Ultra High Energy Particles

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Abstract

We revisit considerations of temporal order in relativistic effects, taking into account Heisenberg's Uncertainty Principle. We then use a formulation of relativistic Quantum Mechanical equations given by Feshbach and Villars to exhibit novel particle antiparticle effects.

1 Introduction

In the context of the collision energies of a few TeV being attained at the LHC in CERN, Geneva we consider some ultra relativistic effects, particularly for the Klein-Gordon (KG) and Dirac equations. Following Weinberg [1] let us suppose that in one reference frame S an event at x_2 is observed to occur later than one at x_1, that is, $x_2^0 > x_1^0$ with usual notation. A second observer S' moving with relative velocity $\vec{v}$ will see the events separated by a time difference

$$x_2'^0 - x_1'^0 = \Lambda_\alpha^0(v)(x_2^\alpha - x_1^\alpha)$$

where $\Lambda_\alpha^\beta(v)$ is the "boost" defined by or,

$$x_2'^0 - x_1'^0 = \gamma(x_2^0 - x_1^0) + \gamma\vec{v} \cdot (x_2 - x_1)$$

and this will be negative if

$$v \cdot (x_2 - x_1) < -(x_2^0 - x_1^0) \tag{1}$$

We now quote from Weinberg [1]:
"At first sight this might seem to raise the danger of a logical paradox.

Suppose that the first observer sees a radioactive decay $A \to B + C$ at x_1, followed at x_2 by absorption of particle B, for example, $B + D \to E$. Does the second observer then see B absorbed at x_2 before it is emitted at x_1? The paradox disappears if we note that the speed $|v|$ characterizing any Lorentz transformation $\Lambda(v)$ must be less than unity, so that (1) can be satisfied only if

$$|x_2 - x_1| > |x_2^0 - x_1^0| \tag{2}$$

"However, this is impossible, because particle B was assumed to travel from x_1 to x_2, and (2) would require its speed to be greater than unity, that is, than the speed of light. To put it another way, the temporal order of events at x_1 and x_2 is affected by Lorentz transformations only if $x_1 - x_2$ is spacelike, that is,

$$\eta_{\alpha\beta}(x_1 - x_2)^\alpha (x_1 - x_2)^\beta > 0$$

whereas a particle can travel from x_1 to x_2 only if $x_1 - x_2$ is timelike, that is,

$$\eta_{\alpha\beta}(x_1 - x_2)^\alpha (x_1 - x_2)^\beta < 0$$

"Although the relativity of temporal order raises no problems for classical physics, it plays a profound role in quantum theories. The uncertainty principle tells us that when we specify that a particle is at position x_1 at time t_1, we cannot also define its velocity precisely. In consequence there is a certain chance of a particle getting from x_1 to x_2 even if $x_1 - x_2$ is spacelike, that is, $|x_1 - x_2| > |x_1^0 - x_2^0|$. To be more precise, the probability of a particle reaching x_2 if it starts at x_1 is nonnegligible as long as

$$(x_1 - x_2)^2 - (x_1^0 - x_2^0)^2 \leq \frac{\hbar^2}{m^2} \tag{3}$$

where $\hbar$ is Planck's constant (divided by 2π) and m is the particle mass. (Such space-time intervals are very small even for elementary particle masses; for instance, if m is the mass of a proton then $\hbar/m = w \times 10^{-14} cm$ or in time units $6 \times 10^{-25} sec$. Recall that in our units $1 sec = 3 \times 10^{10} cm$.) We are thus faced again with our paradox; if one observer sees a particle emitted at x_1, and absorbed at x_2, and if $(x_1 - x_2)^2 - (x_1^0 - x_2^0)^2$ is positive (but less than $\hbar^2/m^2$), then a second observer may see the particle absorbed at x_2 at a time t_2 before the time t_1 it is emitted at x_1".

To put it another way, the temporal order of causally connected events cannot be inverted in classical physics, but in Quantum Mechanics, the Heisenberg

Uncertainty Principle leaves a loop hole. To quote Weinberg again:
"There is only one known way out of this paradox. The second observer must see a particle emitted at x_2 and absorbed at x_1. But in general the particle seen by the second observer will then necessarily be different from that seen by the first. For instance, if the first observer sees a proton turn into a neutron and a positive pi-meson at x_1 and then sees the pi-meson and some other neutron turn into a proton at x_2, then the second observer must see the neutron at x_2 turn into a proton and a particle of negative charge, which is then absorbed by a proton at x_1 that turns into a neutron. Since mass is a Lorentz invariant, the mass of the negative particle seen by the second observer will be equal to that of the positive pi-meson seen by the first observer. There is such a particle, called a negative pi-meson, and it does indeed have the same mass as the positive pi-meson. This reasoning leads us to the conclusion that for every type of charged particle there is an oppositely charged particle of equal mass, called its antiparticle. Note that this conclusion does not obtain in nonrelativistic quantum mechanics or in relativistic classical mechanics; it is only in relativistic quantum mechanics that antiparticles are a necessity. And it is the existence of antiparticles that leads to the characteristic feature of relativistic quantum dynamics that given enough energy we can create arbitrary numbers of particles and their antiparticles".
As can be seen from the above, the two observers S and S' see two different events, viz., one sees, in this example the protons while the other sees neutrons. Moreover, this is a result stemming from (3), viz.,

$$0 < (x_1 - x_2)^2 - (x_1^0 - x_2^0)^2 (\leq \frac{\hbar^2}{m^2}) \tag{4}$$

The inequality (4) points to a reversal of time instants (t_1, t_2) as noted above. However, as can be seen from (4), this happens within the Compton wavelength.
We now consider the KG and Dirac equations in the above context of time "reversals". It is well known that the KG relativistic equation displays the phenomenon of negative energies. The problems of the KG equation can be traced to the second time derivative. To avoid this Dirac considers a first order equation, but here also there were negative energies and he had to further propose his Hole Theory to circumvent this, whereas Pauli and Wieskopf overcame the difficulties by treating the KG equation in a field theoretical sense, where the two degrees of freedom would represent distinctly charged

particles.

2 The Feshbach Villars Formulation

Feshbach and Villars [2] interpreted the KG equation in a single particle rather than field theoretic context. Infact they showed that this (F-V) formulation also applies to the Dirac equation. To see this, we can rewrite the K-G equation in the Schrodinger form, invoking a two component wave function,

$$\Psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}, \tag{5}$$

The $K - G$ equation then can be written as (Cf.ref.[2] for details)

$$\imath\hbar(\partial\phi/\partial t) = (1/2m)(\hbar/\imath\nabla - eA/c)^2(\phi + \chi)$$

$$+(eA_0 + mc^2)\phi,$$

$$\imath\hbar(\partial\chi/\partial t) = -(1/2m)(\hbar/\imath\nabla - eA/c)^2(\phi + \chi) + (eA_0 - mc^2)\chi \tag{6}$$

It will be seen that the components ϕ and χ are coupled in (6). In fact we can analyse this matter further, considering free particle solutions for simplicity. We write,

$$\Psi = \begin{pmatrix} \phi_0(p) \\ \chi_0(p) \end{pmatrix} e^{\imath/\hbar(p\cdot x - Et)}$$

$$\Psi = \Psi_0(p)e^{\imath/\hbar(p\cdot x - Et)} \tag{7}$$

Introducing (7) into (6) we obtain, two possible values for the energy E, viz.,

$$E = \pm E_p; \quad E_p = [(cp)^2 + (mc^2)^2]^{\frac{1}{2}} \tag{8}$$

The associated solutions are

$$\left.\begin{array}{ll} E = E_p & \phi_0^{(+)} = \frac{E_p + mc^2}{2(mc^2E_p)^{\frac{1}{2}}} \\ \psi_0^{(+)}(p): & \chi_0^{(+)} = \frac{mc^2 - E_p}{2(mc^2E_p)^{\frac{1}{2}}} \end{array}\right\} \phi_0^2 - \chi_0^2 = 1,$$

$$\left.\begin{array}{ll} E = -E_p & \phi_0^{(-)} = \frac{mc^2 - E_p}{2(mc^2E_p)^{\frac{1}{2}}} \\ \psi_0^{(-)}(p): & \chi_0^{(-)} = \frac{E_p + mc^2}{2(mc^2E_p)^{\frac{1}{2}}} \end{array}\right\} \phi_0^2 - \chi_0^2 = -1 \tag{9}$$

It can be seen from this that even if we take the positive sign for the energy in (8), the ϕ and χ components get interchanged with a sign change for the energy. Furthermore we can easily show from this that in the non relativistic limit, the χ component is suppressed by order $(p/mc)^2$ compared to the ϕ component exactly as in the case of the Dirac equation [3]. Let us investigate this circumstance further [4, 5].

It can be seen that (6) are Schrodinger equations and so solvable. However they are coupled. We have from them,

$$\dot{\phi} + \dot{\chi} = (eA_0 + mc^2)(\phi + \chi) - 2mc^2\chi \tag{10}$$

In the case if

$$mc^2 >> eA_0 \quad (\text{or}\, A_0 = 0) \tag{11}$$

that is we are dealing with energies or interactions much greater than the electromagnetic, (or in the absence of an external field) we can easily verify that

$$\phi = e^{\imath px - Et} \text{and}\, \chi = e^{\imath px + Et} \tag{12}$$

is a solution.

That is ϕ and χ belong to opposite sign of $E(m \neq 0)$ (Cf. equation (9)). The above shows that the K-G equation mixes the positive and negative energy solutions.

If on the other hand $m_0 \approx 0$, then (10) shows that χ and ϕ are effectively uncoupled and are of same energy. This shows that if ϕ and χ both have the same sign for E, that is there is no mixing of positive and negative energy, then the rest mass m_0 vanishes. A non vanishing rest mass requires the mixing of both signs of energy. Indeed it is a well known fact that for solutions which are localized, both signs of the energy solutions are required to be superposed [3, 6]. This is because only positive energy solutions or only negative energy solutions do not form a complete set.

Interestingly the same is true for localization about a time instant t_0. That is physically, only the interval $(t_0 - \Delta t, t_0 + \Delta t)$ is meaningful. This was noticed by Dirac himself when he deduced his equation of the electron [7]. Strictly speaking, the electron would have the velocity of light, if we work with spacetime points.

In any case both the positive and negative energy solutions are required to form a complete set and to describe a point particle at x_0 in the delta function sense. The narrowest width of a wave packet containing both positive and

negative energy solutions, which describes the spacetime development of a particle in the familiar non-relativistic sense, as is well known is described by the Compton wavelength. As long as the energy domain is such that the Compton wavelength is negligible then our usual classical type description is valid. In particular, the time inversion conditions stemming from equation (3) of Section 1 does not happen.
However as the energy approaches levels where the Compton wavelength can no longer be neglected, then new effects involving the negative energies and anti particles begin to appear (Cf.ref.[2]).
Further, we observe that from (12)

$$t \to -t \Rightarrow E \to -E, \quad \phi \leftrightarrow \chi \tag{13}$$

(Moreover in the charged case $e \to -e$). It can be shown that the Schrodinger equation goes over to the Klein-Gordon equation if we allow t to move forward and also backward in $(t_0 - \Delta t, t_0 + \Delta t)$ (Cf.ref.[5]). Here we have done the reverse of getting the Klein-Gordon equation into two Schrodinger equations. This is expressed by (6).
In any case we would like to reiterate that the two degrees of freedom associated with the second time derivative can be interpreted, following Pauli and Weisskopf as positive and negatively charged particles or particles and anti particles.

3 Remarks and Discussion

We summarize the following:
i) From the above analysis it is clear that a localized particle requires both signs of energy. At relatively low energies, the positive energy solutions predominate and we have the usual classical type particle behaviour. On the other hand at very high energies it is the negative energy solutions that predominate as for the negatively charged counterpart or the anti particles. More quantitatively, well outside the Compton wavelength the former behaviour holds. But as we approach the Compton wavelength we have to deal with the new effects.
ii) To reiterate if we consider the positive and negative energy solutions given by $\pm E_p$, as in (9), then we saw that for low energies, the positive solution ϕ_0 predominates, while the negative solution χ_0 is $\sim (\frac{v}{c})^2$ compared to the

positive solution. On the other hand at very high energies the negative solutions begin to play a role and in fact the situation is reversed with ϕ_0 being suppressed in comparison to χ_0. This can be seen from (9).
iii) We could now express the foregoing in the following terms: It is well known that we get meaningful probability currents and subluminal classical type situations using positive energy solutions alone as long as we are at energies low enough such that we are well outside the Compton scale. As we near the Compton scale however, we begin to encounter negative energy solutions or these anti-particles.
From this point of view, we can mathematically dub the solutions according to the sign of energy $(p_0/|p_0|)$ of these states: $+1$ and -1. This operator commutes with all observables and yet is not a multiple of unity as would be required by Schur's lemma, as it has two distinct eigen values. This is a superselection principle or a superspin with two states and can be denoted by the Pauli matrices. The two states would refer to the positive energy solutions and the negative energy solutions (Cf.refs.[4, 5]).
iv) We could now think along the lines of $SU(2)$ and consider the transformation [8]

$$\psi(x) \to exp[\frac{1}{2}\imath g\tau \cdot \omega(x)]\psi(x). \tag{14}$$

This leads to a covariant derivative

$$D_\lambda \equiv \partial_\lambda - \frac{1}{2}\imath g\tau \cdot \bar{W}_\lambda, \tag{15}$$

as in the usual theory, remembering that ω in this theory is infinitessimal. We are thus lead to vector Bosons $\bar{W}_\lambda$ and an interaction rather like the weak interaction. However we must bear in mind that this new interaction between particle and anti-particle [9] would be valid only within the Compton time, inside this Compton scale Quantum Mechanical bridge.
v) We have already seen that even given the Lorentz transformation, due to Quantum Mechanical effects, there could be an apparent inversion of events, though at the expense of the exact description of either observer. This has been brought out in Section 1 in the case of the observer seeing protons and another seeing neutrons. We now observe that in the above formulation for the wave function

$$\Psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix},$$

ϕ (or more correctly ϕ_0) represents a particle while χ represents an antiparticle. So, for one observer we have

$$\Psi \sim \left(\begin{array}{c} \phi \\ 0 \end{array} \right) \qquad (16)$$

and for another observer we can have

$$\Psi \sim \left(\begin{array}{c} 0 \\ \chi \end{array} \right) \qquad (17)$$

that is the two observers would see respectively a particle and an antiparticle. This would be the same for a single observer, if for example the particle's velocity got a boost so that (17) rather than (16) would dominate after sometime.

Interestingly, just after the Big Bang, due to the high energy, we would expect, first (17) that is antiparticles to dominate, then as the universe rapidly cools, particles and antiparticles would be in the same or similar number as in the Standard Model, and finally on further cooling (16) that is particles or matter would dominate.

vi) We now make two brief observations, relevant to the above considerations. Latest results in proton-antiproton collisions at Fermi Lab have thrown up the Bs mesons which in turn have decayed exhibiting CP violations in excess of the predictions of the Standard Model, and moreover this seems to hint at a new rapidly decaying particle. Furthermore, in these high energy collisions particle to antiparticle and vice versa transformations have been detected.

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