Breakdown of the photon lifetime concept in optical cavities with negative group delay

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The concept of photon lifetime in a cavity is shown to be non-relevant in the case where the cavity round-trip group delay is negative due to the presence of a strong intracavity negative dispersion. Causality is shown to forbid the cavity spectrum from being a single Lorentzian. These features are tested experimentally using intracavity detuned electromagnetically induced transparency in room-temperature metastable helium.

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Since the early works of Sommerfeld [1] and Brillouin [2, 3] on light propagation through resonant atomic systems, slow and fast light have been the subject of considerable research efforts. It is now well established that the group velocity of light can change dramatically in a dispersive medium: slow, fast, or even negative group velocity light can be observed. Moreover, such effects can, under some conditions, occur without any pulse distortion [4]. This has led to much controversy about Einstein's causality and the propagation of a signal in such situations, which has been solved by considering the information as carried by non-analyticity points [5–9].

The question of the lifetime of photons in cavities filled with a dispersive medium is an active subject of research, with consequences on potential applications such as the increase of the sensitivity of gyroscopes using fast light [10–12]. In this context, we have recently confirmed experimentally that in the case of a slow light medium inserted inside an optical cavity, the photon lifetime is governed by the group velocity [13]. In this Letter, we investigate some paradoxes arising from the consideration of a negative dispersion medium inserted inside a cavity, and we show that in such a case the photon lifetime concept is no longer relevant.

Let us consider a pulse of light propagating through a dispersive medium of refractive index $n(\omega)$ and length L_{cell}. If the dispersion is positive, the group velocity v_{g} is positive and the pulse experiences a positive group delay $\tau_{\mathrm{g}} = L_{\mathrm{cell}}/v_{\mathrm{g}}$ during its propagation through the medium (see Fig. 1(a)). On the contrary, when the dispersion is negative enough, v_{g} can become negative, leading to the appearance of a negative group delay τ_{g} through the medium. This can lead to the kind of situation sketched in Fig. 1(b), in which the outgoing pulse leaves the medium before the incident pulse enters it while a pulse propagates in the backward direction inside the medium. In these paradoxical situations, causality has been shown to be ensured by the propagation of non-analyticity points at the speed c of light in vacuum [7–9]. Let us now introduce such a dispersive medium inside a resonant cavity, as shown in Fig. 1(c). In the case of positive dispersion, i.e., slow light, we have recently shown [13] that the lifetime of the photons in the cavity is given as expected by $\tau_{\mathrm{cav}} = \tau_{\mathrm{g}}^{\mathrm{RT}}/\Pi$, where Π stands for the fractional loss per cavity round trip, and $\tau_{\mathrm{g}}^{\mathrm{RT}} = \tau_{\mathrm{g}} + L_{\mathrm{vac}}/c$ is the group delay for one round trip inside the cavity with L_{vac} the length of the empty part of the cavity. In the latter case, the intracavity decaying intensity then invokes the simple picture of a pulse propagating at the group velocity inside the cavity and decaying at each round trip because of losses (see the decaying pulses of Fig. 1(d)). This picture does no longer hold in the case where the intracavity dispersion is negative and is strong enough not only to make the group delay across the cell τ_{g} negative, but also the cavity group

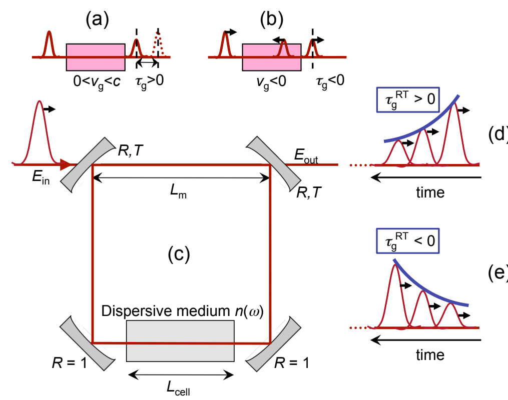

FIG. 1: (color online) Propagation of a light pulse through (a) a slow-light medium (the dotted curve corresponds to a pulse propagating in vacuum), (b) a negative-light medium, (c) a cavity containing a dispersive medium. (d,e) Series of pulses at the output of the cavity in the case of (d) positive and (e) negative round-trip group delay $\tau_{\mathrm{g}}^{\mathrm{RT}}$, when the cavity is excited by an incident pulse.

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round-trip time $\tau_{\mathrm{g}}^{\mathrm{RT}}$ smaller than zero. In this case, if we follow the same picture as in the case of positive dispersion, the pulse that has made one round trip inside the cavity must exit the cavity before the initial pulse, and is even preceded by the pulse that has undergone two round trips inside the cavity, etc. This should lead to an increase of the intensity with time, as shown in Fig. 1(e), which of course looks absurd.

We can try to solve this paradox by calculating the field at the output of the cavity for any incident excitation $E_{\mathrm{in}}(t)$ using simple linear response theory. The positive-frequency part of the output field reads

$$E_{\mathrm{out}}^{(+)}(t) = \int_{-\infty}^{t} \mathrm{d}t' E_{\mathrm{in}}^{(+)}(t')\, R(t-t')\,, \tag{1}$$

where $E_{\mathrm{in}}^{(+)}(t)$ is the positive-frequency part of the input field, and $R(\tau)$ is the response function of the cavity which is zero for $\tau < 0$. We can stress the fact that R is causal by writing it as $R(t) = S(t)H(t)$, where H is the Heavyside step function, and S is the Fourier transform of the cavity transmission $\widetilde{S}(\omega)$ for a monochromatic incident field. Then Eq. (1) simply reads

$$E_{\mathrm{out}}^{(+)}(t) = [E_{\mathrm{in}}^{(+)} * (SH)](t)\,. \tag{2}$$

We consider a cavity like the one in Fig. 1(c). The input and output mirrors are identical, with intensity reflection and transmission coefficients given by R and T, respectively. The two other mirrors are perfectly reflecting. We call L_{m} the length between the input and output mirrors. The cavity transmission for a monochromatic field of angular frequency ω is then given by

$$\widetilde{S}(\omega) = \frac{T \exp\left[\mathrm{i}\frac{\omega}{c} L_{\mathrm{m}}\right]}{1 - R \exp\left[\mathrm{i}\frac{\omega}{c}\left(L_{\mathrm{vac}} + n(\omega) L_{\mathrm{cell}}\right)\right]}\,. \tag{3}$$

If we suppose that ω is close to a resonance frequency ω_p of the cavity, for which $\exp\left[\mathrm{i}\frac{\omega_p}{c}(L_{\mathrm{vac}} + n(\omega_p)L_{\mathrm{cell}})\right] = 1$, then, at first order in $(\omega - \omega_p)/\omega_p$, Eq. (3) becomes

$$\widetilde{S}(\omega) = \frac{T \exp\left[\mathrm{i}\frac{\omega}{c} L_{\mathrm{m}}\right]}{1 - R - \mathrm{i}R(\omega - \omega_p)\tau_{\mathrm{g}}^{\mathrm{RT}}}\,, \tag{4}$$

leading to:

$$\widetilde{S}(\omega) = \left(\frac{T}{R\tau_{\mathrm{g}}^{\mathrm{RT}}}\right) \frac{\exp\left[\mathrm{i}\frac{\omega}{c} L_{\mathrm{m}}\right]}{\frac{\gamma_{\mathrm{cav}}}{2} - \mathrm{i}(\omega - \omega_p)}\,, \tag{5}$$

where the cavity decay rate is given by

$$\gamma_{\mathrm{cav}} = 2(1-R)/R\,\tau_{\mathrm{g}}^{\mathrm{RT}} \simeq \Pi/\tau_{\mathrm{g}}^{\mathrm{RT}} = 1/\tau_{\mathrm{cav}}\,. \tag{6}$$

We have assumed that $1 - R \ll 1$. In order to predict what a measurement of the photon lifetime should give, let us now consider the response of this cavity to a laser field at frequency ω_l which is turned off at $t = 0$:

$$E_{\mathrm{in}}^{(+)}(t) = E_0[1 - H(t)]\mathrm{e}^{-\mathrm{i}\omega_l t}\,. \tag{7}$$

In the case of a cavity with a positive round-trip group delay $\tau_{\mathrm{g}}^{\mathrm{RT}}$, Eqs. (2), (5) and (7) lead to

$$E_{\mathrm{out}}^{(+)}(t) = \frac{S_0 E_0 \mathrm{e}^{-\mathrm{i}\omega_l(t - \frac{L_{\mathrm{m}}}{c})}}{\frac{\gamma_{\mathrm{cav}}}{2} - \mathrm{i}(\omega_l - \omega_p)}\,, \text{ if } t \le \frac{L_{\mathrm{m}}}{c}\,, \tag{8}$$

$$E_{\mathrm{out}}^{(+)}(t) = \frac{S_0 E_0 \mathrm{e}^{-\mathrm{i}\omega_p(t - \frac{L_{\mathrm{m}}}{c})}}{\frac{\gamma_{\mathrm{cav}}}{2} - \mathrm{i}(\omega_l - \omega_p)} \mathrm{e}^{-\frac{\gamma_{\mathrm{cav}}}{2}(t - \frac{L_{\mathrm{m}}}{c})}\,, \quad \text{if } t \ge \frac{L_{\mathrm{m}}}{c}\,, \tag{9}$$

with $S_0 = T/R\tau_{\mathrm{g}}^{\mathrm{RT}}$, which is the standard solution for a decaying cavity, in agreement with the observations of Ref. [13]. On the contrary, in the case of a negative light cavity for which $\tau_{\mathrm{g}}^{\mathrm{RT}} < 0$, Eqs. (2), (5) and (7) lead to

$$E_{\mathrm{out}}^{(+)}(t) = \frac{S_0 E_0 \left(\mathrm{e}^{-\mathrm{i}\omega_l(t - \frac{L_{\mathrm{m}}}{c})} - \mathrm{e}^{-(\mathrm{i}\omega_p + \frac{\gamma_{\mathrm{cav}}}{2})(t - \frac{L_{\mathrm{m}}}{c})}\right)}{\frac{\gamma_{\mathrm{cav}}}{2} - \mathrm{i}(\omega_l - \omega_p)}\,, \quad \text{if } t \le \frac{L_{\mathrm{m}}}{c}\,, \tag{10}$$

$$E_{\mathrm{out}}^{(+)}(t) = 0\,, \text{ if } t \ge \frac{L_{\mathrm{m}}}{c}\,, \tag{11}$$

which, once again, clearly violates causality.

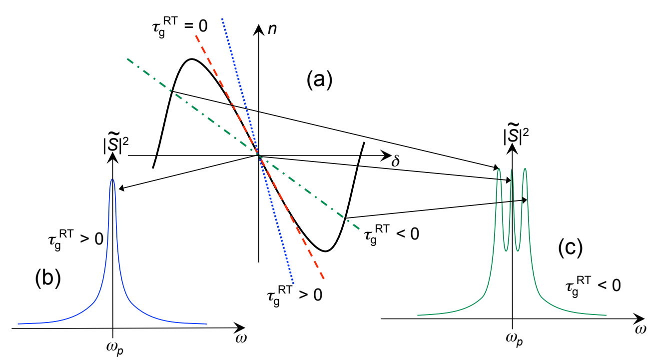

FIG. 2: (color online) (a) Continuous line: typical negative dispersion curve. The intersections of this curve with the dashed ($\tau_{\mathrm{g}}^{\mathrm{RT}} = 0$), dotted ($\tau_{\mathrm{g}}^{\mathrm{RT}} > 0$), and dot-dashed ($\tau_{\mathrm{g}}^{\mathrm{RT}} < 0$) curves determine whether the resonance is (b) single peaked or (c) multi-peaked.

In deriving Eqs. (10) and (11), the only hypothesis that we have made is that the cavity transmission given by Eq. (3) could be reduced to a single Lorentzian peak as given in Eq. (4). This hypothesis is valid as long as the spectrum of the incident field given by Eq. (7) is contained in a single cavity transmission peak and all frequencies experience the same group index. In order to examine this condition, we consider a typical negative dispersion curve as given by the continuous line in Fig. 2(a). Let us suppose, without any loss of generality, that the inflection point of the dispersion curve occurs at the empty cavity resonance frequency ω_p, meaning that $\frac{\omega_p}{c}(L_{\mathrm{vac}} + n(\omega_p)L_{\mathrm{cell}}) = 2p\pi$, where p is an integer. Let us try to determine whether extra resonance peaks, due to negative dispersion, could occur in the vicinity of the peak at ω_p. If $\omega_p + \delta$ is the angular frequency of such an extra peak, the resonance condition reads

$$\frac{\omega_p + \delta}{c}(L_{\rm vac} + n(\omega_p + \delta)L_{\rm cell}) = 2p\pi \,. \qquad (12)$$

To first order in δ/ω_p, this condition is equivalent to

$$n(\omega_p + \delta) - n(\omega_p) = -\frac{L_{\rm vac} + n(\omega_p)L_{\rm cell}}{\omega_p L_{\rm cell}}\delta \,. \qquad (13)$$

The left-hand side of Eq. (13) versus δ is the continuous line in Fig. 2(a). The right-hand side is a straight line, as shown by the dotted, dashed, and dot-dashed lines in Fig. 2(a). One can see that the shape of the resonance, namely, the existence of no other solution than $\delta = 0$, leading to a single peak as in Fig. 2(b), or the existence of two other resonance frequencies for $\delta \neq 0$, leading to two extra resonance peaks as in Fig. 2(c), depends on the relative values of the slopes of the dispersion curve and the line corresponding to the right-hand side of Eq. (13). In particular, the condition for the existence of two extra solutions reads, at first order in δ:

$$-\left.\frac{{\rm d}n}{{\rm d}\omega}\right|_{\omega_p} > \frac{L_{\rm vac} + n(\omega_p)L_{\rm cell}}{\omega_p L_{\rm cell}} \,, \qquad (14)$$

which is equivalent to $\tau_{\rm g}^{\rm RT} < 0$. We thus reach the following conclusion: the fact that the group delay for one round trip inside the cavity is negative leads to the existence of satellite peaks around the resonance considered. This negates the approximation used to obtain Eq. (4) and explains why the non-causal situation described above can actually never be reached. Figure 2(c) illustrates how this condition results in thr existence of two extra peaks for the cavity resonance labeled by the integer p. Note that in the case of slow light, the slope of the dispersion curve in Fig. 2(a) would be reversed, allowing only one intersection with the continuous line and thus forbidding the existence of extra resonance peaks.

Let us be more specific about the situation in which a medium can exhibit a strong negative dispersion. A very popular example of negative dispersion is provided by a gain doublet [6, 7, 14–17]. Figure 3(a) shows the transmission $|\widetilde{S}(\omega)|^2$ of the cavity versus detuning in that case. The dashed curve corresponds to the empty cavity, which is 2.45 m long with 29% losses per round trip. We now suppose that a gain-doublet medium is inserted inside the cavity. The two gain peaks are separated by 1.5 MHz. We suppose that they are located symmetrically with respect to the cavity resonance. The gain maxima correspond to 28% per round trip and the full width at half maximum of each peak is 800 kHz. In these conditions, the group delay for one round trip inside the cavity is $\tau_{\rm g}^{\rm RT} = -3.3\,$ns. The corresponding intensity transmission spectrum of the cavity is reproduced as a continuous line in Fig. 3(a). One can clearly see the two transmission peaks corresponding to the conjugated effects of the two

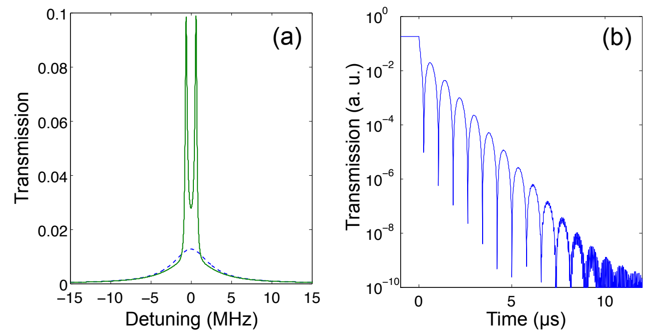

FIG. 3: (color online) (a) Theoretical cavity transmission versus detuning in the presence of (continuous line) and without (dashed line) an intracavity gain doublet creating negative light. (b) Corresponding decay of the intracavity intensity when the incident field is turned off at $t = 0$.

additional resonance peaks and of the two gain maxima. The main difference with respect to Fig. 2(c), which was computed using only the real part of the dispersion and by artificially setting the imaginary part to zero (no gain or absorption) is that there is no central transmission peak. This is consistent with the fact that there is no gain peak at zero detuning. One can also notice in this spectrum that the two lateral peaks are slightly shifted towards the line center with respect to the positions of the atomic resonances, which is consistent with the fact that these gain peaks are located in a positive dispersion spectral region.

Using Eqs. (2), (3) and (7), we calculate the temporal evolution of the intensity $|E_{\rm out}^{(+)}(t)|^2$ at the output of the cavity when the incident field is suddenly turned off at $t = 0$. Such a decay is represented in Fig. 3(b) on a logarithmic scale. It is clearly non-exponential. It consists of a fast decay by two orders of magnitude, followed by oscillations which correspond to beat notes between the two peaks of the transmission spectra. It is an illustration of the general principle of Fig. 2: any intracavity negative dispersion effect which is strong enough to make the round-trip group delay negative will cause secondary transmission peaks to emerge that will make the cavity decay non-exponential, forbidding one to define a photon lifetime for this cavity.

In order to give an experimental illustration, we use another system in which a large negative group delay can be achieved: detuned electromagnetically induced transparency (EIT) in a hot vapor of metastable ^{4}He atoms [18]. We use a 6-cm long cell filled with 1 Torr of helium at room temperature. Some of these atoms are excited to the $^3S_1$ metastable state using an RF discharge at 27 MHz. Metastable helium is well known for exhibiting a pure three-level Λ system when excited at the 1.083 μm transition between the 2^3S_1 and 2^3P_1 energy levels using circularly polarized light. Light at 1.083 μm is provided by a single-frequency diode laser. The frequencies and

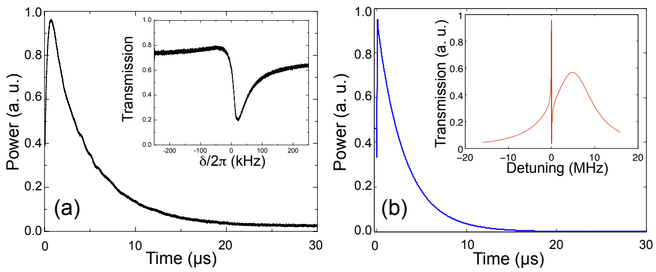

FIG. 4: (color online) (a) Experimental decay of the intracavity intensity when the incident probe field is turned off at $t = 0$. Inset: Transmission of the cell versus Raman detuning δ. (b) Corresponding theoretical cavity decay. Inset: Calculated cavity transmission profile.

Rabi frequencies of the coupling and probe beams used in our experiment are driven by two acousto-optic modulators. The cell is inserted inside a 2.4-m long triangular ring cavity made of two plane mirrors with 2% transmission and a high reflectivity concave mirror with a 5-m radius of curvature. A telescope expands the coupling beam diameter up to 1 cm, which is much larger than the probe beam diameter. The cavity is resonant only for the probe field as two polarization beam-splitters drive the coupling beam inside and outside the cavity [13].

With a coupling power of 5 mW, detuned by 1.44 GHz from the maximum of the Doppler profile of the transition, the evolution of the cell transmission versus Raman detuning δ exhibits the asymmetric Fano-like profile shown in the inset of Fig. 4(a). The large one-photon detuning transforms the transparency peak typical of EIT into an asymmetric absorption peak [19]. Consequently, in the vicinity of the transmission minimum, the system exhibits strong negative dispersion. With our experimental parameters, we measure, using propagation of an intensity modulation, a negative group delay of $\tau_{\mathrm{g}} \approx -4\,\mu$s for our 6-cm long cell. Once this cell is inserted inside the cavity, we still apply the same coupling field to the atom. The probe field, which has the same frequency as the coupling field ($\delta = 0$), is incident on the cavity input mirror. We slowly scan the length of the cavity using a piezoelectric actuator that carries one of the mirrors. When the cavity is at resonance with the probe, we abruptly turn off the probe field using an acousto-optic modulator. We then observe the evolution of the intensity at the output of the cavity (see Fig. 4(a)). One can see that the intensity starts increasing, on a time scale shorter than 1 μs, before decreasing. This evolution is clearly non-exponential, showing once more that a negative cavity round-trip group delay leads to a non-exponential decay of the intracavity photons. Notice here that the cavity length being 2.4 m, the cavity round-trip group delay $\tau_{\mathrm{g}}^{\mathrm{RT}}$ is also of the order of $-4\,\mu$s.

These experimental results are consistent with the theoretical calculations based on Eqs. (2), (3) and (7) using the expression for inhomogeneously broadened detuned EIT as in Ref. [18]. With a coupling Rabi frequency of 11 MHz and a Raman coherence lifetime of $13\,\mu$s, we obtain the curve of Fig. 4(b), which is similar to the experimental result of Fig. 4(a): when the incident intensity is turned off, the intensity at the output of the cavity starts increasing before decreasing. This typical non-exponential decay constitutes one more illustration of the behavior described in Fig. 2.

In conclusion, we have demonstrated the fact that it is impossible to obtain a negative group delay for one round trip inside a resonant cavity while keeping an exponential decay of the intracavity intensity. We have shown that this result respects causality in negative group-delay cavities. We have illustrated this remarkable result both numerically and experimentally, by using negative velocity light induced by a gain doublet and detuned EIT in a metastable vapor, respectively. Our result is consistent with the fact that slow light is usually associated with a transparency peak, which reduces the bandwidth to be considered. On the contrary, fast and negative group velocity light appears in the case of an absorption peak, leading to the possibility of many frequencies playing a role, and thus the lifetime of photons in such a cavity can no longer be simply defined. This should have interesting consequences on the spontaneous emission rate of atoms placed in such a cavity [20], with application to the spontaneous emission noise of lasers based on such negative light cavities.

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