factor of intermediate allowed level (IL) to VB transition；$I_{2d}=\beta_2/s_2$ is the so-called dark irradiance，β_1 and β_2 are the thermoionization probability constants for the transitions of VB-IL and IL-CB；s_1 and s_2 are photoexcitation crosses；I_1 is the intensity of the gating beam，which is kept constant. I_2 is the total intensity of the N optical beams and can be expressed

$$I_2=(n_e/2\eta_0)\sum_{l=1}^{N}|\varphi_l|^2 \qquad (3)$$

By substituting Eq. (2) into Eq. (1)，and after appropriate normalization，we find that the normalized envelopes U_1，U_2，$\cdots$，U_N of the N optical beams satisfy the following dynamical evolution equations

$$\mathrm{i}\frac{\partial U_j}{\partial \xi}+\frac{1}{2}\frac{\partial^2 U_j}{\partial s^2}+$$
$$\alpha\eta\frac{\sum\limits_{l=1}^{N}|U_l|^2(1+\sigma+\sum\limits_{l=1}^{N}|U_l|^2)}{1+\sum\limits_{l=1}^{N}|U_l|^2}U_j=0 \qquad (4)$$

where $U_j=(2\eta_0 I_{2d}/n_e)^{-1/2}\varphi_j$，$\eta_0=(\mu_0/\varepsilon_0)^{1/2}$，$\alpha=(k_0 x_0)^2(n_e r_{33}/2)E_p$，$\sigma=\gamma_1 N_A/\beta_2$，$\eta=\dfrac{\beta_2}{s_1 I_1+\beta_1}$. ξ and s are dimensionless coordinates defined as $\xi=z/(kx_0^2)$，$s=x/x_0$ respectively，x_0 is an arbitrary spatial scale.

2 Solutions of grey PV soliton families

Take the similar methods to Refs. [4，17]，the solution of Eq. (4) can be expressed

$$U_j(s,\xi)=\rho^{1/2}c_j y(s)\exp\left[\mathrm{i}\left(\nu\xi+J\int\frac{\mathrm{d}s}{y^2(s)}\right)\right] \quad (5)$$

where U_j is the normalized envelope，J is an appropriate real constant to be determined，$y(s)$ is a normalized real function bounded between $|y(s)|\leqslant$

1 and satisfy the following conditions：$y(s\rightarrow \pm\infty)=1$，$y^2(0)=m$，$(0<m<1)$，$m$ is the greyness parameters，$\dot{y}(0)=0$，$y^{(n)}(\infty)=0(n\geqslant 1)$. c_j^2 is the ratio of the intensity of jth soliton component to the total intensity of the soliton families，i. e. c_j^2 satisfies $c_j^2=I_j/\sum\limits_{l=1}^{N}I_l$ and $\sum\limits_{l=1}^{N}c_l^2=1$. Substitution Eq. (5) into (4) leads to

$$\frac{\mathrm{d}^2 y}{\mathrm{d}s^2}-2\nu y-\frac{J^2}{y^3}+2\alpha\eta\sigma y+2\alpha\eta\rho y^3-\frac{2\alpha\eta\sigma y}{1+\rho y^2}=0 \quad (6)$$

According to the boundary conditions of $y(s\rightarrow\infty)$，we obtain

$$J^2=-2\nu+2\alpha\eta\rho\frac{1+\rho+\sigma}{1+\rho} \qquad (7)$$

Integrating Eq. (6) obtain

$$\left(\frac{\mathrm{d}y}{\mathrm{d}s}\right)^2=2(\nu-\alpha\eta\sigma)(y^2-1)+J^2\left(1-\frac{1}{y^2}\right)+$$
$$\frac{2\alpha\eta\sigma}{\rho}\ln\left(\frac{1+\rho y^2}{1+\rho}\right)-\alpha\eta\rho(y^4-1) \qquad (8)$$

According to the boundary conditions of $y(s\rightarrow 0)$，we obtain

$$\nu=\alpha\eta\left[\rho+\sigma+\frac{\rho m}{2}-\frac{\sigma}{(\rho+1)(1-m)}-\right.$$
$$\left.\frac{\sigma m}{\rho(1-m)^2}\ln\left(\frac{1+\rho m}{1+\rho}\right)\right] \qquad (9)$$

From Eqs. (7) and (9)，we have

$$J^2=-2\alpha\eta m\sigma\left[\frac{\rho}{2\sigma}-\frac{1}{(\rho+1)(1-m)}-\right.$$
$$\left.\frac{1}{\rho(1-m)^2}\ln\left(\frac{1+\rho m}{1+\rho}\right)\right] \qquad (10)$$

We can prove that the quantity in the square brackets of the equality (10) is positive，so that we can deduce that $\alpha<0$，which indicates that the grey PV soliton families can be formed only when the PV field is in the opposite direction with respect to the optical c axis of the materials.

Further integration of Eq. (8) leads to

$$s=\pm\int_{\sqrt{m}}^{y}\left\{-2\alpha\eta\left[\left(\rho+w-\frac{\sigma}{1+\rho}\right)(1-y^2)+\frac{\rho}{2}(y^4-1)-\right.\right.$$
$$\left.\left.w\left(\frac{1}{y^2}-1\right)-\frac{\sigma}{\rho}\ln\left(\frac{1+\rho y^2}{1+\rho}\right)\right]\right\}^{-1/2}\mathrm{d}y \quad (11)$$

where

$$w=m\left[\frac{\rho}{2}-\frac{\sigma}{(1+\rho)(1-m)}-\right.$$
$$\left.\frac{\sigma}{\rho(1-m)^2}\ln\left(\frac{1+\rho m}{1+\rho}\right)\right] \qquad (12)$$

From Eq. (11) we can obtain the normalized grey soliton profile by numerical integration. In this case，the soliton family components can then be simply obtained through Eq. (5).

3 Numerical simulations

To illustrate our results，we consider the LiNbO₃ crystal with following parameters[11]：

$r_{33} = 30 \times 10^{-12}$ mV^{-1}，$E_p = -4 \times 10^6$ Vm^{-1}，$n_e = 2.2$，$\lambda_0 = 0.5$ μm，$x_0 = 10$ μm. From these parameters，we can calculate $\alpha = -22.2$. Moreover，we take $\eta = 1.5 \times 10^{-4}$，$\sigma = 10^4$，$\rho = 10$，$m = 0.5$. Fig. 1 depicts the intensity profiles of the grey PV soliton families with five components ($N=5$). It is note that we can obtain the grey-grey soliton pairs at $N=2$(Fig. 2).
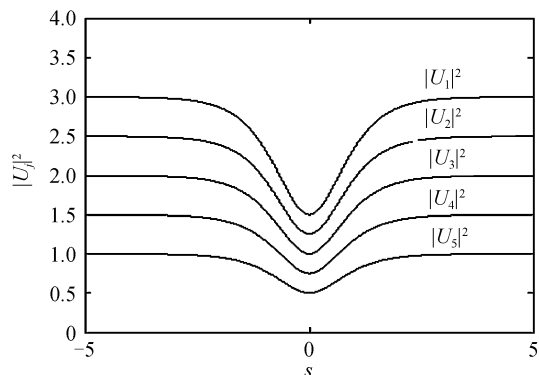

Fig. 1　Intensity profiles of grey PV soliton families with five components ($c_1^2 = 0.30$，$c_2^2 = 0.25$，$c_3^2 = 0.20$，$c_4^2 = 0.15$，$c_5^2 = 0.1$)
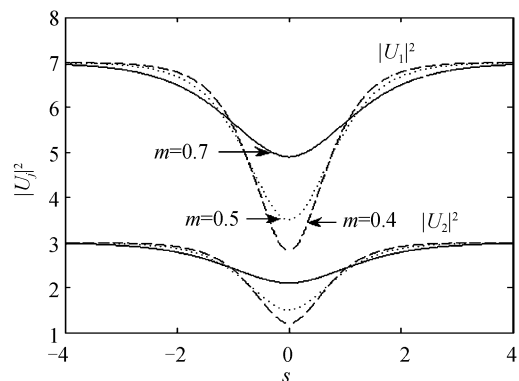

Fig. 2　Intensity profiles of grey-grey PV soliton pairs at $N=2$($c_1^2 = 0.70$，$c_2^2 = 0.3$)

Moreover，when the greyness of soliton takes zero，i. e.，$m=0$，the incoherently coupled grey PV soliton families degenerate to dark PV soliton families (Fig. 3) or soliton pairs (Fig. 4) automatically.
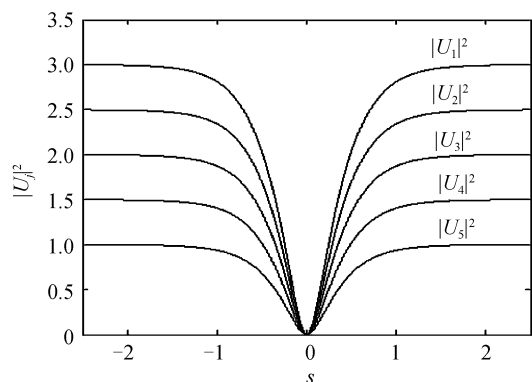

Fig. 3　Intensity profiles of dark PV soliton families with five components ($c_1^2 = 0.30$，$c_2^2 = 0.25$，$c_3^2 = 0.20$，$c_4^2 = 0.15$，$c_5^2 = 0.1$)
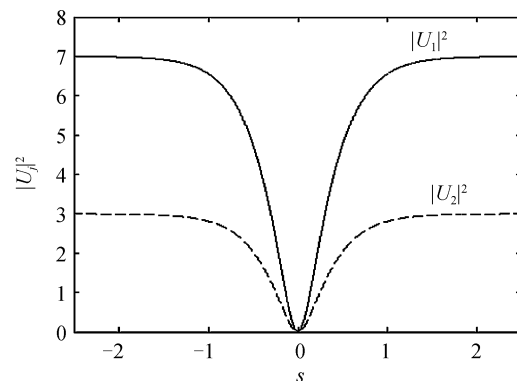

Fig. 4　Intensity profiles of dark-dark PV soliton pairs at $N=2$($c_1^2 = 0.70$，$c_2^2 = 0.3$)

4　Conclusion

We have provided the incoherently coupled grey PV soliton families that can exist in two-photon PV-PR materials in the steady state and under open-circuit conditions. Such soliton families can be established provided that the incident beams have the same polarization，wavelength，and are mutually incoherent. The grey PV soliton families reduce to grey-grey PV soliton pairs when they contain only two components. Moreover，the grey PV soliton families also reduce to dark PV soliton families or soliton pairs when the greyness of soliton takes zero.

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基于双光子光折变效应的非相干耦合灰光伏孤子族

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摘　要：为了研究开路条件下双光子光伏光折变材料中的灰光伏孤子族，建立了光伏孤子族的动态演化方程，给出了非相干耦合灰光伏孤子族的数值解. 采用数值模拟的方法，求解孤子族的数值表达式. 结果表明：这种孤子族是由多束偏振方向和波长都相同的互不相干光束耦合形成的. 当孤子族只有两个分量时，灰光伏孤子族可以退化成灰-灰光伏孤子对；当孤子的灰度参量取零时，灰光伏孤子族退化成暗光伏孤子族，灰-灰光伏孤子对退化成暗-暗光伏孤子对. 文中采用的光伏光折变材料是 $LiNbO_3$.

关键词：非线性光学；双光子光折变效应；灰光伏孤子族