Subtlety in the Use of Maxwell's Equation and a New Electromagnetic Wave in Electron Plasmas

H. Saleem

National Centre for Physics, Quaid-i-Azam University Campus, Islamabad, Pakistan

Abstract

The ambiguity involved in the use of Maxwell's equation particularly in electron plasmas is discussed. It is pointed out that in the slow time scale perturbations the displacement current is ignored but it does not imply that the electron density fluctuations vanish. The contradictions in the assumptions and approximations used in the literature on this subject are discussed. A new low frequency electromagnetic wave is described which is a normal mode of non-uniform magnetized electron plasmas. This wave can couple with plasma hybrid oscillations if ion dynamics is taken into account. It is stressed that the electron magnetohydrodynamics (EMHD) model seems to be simple but in fact its use is subtle and its scope is very limited.

PACS Numbers: 52.27.-h; 52.35.-g; 52.35.Hr; 52.35.Lv

It is well-known that the Maxwell's equation reduces to Ampere's law when displacement current is neglected in the slow time scale phenomena. The divergence of current vanishes and hence several researchers conclude that the density fluctuations in a charged particle system disappears. In electron-ion plasma the quasi-neutrality is used as for example in the case of Alfven waves but it is justified. On the other hand, if ions are considered to be stationary, then the electron density fluctuations do not appear due to Ampere's law in previous investigations both in unmagnetized [1 - 6] and magnetized [7 -9] plasmas and here it is incorrect. The forms of transverse waves in cartesian geometry have serious flaws.

A great deal of literature exists on the topic of magnetic field generation on laser-plasma [1 -6] and cosmological [10] scales assuming the system to be unmagnetized initially. On the other hand, the efforts have also been made to find out some mechanism for the generation of magnetic fluctuations parallel to external magnetic field which can be very important in plasma opening switches (PoS) [7, 8]. Thermomagnetic instability in laser plasmas has also been investigated assuming electron density to be fixed in magnetized inhomogeneous plasma [9].

It is a misconception that the zero displacement current means the zero density fluctuation in electron plasmas. Based on this conclusion drawn from Ampere's law several research papers and review articles have appeared in journals and books which need to be corrected.

For the study of magnetic field generation and plasma switches a simpler single fluid model called electron magnetohydrodynamics (EMHD) was presented [11]. In this model the electron inertia term in equation of motion is neglected and Ampere's law is used assuming the time scale $|\partial_t| << \omega_{pe}, ck$. Moreover, the ions are assumed to be static in the limit $\omega_{pi} << |\partial_t|$ where $\omega_{pj} = (4\pi n_0 e^2/m_j)^{\frac{1}{2}}$ is the plasma oscillation frequency of jth species, k is wave number and c is velocity of light. In magnetized plasmas, EMHD is supposed to be valid for $\Omega_i << |\partial_t| << \Omega_e$ where $\Omega_j = eB_0/m_j c$ is the gyrofrequency of the jth species.

The EMHD model contains several discrepancies and contradictions. But it has been continuously used for the last several decades. The case of magnetized plasmas is more important because of its laboratory applications. The previous concept of negligible density fluctuations associated with low frequency perturbation in the electron plasma is still being followed [12]. In some works the nonlinear whistler wave in pure electron plasmas have been studied. Since these waves are basically pure transverse in nature, therefore, the electron density perturbations in nonlinear stage may be neglected [13], but this point also needs to be checked carefully.

Let us choose the wave geometry the same as was considered in the so called magnetic drift wave (MDW) in Refs. [7, 8]. The constant external magnetic field is $\hat{\mathbf{B}}_0 = B_0\hat{\mathbf{z}}$, the density gradient is $\nabla n_{j0} = -\hat{\mathbf{x}}\frac{dn_0}{dx}$ and perturbation is proportional to $e^{i(k_y y - \omega t)}$ while the subscript naught (0) denotes equilibrium quantities. The only difference is that we assume the density gradient along negative x-axis in resemblance with the well-known drift wave case while in the derivation of the dispersion relation for MDW it was assumed along positive x-axis.

Now we analyse the physical model in detail based on EMHD equations which contains fundamental errors. In the limit $\Omega_i << \omega$ the ions are assumed to be stationary and for $\omega << \omega_{pe}, ck, \Omega_e$, the displacement current is neglected in the Maxwell's equation,

$$\nabla \times \mathbf{B} = e\frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\partial_t \mathbf{E} \tag{1}$$

which then becomes Ampere's law,

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} \tag{2}$$

In the case of pure electron plasma we have $\mathbf{J}_1 = -en_{e0}\mathbf{v}_{e1}$ where the subscript one (1) denotes linearly perturbed quantities. Since Ampere's law implies $\nabla.\mathbf{J}_1 = 0$, therefore in the derivation of dispersion relation of MDW it is deduced that $n_{e1} = 0$. Using the limit $|\partial_t| << \Omega_e$, electron acceleration term is also neglected in the equation of motion which then becomes very simple as,

$$\mathbf{E}_1 = \frac{c}{B_0}\mathbf{v}_{e1} \times \hat{\mathbf{z}} \tag{3}$$

The magnetic fluctuation is assumed to be along z-axis and hence the Faraday's law

$$\nabla \times \mathbf{E}_1 = -\frac{1}{c}\partial_t \mathbf{B}_1 \tag{4}$$

requires $\mathbf{E}_1 = E_1\hat{\mathbf{x}}$ which implies $\nabla.\mathbf{E}_1 = 0$ and therefore the wave seems to be pure transverse. Equation (2) gives,

$$\nabla.\mathbf{v}_{e1} = \frac{c}{4\pi e n_0}\frac{\nabla n_0}{n_0}.(\nabla \times \mathbf{B}_1) \tag{5}$$

and hence $\nabla.\mathbf{v}_{e1} \neq 0$.

The final dispersion relation under local approximation turns out to be [8],

$$\omega = \lambda_e^2 k_y^2 \left(\frac{\kappa_n}{k_y}\Omega_e\right) = k_y v_A \left(\frac{c}{\omega_{pi} L_n}\right) \tag{6}$$

where $v_A = B_0/\sqrt{4\pi n_0 m_e}$ is the electron Alfven speed, $\omega_{pi} = (4\pi n_0 e^2/m_i)^{\frac{1}{2}}$ is the ion plasma oscillation, $L_n = 1/\kappa_n$, $\kappa_n = \left|\frac{1}{n_0}\frac{dn_0}{dx}\right|$ and $\lambda_e = c/\omega_{pe}$.

A very trivial but crucial point has been overlooked in the above assumptions and approximations. The similar treatment in case of unmagnetized plasmas has also been followed by Jones [4] and several others. He also derived a so called new low frequency transverse electromagnetic wave in unmagnetized electron plasmas using $\nabla.\mathbf{E}_1 = 0$ and $\nabla.\mathbf{v}_{e1} \neq 0$. Later on, several linear and nonlinear investigation of this mode within the local approximation were performed.

Most of these authors have used the electron magnetohydrodynamics (EMHD) set of equations [11]. The EMHD model was presented to reduce the time and spacial scales of two component electron-ion plasma to a single fluid which could explain some important phenomena. The assumptions and approximations, which seem to be very reasonable apparently, are in fact self-contradictory and erroneous. Let us analyse the above simple linear model and look into the physical picture in some detail. It will help us in finding out the interesting and important clear results which will be very useful for future studies and applications. Equation (5) implies $\nabla.\mathbf{v}_{e1} \neq 0$ and Eq. (2) requires,

$$\nabla n_0.\mathbf{v}_{e1} + \nabla.\mathbf{v}_{e1} = 0 \tag{7}$$

Thus the electron velocity has two non-zero components v_{ex1} and v_{ey1}. If it is so, then Eq. (3) yields $E_{1y} \neq 0$ and hence $\nabla.\mathbf{E}_1 \neq 0$ which is a contradiction to the initial assumption that the MDW is pure transverse. Two important points need attention

1. Ampere's Law does not necessarily imply that electron density fluctuations are zero.

2. Since $\kappa_n << k_y$ under local approximation, therefore ω can be near lower hybrid oscillations $(\Omega_e \Omega_i)^{\frac{1}{2}}$. Thus the ion dynamics cannot be ignored particularly in the case of hydrogen plasma.

First we present a model for pure electron plasma which is more useful for heavier ion plasmas where the approximation $\omega_{pi}, \Omega_i << \omega$ seems to be more logical and ions can be assumed to be static. The experiments have been performed to produce pure pair-ion fullerene plasmas [14, 15, 16] in Japan. However, a criterion for the pure pair-ion plasma has been defined [17] and it shows that the fullerene plasma produced in experiments was not pure pair-ion plasma. On the other hand, it has been learnt that there are plans to study electron shear flow effects in barium plasma in USA [18]. Since we are in the lower hybrid range of frequencies $\Omega_i << \omega << \Omega_e$, therefore we retain the electron inertia term in equation of motion which for $\mathbf{E} = (E_{1x}, E_{1y}, 0)$ yields,

$$v_{e1x} = \frac{e}{m_e \Omega_e^2}\{\iota\omega E_{1x} + \Omega_e E_{1y}\} \tag{8}$$

$$v_{e1y} = \frac{e}{m_e \Omega_e^2}\{\iota\omega E_{1y} - \Omega_e E_{1x}\} \tag{9}$$

Then Poisson equation becomes,

$$\left\{\left(1 + \frac{\Omega_e^2}{\omega_{pe}^2}\right)\omega + \Omega_e \frac{\kappa_n}{k_y}\right\}\iota k_y = \Omega_e E_{1x} \tag{10}$$

and Ampere's law yields,

$$E_{1y} = (\lambda_e^2 k_y^2)\frac{\Omega_e}{\omega}\iota E_{1x} \tag{11}$$

Then Eqs. (10) and (11) give a new low (or hybrid) frequency electromagnetic wave in non-uniform electron plasmas as,

$$\omega = -\frac{\lambda_e^2 k_y^2}{\left\{1 + \lambda_e^2 k_y^2 (1 + \frac{\Omega_e^2}{\omega_{pe}^2})\right\}}\left(\frac{\kappa_n}{k_y}\Omega_e\right) \tag{12}$$

The term $\mu = \lambda_e^2 k_y^2 \left(1 + \frac{\Omega_e^2}{\omega_{pe}^2}\right)$ is crucial. The factor $\lambda_e^2 k_y^2$ appears due to electron inertia term in equation of motion and $\lambda_e^2 k_y^2 \left(\frac{\Omega_e^2}{\omega_{pe}^2}\right)$ appears due to density fluctuations (or compressibility) in Poisson equation. In heavier ion plasmas Eq. (12) can be very important and even in hydrogen plasma it can be valid if $\kappa_n/k_y \sim (10^{-1})$, $\lambda_e^2 k_e^2 \sim (1)$ and $\Omega^2/\omega_{pe}^2 \lesssim 1$ so that $\Omega_i << \omega$ remains valid. However, in hydrogen plasmas, the frequency of this mode will be closer to $(\Omega_e \Omega_i)^{\frac{1}{2}}$ and therefore it is preferable to include ion dynamics. Ion momentum equation gives,

$$v_{i1x} = \frac{e}{m_i} \frac{1}{(\omega^2 - \Omega_i^2)} (\iota \omega E_{1x} - \frac{e}{m_i} E_{1y}) \tag{13}$$

$$v_{i1y} = \frac{e}{m_i} \frac{1}{(\omega^2 - \Omega_i^2)} (\iota \omega E_{1y} - \Omega_i E_{1x}) \tag{14}$$

Then the Poisson equation becomes,

$$\left[\left(1 + \frac{\omega_{pe}^2}{\Omega_e^2}\right)\omega^3 + \left(\frac{\omega_{pe}^2}{\Omega_e}\frac{\kappa_n}{k_y}\right)\omega^2 - \omega_{pi}^2 \omega\right] \iota k_y E_{1y}$$

$$= \left[\omega_{pi}^2(\Omega_i k_y - \omega \kappa_n) + \frac{\omega_{pe}^2}{\Omega_e^2}(\Omega_e k_y)\omega^2\right] E_{1x} \tag{15}$$

In the electrostatic case $E_{1x} = 0$ it reduces to,

$$\omega^2 + \frac{\kappa_n}{k_y}\Omega_e - \Omega_i \Omega_e = 0 \tag{16}$$

In the electromagnetic case we obtain,

$$\left\{1 + \lambda_e^2 k_y^2 \left(1 + \frac{\Omega_e^2}{\omega_{pe}^2}\right)\right\}\omega^2 + \lambda_e^2 k_y^2 \left(\frac{\kappa_n}{k_y}\Omega_e\right)\omega - \lambda_e^2 k_y^2 (\Omega_e \Omega_i) = 0 \tag{17}$$

The electromagnetic wave described in Eq. (12) couples with lower hybrid oscillations in Eq. (17) due to ions contribution in the limit $\omega^2 \simeq \Omega_e \Omega_i$, $\omega \kappa_n \simeq \Omega_i k_y$ and $\lambda_e^2 k_y^2 \sim (1)$.

The new electromagnetic wave described by dispersion relation (12) will have lot of applications in plasma dynamics for example in plasma switches and in plasma transport in tokamaks. It is partially longitudinal and partially transverse.

To summarize, the ambiguity in the case of Ampere's law created by a great deal of literature on EMHD models presented for the generation of magnetic fields has been clarified. It has been pointed out that when the divergence of transverse current is zero, it does not mean that longitudinal current is zero. Therefore, the Ampere's law does not necessarily imply that there are no density perturbations in the charged particle system. Coupling of Ampere's law with Poisson equation gives a new partially longitudinal and partially transverse wave which exists in non-uniform magnetized electron plasmas. This mode has been overlooked so far in the plasmas, and in our point of view it plays important role in plasma dynamics.

References

[1] J. A. Stamper, K. Papadapoulos, R. N. Sudan, S. O. Dean, E. A. McLean, and J. M. Dawson, Phys. Rev. Lett. **26**, 1012 (1971).

[2] L. A. Bol'shov, Yu. A. Dreizin, and A. M. Dykhne, Pis'ma Zh. Edsp. Teor. Fiz. **19**, 288 (1974) [JETP Lett. **19**], 168 (1974).

[3] K. A. Brueckner and S. Jorna, Rev. Mod. Phys. **46**, 325 (1974).

[4] R. D. Jones, Phys. Rev. Lett. **51**, 1269 (1983).

[5] J. A. Stamper, E. A. McLean and B. H. Ripin, Phys. Rev. Lett. **40**, 1177 (1978).

[6] A. Raven, O. Willi, and R. T. Rumsby, Phys. Rev. Lett. **41**, 554 (1978).

[7] J. D. Huba, Phys. Fluids B3(12), 3217 (1991).

[8] J. D. Huba, J. M. Grossmann, and P. F. Ottinger, Phys. Plasmas **1** (10), 3444 (1994).

[9] A. Frushtman and H. R. Strauss, Phys. Fluids **B4** (**6**), 1397 (1992).

[10] A. Lazarian, Astron. Astrophys. **264**, 326 (1992).

[11] A. A. Kingssep, K. V. Chukbar, and V. V Yan'Kov, in Reviews of Plasma Physics, edited by B. B. Kadomtsev (Consultant Bureau, New York 1990), Vol. **16**, p. 243.

[12] B. Eliasson, P. K. Shukla V. P. Pavlenko, Phys. Plasmas **16**, 042306 (2009).

[13] B. Eliasson and P.K. Shukla, Phys. Rev. Lett. 99, 205005 (2007).

[14] W. Oohara and R. Hatakeyama, Phys. Rev. Lett. **91**, 205005 (2003).

[15] W. Oohara, D. Data and R. Hatakeyama, Phys. Rev. Lett. **95**, 175003 (2005).

[16] W. Oohara, Y. Kuwabara, and R. Hatakeyama, Phys. Rev. **E 75**, 056403 (2007).

[17] H. Saleem, Phys. Plasmas **14**, 014505 (2007).

[18] M. Koepke, private communications (2006). Laboaroty experimets on the parallel electron velocity shear could potentially be done with the WVU Q Machine at West Virginia University USA.