

## MONETARY POLICY WITH INVESTMENT–SAVING IMBALANCES

Roberto Tamborini\*  
University of Trento

(October 2008; revised June 2009)

### ABSTRACT

Financial instability is the new challenge for monetary policy. Most studies indicate that financial crises follow prolonged unwinding of investment–saving imbalances (ISI). These phenomena are not contemplated by the standard theoretical framework of continuous intertemporal equilibrium. This paper’s aim is to take a first step into the analysis of monetary policy in the context of ISI. First, a dynamic model of a flex-price, competitive economy is presented where ISI are allowed to develop. Second, upon introducing different types of Taylor rules, some indications for the conduct of monetary policy emerge, which are at variance with the standard view.

### 1. INTRODUCTION

The age of ‘Great Moderation’—sustained growth and employment with low and stable inflation—that blessed most of the world economy throughout the 1990s led the economic profession to think that the right theoretical foundations for macroeconomic policies had finally been found, namely the so-called ‘New Neoclassical Synthesis’ (NNS) (Blanchard, 2000). This is the theoretical apparatus that has completely replaced the Old Keynesian-Neoclassical Synthesis of the 1950–60s on the modern methodological grounds of dynamic stochastic general equilibrium (DSGE),

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\* I wish to thank Andrea Fracasso, Axel Leijonhufvud, Michael Trautwein, Massimo Molinari, Ronny Mazzocchi, Stefano Schiavo, Vela Velupillai and two referees of this Journal for helpful discussion and comments. I remain fully responsible for this paper. Financial support by the Italian Ministry of University and Research (PRIN grant No. 2007HA3S722\_004) is gratefully acknowledged.

delimited within the ‘triangle’ of *intertemporal equilibrium*, *monopolistic competition* and *sticky prices* (Blanchard, 1997). The last two are the sole elements that also qualify this apparatus as ‘New Keynesian’ in opposition to the ‘New Classical’ one based on perfect competition and flexible prices.

Unexpectedly, since the turn of the century the *most developed* countries in the world have been prone to repeated financial crises of major order of magnitude and widespread macro-consequences. While the depth, extent and duration of the latest event—the gigantic ‘subprime’ mortgage market failure in the USA—are still largely hidden from view, it is already clear that the theoretical system of the Great Moderation has been seriously shaken by the abrupt demise of those historical conditions. One reason is that many observers have attributed distinct responsibilities to monetary policy (especially in the USA), not so much as the result of misbehaviour by monetary authorities than as of unforeseen consequences of the prescriptions distilled from the NNS.

The search for where the fault lies and how to fix it is now afoot. There is broad consensus that a critical issue concerns the nexus between monetary policy and financial markets.<sup>1</sup> The move of central banks away from control of monetary aggregates towards interest-rate management, which is the hallmark of the NNS, has placed monetary policy at the very core of the transmission mechanisms between financial markets and the real economy. The paradox is that, while these connections had been at the centre of the stage in macroeconomics throughout the 20th century, they have suddenly disappeared with the advent of NNS monetary-policy modelling. Two approaches to the problem are being pursued, one ‘internal’ and the other ‘external’ with respect to the existing framework.

The former mainly consists in works that investigate how the optimal design of monetary policy should be modified in order to prevent/counteract the surge of asset-price bubbles, typically modelled as exogenous processes grafted onto the standard NNS model. The question is whether interest-rate rules should or should not include asset prices, or other related indicators as well, alongside consumer goods prices (examples are Cecchetti *et al.*, 2000a; Bernanke and Gertler, 2001; Bordo and Jeanne, 2002; Bean, 2003; Christiano *et al.*, 2007a). Other works address the same question also seeking to refine the standard NNS model by adding a monetary, banking or financial sector (e.g. Iacoviello, 2005; Christiano *et al.*, 2007b; Goodfriend and McCallum, 2007; Canzoneri *et al.*, 2008).

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<sup>1</sup> Some reference works are Cecchetti *et al.* (2000b), Kaufman (2001), Borio and Lowe (2002), Crockett (2003), Hunter *et al.* (2003), Leijonhufvud (2008).

The ‘external’ approach to the problem takes the more radical view that the NNS framework is unable to withstand the patterns of events that should be explained. On this view, the problems stem from the DSGE methodology straitjacket worn by the NNS. As recognized by Lucas, the benefits of this methodology have come with their costs:

The problem is that the new theories, the theories embedded in general equilibrium dynamics [. . .] *don't let us think* about the US experience in the 1930s or about financial crises and their consequences [. . .] We may be disillusioned with the Keynesian apparatus for thinking about these things, but it doesn't mean that this replacement apparatus can do it either. (Lucas, 2004, p. 23)

Two main issues are under the spotlight in this connection. The first concerns the perfectly informed, unboundedly rational representative agent in comparison with alternative empirical hypotheses put forward by ongoing research in behavioural economics and finance (e.g. Shiller, 2000; Akerlof, 2007; De Grauwe, 2008; Delli Gatti *et al.*, 2008). The second issue—which is not alternative to, though not necessarily connected with, the first—relates to the exclusive focus on states of the economy characterized by intertemporal equilibrium (e.g. van der Ploeg, 2005; Leijonhufvud, 2008). Financial imbalances provoking the eruption of financial crises almost by definition imply misallocation of resources along the time dimension of the economy. The typical macroeconomic pattern is one of excess investment over saving triggered by wrong prices of financial resources that turn out to be unsustainable in the long run (e.g. Borio and Lowe, 2002; Borio, 2008). Hence it is argued that there cannot be any deep understanding of these phenomena, let alone a search for best monetary policy responses, until we endeavour to *think* about an economy operating in intertemporal disequilibrium.

This latter argument seems compelling from the theoretical point of view. As a methodological premise, it is worth recalling that ever since the origins of macroeconomics with Wicksell, and until the advent of the DSGE methodology in its various incarnations, inflation, unemployment and business cycles had been thought and taught mainly as *problems* related to intertemporal disequilibrium originating from investment–saving imbalances (ISI) due to some form of malfunctioning of capital markets (Leijonhufvud, 1981). It is also noteworthy that earlier ‘New Keynesians’ had the explicit aim to find microeconomic foundations for the original ideas of Wicksell and Keynes in *this* field (e.g. Greenwald and Stiglitz, 1987, 1993; Stiglitz, 1992). Nonetheless, the intertemporal disequilibrium implications of capital market failures have generally been ignored, and what is now regarded as the

New Keynesian model dispenses with capital market imperfections altogether.<sup>2</sup>

In this perspective, Woodford's (2003) claim on the Wicksellian roots of the NNS (see in particular chapter 1) is puzzling.<sup>3</sup> Puzzlement comes from the fact that in the NNS standard model, whenever the market real interest rate deviates from the natural rate (households' rate of time preference), households reallocate resources towards present/future consumption along a new *intertemporal equilibrium* path with an equivalent impact on aggregate demand. This is a consistent transmission mechanism as long as there are no capital goods (Woodford, 2003, section 4.1). But when there are capital goods to be purchased, and there is a market for loanable funds made by independent borrowers and lenders, ruling ISI out of the theory by assumption constitutes a major theoretical divide with Wicksell as well as with all the leading business cycle scholars of the early 20th century, including Keynes.

More importantly, ISI are a *logical implication* in *any* theory based on the distinction between the market real interest rate and the natural rate, which is, by definition, the interest rate that equates saving and investment in general equilibrium. That this distinction may be more than a theoretical curiosity is testified by the current debate about the Fed's responsibility in the financial crisis for keeping the interest rate 'too low for too long' (e.g. Taylor, 2009). In the NNS framework itself, this sentence can only mean that the nominal interest rate and the anticipated inflation rate resulted in a market real interest rate 'too low' *with respect to the natural rate*. Yet the ignored consequence of the market real interest rate being lower (higher) than the natural rate is that households wish to save less (more) whereas firms wish to invest more (less): neither side of the market can achieve intertemporal equilibrium of plans (Wicksell, 1901, II, p. 193). The problem here lies, not in sticky goods prices, but in a capital market failure. This problem is not even contemplated in the NNS models that endogenize investment (e.g. Casares and McCallum, 2000; Woodford, 2003, chapter 5).

With a view to improving and extending the theoretical framework of monetary policy, my aim with this paper is to take a first step into the analysis

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<sup>2</sup> A few important works based on imperfect capital markets, such as those on 'financial fragility' and the 'financial accelerator' (Bernanke and Gertler, 1989, 1990; Bernanke *et al.*, 1996), are no doubt an integral part of mainstream modern macroeconomics. However, they do not contemplate the possibility of ISI and their intertemporal implications, and at any rate they have exerted little impact on the NNS theory of monetary policy. On the evolution of macroeconomics with imperfect capital markets see also Tamborini (2008).

<sup>3</sup> In Woodford's view this parentage was so important that he gave his book the same title, *Interest and Prices*, as Wicksell's (1898b).

of macroeconomic processes with ISI in a way that allows monetary policy to be consistently studied in connection with these phenomena. With two caveats. First, the stage of treatment in this paper is still remotely connected with the full-fledged financial structure of the economy and it is therefore only propaedeutic to proper study of financial crises. To repeat, the focus is only on the origin and policy implications of one (the first?) of the genes of full-blown financial crises, namely ISI. Second, the model presented here at best draws inspiration from Wicksell's (and Keynes's) idea of the role of ISI in the business cycle, but it is not meant to reproduce Wicksell's (or Keynes's) complex theories in their entirety or integrity. Indeed, the modelling strategy that I have chosen starts from the opposite side of the spectrum, the NNS model, and departs from it as much as is necessary and sufficient for ISI to develop.<sup>4</sup> In my view, this incremental (or perhaps minimalist) strategy, favouring direct comparison with the NNS, helps identify its limits as well as the gains for monetary policy guidance that can be obtained by moving towards the realm of intertemporal disequilibrium. The paper is organized into three parts of which I give here a non-technical preview.

First, section 2 presents the model. The case of ISI is addressed by inverting the analytical structure of the NNS standard model from one with imperfect consumer goods markets and a perfect capital market into one with imperfect capital market and a perfectly competitive, flex-price market for the economy's homogenous output. Adding goods market imperfections and sticky prices may enrich the model for empirical purposes, but, as will be seen, it is not necessary theoretically as is claimed by the NNS. The capital market deals with corporate bonds representative of physical capital, with the central bank taking part to transactions by means of open market operations. This is also the (simplified) channel whereby the central bank has the power to peg the nominal interest rate at which transactions take place.<sup>5</sup> Granted this, it is still the case that, as in the NNS model, the economy rests in intertemporal equilibrium as long as the market real interest rate (the

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<sup>4</sup> The present contribution, therefore, also differs from that put forward by Chiarella and Flaschel (1996). They make reference to Wicksell and consider ISI as a consequences of posited 'independent saving and investment functions' (p. 331), whereas the present model is based on microfounded interdependent functions via market real interest rate. Chiarella *et al.* (2005), in spite of the common reference to out-of-equilibrium processes, and of some similar dynamic structures (see especially section 1.3, and chapter 8), focus on various forms of delayed adjustment of nominal prices and wages on goods and labour markets that are not posited here.

<sup>5</sup> This also seems a more sensible channel than the one posited by the NNS model (see Woodford (2003, section 2.1) and Boianovsky and Trautwein (2004) for a critique). However, an obvious simplification here is that the central bank is taken to represent the banking system as a whole, whereas the private banking sector, which plays the key role in Wicksell (but not in Keynes), is not made explicit (see also footnote 7 below).

difference between the nominal interest rate set by the central bank and the expected inflation rate) is equal to the natural rate, which equates saving (households' purchases of bonds) and investment (firms' sales of bonds).

The capital market is imperfect in that the central bank may fail to manage the nominal interest rate consistently with the natural rate. An up-to-date justification may be that the natural rate is unobservable to the central bank, and it has no direct information about it.<sup>6</sup> As long as the central bank pegs the nominal interest rate so that the market real interest rate differs from the natural rate, ISI arise with firms that are allowed to invest more (or less) than households wish to save by the central bank's extra purchases (or extra sales) of bonds.<sup>7</sup> As a result, the model generates endogenous, dynamic processes of both output and the general price level that deviate from their intertemporal equilibrium values. These 'gaps' persist as long as the original one between the market real interest rate and the natural rate persists.<sup>8</sup>

Then, section 3 introduces monetary policy in the form of interest-rate rules. The question to be addressed is whether this approach to monetary policy is effective in stabilizing the economy in the event of ISI. First, I examine a Taylor rule obtained from a standard, 'well-behaved' optimization problem of the central bank. On the one hand, this rule consistently prescribes that the central bank should have a nominal interest-rate target given by the natural rate plus the inflation target. In fact, this is exactly the same condition that grants intertemporal general equilibrium as discussed above. However, in the absence of direct information on the natural interest rate, the interest-rate target may be wrong, and following this rule boosts ISI and fails to stabilize the economy. In contrast, it is shown that, once ISI are in motion, the economy can be stabilized by means of a more parsimonious rule, i.e. an 'adaptive' rule whereby the nominal interest rate is simply raised or lowered in response to the cyclical condition of output and inflation, with no interest-rate target whatsoever. Stabilization occurs only under some conditions, however. As a consequence of ISI, the dynamic properties that the rule

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<sup>6</sup> Wicksell (1898a) himself was well aware that the critical challenge for monetary (and banking) policy lies in the natural interest rate being subject to unobservable shocks and fluctuations (pp. 82 ff.). Up-to-date econometric research is by no means encouraging on the possibility that central banks can ever obtain all information necessary to target the natural rate precisely (see, for example, D'Amato, (2005), Garnier and Wilhelmssen (2005) and Caresma *et al.* (2005) for recent surveys). A growing literature is now concerned with monetary policy under imperfect information (e.g. Sims, 1998; Sargent, 1999; Orphanides and Williams, 2002, 2006; Primiceri, 2006), although little attention has been devoted to the case of the natural interest rate.

<sup>7</sup> This is the role played by banks in Wicksell's original theory, where they can allow excess investment or excess saving to develop by dishoarding or hoarding reserves at the current interest rate (see also Leijonhufvud, 1981).

<sup>8</sup> As to the connection of this result with Wicksell see Leijonhufvud (1981).

should display in order to stabilize the economy differ from those within the NNS model in some important respects. In particular,

- the central bank's *bounded responsiveness* to excess inflation/deflation (or the irrelevance of the so-called 'Taylor principle');
- the emergence of a trade-off, not between output and inflation, but between *small gaps and smooth paths* in the design of the rule as a convergence control tool.

The model also warns that a crucial role is played by the cyclical sensitivity of inflation. If this is very low (the so-called 'missing inflation' problem), identifying and correcting ISI via inflation gaps may become rather difficult, and the ongoing disequilibrium process may persist for a long time. Finally, section 4 summarizes and concludes.

## 2. A MODEL OF ISI

In this section I introduce the same general-equilibrium framework underlying the NNS standard model where perfect capital market, monopolistic competition and the sticky price assumption are replaced by imperfect capital market, perfect competition and flexible prices. For the sake of concreteness and comparison with the standard NNS model, I also posit common well-behaved functional forms for the production function and the utility function.

The supply side of the economy is characterized by:

- a constant-return-to-scale production function such that output  $Y_t$  is a homogeneous good that can be either consumed or used as input (capital),

$$Y_t = AK_t^a L_t^b, \quad a + b = 1 \quad (1)$$

where  $L_t$  is the current input of labour, and  $K_t$  is the available capital stock;

- a capital accumulation technology such that the share of output transformed into capital at time  $t$  takes one period of time to become operative, and the depreciation rate of capital is 100% per period, that is

$$K_{t+1} = I'_t + (1 - \delta)K_t \quad (2)$$

where  $I'_t$  is gross investment inclusive of capital replacement, and  $\delta = 1$  is the depreciation rate;

- firms that are price takers and seek to maximize their expected profit stream, given (1) and the gross income distribution constraint

$$Y_t = w_t L_t + R_t K_t \quad (3)$$

where  $w_t$  is the real wage rate, and  $R_t \equiv (1 + r_t)$  is the real gross return to be paid on the capital stock purchased at time  $t - 1$  and operative at time  $t$ .

The demand side of the economy consists of households that claim on the economy's capital stock, supply their whole labour force  $L$  inelastically, and choose a consumption plan in order to maximize their lifetime expected utility

$$U(C_t) + E_t[\sum_S \Theta^{-s} U(C_{t+s})] \quad s = 1, \dots, \infty \quad (4)$$

with period utility

$$U(C_t) = \ln C_t$$

given each period's budget constraint

$$C_t + S'_t = Y_t \quad (5)$$

where  $S'_t$  is gross saving,  $\Theta \equiv 1 + \theta$  and  $\theta > 0$  is the rate of time preference.

All the previous variables are in real terms, while factors and output are traded at nominal prices denominated in a single unit of account. As far as capital is concerned, firms can finance investment out of household saving by issuing one-period bonds bearing a nominal interest rate  $i_t$ . Consistently with physical capital, bonds are time-indexed by their maturity, i.e.  $t + 1$  denotes bonds issued at time  $t$  with maturity at  $t + 1$ . Note, therefore, that the market real interest rate relevant to the saving–investment decisions in period  $t$  is given by  $R_{t+1} \equiv (1 + r_{t+1}) = E_t[(1 + i_t)/(1 + \pi_{t+1})]$ , where  $\pi_{t+1}$  is the rate of inflation of the output price  $P_t$  (whereas the actual real interest rate that households earn in each  $t$  is given by  $(1 + i_{t-1})/(1 + \pi_t)$ ).

Consequently, households' budget constraint can also be written as

$$B_{t+1} = H_t + R_t B_t - C_t \quad (6)$$

where  $B_t$  is the outstanding real stock of bonds, and  $H_t \equiv w_t L_t$  is labour income. Note, therefore, that  $B_{t+1} \geq B_t$  occurs if  $H_t + R_t B_t - C_t \equiv S'_t \geq B_t$ , i.e. if gross saving is equal to (greater than) the existing (expiring) stock of bonds.

The economy includes a central bank that exerts some leverage on the nominal interest rate  $i_t$  by trading bonds. Let us assume the following unconditional policy scheme:



- at the beginning of each  $t$  the central bank announces a target inflation rate  $\pi^*$  and pegs the nominal interest rate  $i_t$ ;
- all agents form their expected inflation rate,  $E_t\pi_{t+1}$ , consider their objective functions and constraints, and make their plans accordingly;
- exchanges occur in all markets, and  $[Y_t, \pi_t]$  are realized.

In other words, the central bank is allowed to behave as an intermediary that stands ready to buy and sell bonds at the real interest  $R_{t+1} = E_t[(1 + i_t)/(1 + \pi_{t+1})]$ . At the same time, a well-known issue in the theory of monetary policy arises: Is the policy stance  $[i_t, \pi^*]$  chosen by the central bank consistent with the economy’s intertemporal equilibrium? Here this problem will be addressed by backward reasoning. Let agents believe in the central bank’s inflation target,  $E_t\pi_{t+1} = \pi^*$ : then, the problem is the conditions such that  $\pi^*$  is indeed the rational expectation of the inflation rate, or else, the conditions such that the actual inflation rate does converge to  $\pi^*$  over time.<sup>9</sup> In this section, for the sake of analysis of the properties of the model, the nominal interest rate will be kept invariant. At this stage, this can be seen as the only relevant nominal rigidity in the economy. It is only instrumental to tracking the economy’s intertemporal path as households and firms save and invest at the given real rate  $R_{t+1}$  *vis-à-vis* the natural rate  $R^*$  that would ensure intertemporal equilibrium. It will be removed in the next section upon introducing a feedback rule of the central bank.

The first-order conditions for households’ and firms’ optimal plans include, respectively,

$$U'(C_t) = E_t[U'(C_{t+1})R_{t+1}/\Theta] \tag{7}$$

$$F'(K_t + 1) = R_{t+1} \tag{8}$$

The posited production and utility functions ensure that there exists an equilibrium between saving (demand for bonds) and investment (supply of bonds). We can thus consider the steady-state solution where the employment of labour is normalized to 1,  $L^* = 1$  ( $F'_L(1) = w$ ), and  $R_{t+1} = \Theta \equiv R^*$ ,  $F'_K(K^*) = R^*$ , so that for all  $t$ ,  $C_t = C^*$ ,  $B_t = K_t = K^*$ ,  $Y_t = Y^* = H^* + R^*K^*$ .

<sup>9</sup> In fact, this approach is also usual in policy games where the central bank moves first. Indeed, this is the origin of Wicksell’s intuition of the central bank as ‘manager of expectations’ highlighted by Woodford, and it is the key problem of monetary policy from the Wicksellian point of view. A cost of this approach is that it precludes analysis of endogenous expectation formation schemes that were deemed critical by Wicksell himself and no doubt represent a challenging aspect of ISI processes. An example is developed in Tamborini (2008). However, as stated from outset, the scope of this paper is limited to the first-stage, intertemporal equilibrium consistency problem of the central bank’s policy stance  $[i_t, \pi^*]$ .

Note that  $S'_t = I'_t = K^*$ , and once account is taken of capital replacement, net investment and saving are nil.<sup>10</sup> Hence  $R^*$  is the natural interest rate, and the Fisher equation,  $(1 + i_t) = R^*(1 + \pi^*)$ , should hold for all  $t$ .

The capital market is affected by a tiny imperfection in that the central bank has no direct information on the natural rate. Hence let us now consider any period  $t$  in which the central bank has announced  $[\pi^*, i_t]$  and it happens that  $R_{t+1} \neq R^*$ .<sup>11</sup> The model is fully worked out in Appendix A, and here I provide a brief non-technical rendition.

As long as  $R_{t+1} \neq R^*$ , *ceteris paribus*, there is excess investment/saving in the capital market: that is to say, the demand for bonds differs from supply. In order to maintain the peg, however, the central bank should be ready to fill any gap between demand and supply of bonds in the private sector (i.e. buy extra-bonds if  $I_t > S_t$  and sell extra-bonds if  $S_t > I_t$ ).<sup>12</sup> As a result, households and firms are allowed to save and invest as much as they want; however, the underlying consumption and production plans are not mutually consistent, resulting in present as well as future aggregate demand–supply imbalances. More precisely, the flaw lies in ‘wrong budgeting’ in that the real value of bonds in the economy does no longer match the real stock of capital. Hence trading at the false interest rate in the capital market propagates disequilibrium across markets and over time.

The next question is: if we want the economy to obey the principle of market clearing, how can the demand–supply imbalances be corrected? In words, the answer is that for a given ‘interest-rate gap’ at time  $t$ , measured by  $R_{t+1}/R^*$ , there should be a re-combination in the vectors of *present and future output and general price level* such that households’ ‘wrong budgeting’ is corrected *ex post*, and the actual trades in the output market are made mutually consistent. Technically speaking, these new vectors solve the intertemporal optimization problem of households and firms, given  $R_{t+1} \neq R^*$  and output market clearing at all dates. The thrust of the model is that, as a consequence of trading at a false interest rate at any point in time,

<sup>10</sup> Abstracting from technical progress or technical shocks, this is in fact a Sidrausky-type steady state, where the key allocational variable in the capital market is precisely the rate of intertemporal preferences in consumption,  $\Theta$ .

<sup>11</sup> Do private agents always know the true natural interest rate? This is an interesting and important question, but here it turns out to be less relevant than in a standard set-up with a single representative saver–investor. In fact, all the ISI model needs is that households optimize upon observing  $R_{t+1}/\Theta$  and that firms optimize *vis-à-vis*  $R_{t+1}/F'_K(K_{t+1})$ . Each agent need not know whether  $R_{t+1}$  is the natural rate or not.

<sup>12</sup> As recalled previously, this seems in fact Wicksell’s own idea of bank intermediation as the origin of ISI. In the context of modern theories of imperfect capital markets, this is a scheme of ‘trading at false price’ due to imperfect information of intermediaries (more on this point in Tamborini, 2008). Note that this scheme does *not* imply rationing.

all markets do clear continuously, but *both current and future* output and inflation rates will differ from those ( $Y^*$ ,  $\pi^*$ ) that would result in the intertemporal equilibrium associated with the natural interest rate  $R^*$ . The assumed functional forms yield simple and manageable results for the time sequence of these ‘gaps’ of output  $\hat{Y}_{t+s} \equiv Y_{t+s}/Y^*$  and inflation  $\hat{\Pi}_{t+s} \equiv (1 + \pi_{t+s})/(1 + \pi^*)$  (see Appendix A). These can also be easily transformed into log-linear equations directly comparable with the standard NNS model. The resulting ‘ISI model’ is the following:

$$\hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{i}_t \tag{9}$$

$$\hat{\pi}_{t+1} = \beta \hat{y}_{t+1} \tag{10}$$

with  $\hat{y}_t \equiv \ln \hat{Y}_t \approx y_t - y^*$ ,  $\hat{\pi}_{t+1} \equiv \ln \hat{\Pi}_{t+1} \approx \pi_{t+1} - \pi^*$ ,  $\hat{i}_t \equiv i_t - i^*$ .

Consider an interest-rate gap at time  $t$ ,  $R_{t+1}/R^*$ . Then,  $\ln R_{t+1}/R^* \approx i_t - \pi^* - r^*$ . The variable  $i^* \equiv r^* + \pi^*$  is often called the ‘non-accelerating inflation rate of interest’ (NAIRI), and can be used as direct benchmark for the nominal rate  $i_t$  (of course,  $i_t = i^*$  is exactly equivalent to the equality between the market real interest rate and the natural one considered so far).

Given  $\hat{i}_t \neq 0$ , output gaps will open up at time  $t$  and  $t + 1$ : equation (9) describes output dynamics *off* the IS schedule that would result in the natural-rate intertemporal equilibrium. The output gap in  $t + 1$  is measurable as a proportion  $-\alpha$  of the interest-rate gap, and a proportion  $\rho$  of the uncorrected output gap, as of time  $t$ .<sup>13</sup> It is worth noting that (9) generates time series of output gaps that, *ex post*, display two main features: (a) *dependence on the lagged value* of interest-rate gaps, and (b) some degree of *serial correlation or ‘inertia’*. These two features mark a substantial difference with the standard NNS ‘forward-looking’ formulation of the IS equation, where the current gap between the market and the natural interest rate only impinges upon the *current* output gap, whereas the *future* (expected) output gaps only depend on the *future* (expected) interest-rate gaps.<sup>14</sup> Yet a dynamic structure such as (9) is consistent with recurrent empirical estimates of IS equations (see, for example, Laubach and Williams, 2003; Caresma *et al.*,

<sup>13</sup> This is, in fact, ‘spurious’ correlation due to the fact that both  $\hat{Y}_t$  and  $\hat{Y}_{t+1}$  depend on the common factor  $R_{t+1}/R^*$  (see Appendix A).

<sup>14</sup> The standard log-linearized formulation of the IS equation is

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r^*)$$

where  $x_t$  is a measure of the output gap (the difference between current output and the flex-price DSGE level of output), and  $\sigma$  is the intertemporal elasticity of substitution of private expenditure. Clearly,  $x_t$  and  $E_t x_{t+1}$  are serially independent.

2005; Garnier and Wilhelmsen, 2005; Orphanides and Williams, 2006). These empirical regularities are not easily accommodated in the NNS model unless it is filled with additional *ad hoc* ‘frictions’, usually interpreted as the outcome of ‘backward-looking’ behaviour of agents.<sup>15</sup> Here they result directly from forward-looking agents *and* the fact that intertemporal disequilibrium connects each period’s variables in a way that is not captured by the NNS model.

Equation (10) describes the associated price/output dynamics *off* the AS curve associated with the natural-rate intertemporal equilibrium, where the structural parameter  $\beta = alb$ , determined by the production-function parameters, determines the deviation of current inflation from the expected inflation rate necessary for competitive firms to supply one unit of profit-maximizing output above/below potential. There are two key points in this equation that distinguish it from the AS of the NNS model, apart from firms being price takers. First, price stickiness is not an issue: the price level changes as much as is necessary to equate demand and supply at the given expected inflation rate. Of course, different microeconomic assumptions and values of the parameter  $\beta$  may lead to different quantitative results and different combinations of price-quantity changes (see also section 3.2 below). Second, excess inflation is, however, a disequilibrium phenomenon in the expectational sense. As long as, for the given pair  $[i_t, \pi^*]$  set by the central bank, there exist interest-rate gaps, and hence output gaps, for the goods market to clear the economy *must* be off the price level path indicated by  $\pi^*$ .<sup>16</sup>

System (9)–(10) is a non-homogeneous system of two first-order difference equations in the three gaps  $[\hat{y}_{t+1}, \hat{\pi}_{t+1}, \hat{i}_t]$ . Consequently, we have the following proposition.

*Proposition 1:* A permanent interest-rate gap determines permanent output and inflation gaps. Conversely, the output and inflation gaps are nil only if the interest-rate gap is also nil.

*Proof:* Appendix B.

For example, suppose that  $\hat{i}_0 < 0$ . The model reproduces a prototypical Wicksellian ‘cumulative process’ (e.g. Wicksell, 1898a, pp. 77 ff.). If the system starts at a constant price level ( $\pi_0 = \pi^* = 0$ ), the general price level

<sup>15</sup> Examples of inertial market frictions are provided in Woodford (2003, chapter 5), Aghion *et al.* (2004, Part I). Chiarella *et al.* (2005, chapters 1 and 8) offer a thorough discussion.

<sup>16</sup> This was in fact the fundamental idea behind Wicksell’s ‘cumulative processes’ in the price level, that is to say, the symptom that excess investment (excess saving) is being accommodated at the wrong market rate and the economy driven out of the intertemporal equilibrium path (e.g. Wicksell, 1898a, pp. 75 ff.)

will rise indefinitely. If  $\rho \in [0, 1]$  it will rise *at a constant rate*.<sup>17</sup> For any  $\pi_0 = \pi^* > 0$ , the result is that the rate of change of the price level will deviate from  $\pi^*$  forever. Although the market clears, the result is not an intertemporal equilibrium. If instead  $\hat{i}_0 > 0$ , the model reproduces a Keynesian story, with output settling down below potential. The model, however, corrects the simplistic view that Wicksellian processes affect only the price level whereas Keynesian processes affect only output.

There are three essential points and messages for monetary policy. First, as anticipated above, excess inflation is a disequilibrium phenomenon in the expectational sense. Second, changes in the price level alone, no matter how large or quick they may be, are unable to drive the economy onto the right path as long as they do not affect the nominal interest rate. Third, and consequently, the Wicksellian legacy that setting an inflation target and pegging the nominal interest rate unconditionally may not be a consistent policy is confirmed (unless the peg is by fluke exactly the NAIRI: Leijonhufvud, 1981; McCallum, 1986).

### 3. MONETARY POLICY

The previous conclusions elicit a conception of monetary policy as a visible hand possibly keeping the interest rate on the right track. The core of the modern theory of monetary policy, in fact, concerns the definition of a class of interest-rate rules that support a determinate rational-expectations equilibrium consistent with the targets of inflation and output. It has been demonstrated that the Taylor rule, under some conditions, belongs to this class of rules (Woodford, 2003). This section re-examines the Taylor rule and its properties in the context of the ISI model presented above.<sup>18</sup>

#### 3.1 *The non-optimality of the optimal Taylor rule with imperfect information*

In the NNS, increasing emphasis has been placed on the design of optimal monetary policy rules with reference to some welfare benchmark of the economy. Generally, it is shown that Taylor-type interest-rate reaction

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<sup>17</sup> More complex dynamic patterns may arise from different expectational mechanisms than the one assumed here. On this point, which is however not crucial here, see, for example, Chiarella *et al.* (2005), Tamborini (2008).

<sup>18</sup> The relevant models with proofs are developed in Appendix B.

functions can be derived from a central bank's optimal control problem such as the following (see, for example, Clarida *et al.*, 1999):

$$\max L_t = - \sum_{s=0}^{\infty} \frac{1}{2} [(y_{t+s} - y^*)^2 + \eta(\pi_{t+s} - \pi^*)^2] \quad (11)$$

$$\text{s.t. } (\pi_t - \pi^*) = \beta(y_t - y^*)$$

In this formulation, the central banker aims at minimizing the absolute value of the inflation and output gaps along the dynamic path of the system, with  $\eta$  measuring the relative degree of inflation (variation) aversion. By applying the same procedure as Clarida *et al.* (1999), one obtains

$$i_t = i^* + \phi(y_t - y^*) + \gamma(E_t[\pi_{t+1}|i_{t-1}] - \pi^*) \quad (12)$$

with  $\phi = \rho/\alpha$  and  $\gamma = \eta\beta/\alpha$ .

The most important features of this formulation are the following. First, there is an explicit target for the interest rate, which is just the NAIRI  $i^*$ . Second, the coefficients  $\phi$  and  $\gamma$  are not arbitrary, but are determined by the structural parameters of the economy and of the central bank's loss function. Third, the informational inflation rate used to assess the cyclical position of the economy is  $E_t[\pi_{t+1}|i_{t-1}]$ , the forecast of the inflation rate in the absence of policy interventions (see also Svensson, 1997; Woodford, 2003, chapter 8).

Let us first examine the dynamic properties of the economy under rule (12). If we shift the term  $i^*$  to the left-hand side, use the structural model to solve for  $E_t[\pi_{t+1}|i_{t-1}]$ , and move one period forward, we can add this equation to (9)–(10) to obtain a homogeneous system in the three gaps  $[\hat{y}_{t+1}, \hat{\pi}_{t+1}, \hat{i}_{t+1}]$ . It is a general property of homogenous systems that a steady-state solution with zero gaps exists. Therefore, since the optimal Taylor rule is simply a homogenous transformation of the interest-rate variable, it is sufficient to grant the existence of a zero-gaps steady state. In other words: the optimal Taylor rule prescribes the central bank to target the NAIRI; as a result,  $\hat{i}_t = 0$  in all  $t$ , whatever the value of the natural rate, which implies  $[\hat{y}_{t+1}, \hat{\pi}_{t+1}] = 0$  in all  $t$ . Therefore, the economy is constantly kept in the zero-gaps steady state and no ISI process would ever arise.

On the one hand, this result may be interpreted in the sense that the optimal Taylor rule gives the central bank the right prescription and indicator in order to prevent ISI. On the other hand, the same result may be regarded as trivial or empty; in fact, in order to target the NAIRI the central bank should know the true natural interest rate at any point in time, but ISI may arise just because central banks do not have direct information about the natural rate. Hence the relevant question is whether the optimal Taylor rule

is also ‘robust’, i.e. whether it can lead the economy to the zero-gaps steady state even though the informational requirement of the central bank is relaxed and ISI unfold. Let us thus turn to the assumption made in our basic model, namely that the central bank has no direct information on the natural interest rate, or let  $i^*$  be replaced by  $\tilde{i}$ . The following proposition holds:

*Proposition 2:* The adoption of an interest-rate target in the Taylor rule implies that, if the central bank has no direct information on the natural rate and the interest-rate target is wrong, the system will never converge to the zero-gaps steady state.

*Proof:* Appendix B.

The intuition is that a central bank with a wrong NAIRI in the Taylor rule, which implies that at any point in time it may be the case that  $\hat{i}_t \neq 0$ , brings the system back to the case with an exogenously pegged nominal interest rate that is not consistent with the natural-rate intertemporal equilibrium.

The conclusion is therefore that the optimal Taylor rule is not robust in the face of a small informational error of the central bank. A consistent implication might be that, in all relevant cases, the optimal Taylor rule ought to be switched off. Yet a rule that is switched off in particular circumstances is no longer a rule in the sense of the modern normative theory of central banking. Moreover, if the central bank is misinformed about the natural interest rate, it is also unlikely to realize that the rule should be switched off. The argument that the central bank will sooner or later realize that the economy displays permanent output and inflation gaps is not so obvious. For in reality output and inflation are hit by their own shocks to which the central bank will respond by shifts of  $i_t$  around  $\tilde{i}$ , so that it may not be so easy to detect that output and inflation are not on their intertemporal equilibrium trend.<sup>19</sup>

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<sup>19</sup> It has been argued that under suitable learning rules (and stochastic processes), the central bank can learn the true value of the natural rate(s) and update the rule in real time, with the process converging towards a ‘self-confirming’ equilibrium which generates optimal policy responses (e.g. Sims, 1998; Sargent, 1999; Primiceri, 2006). The estimates and simulations of the NNS model with mismeasurement of natural rates and central bank’s learning presented by Orphanides and Williams (2002, 2008) do not lend empirical support to the convergence prediction. Instead, the simulation presented by Primiceri (2006) concerning the post-war learning process of the Fed about the parameters of the US Phillips curve suggests that, in the long run, the process has eventually been successful. However, the ‘long run’ covers the 15 years between the late 1960s and the early 1980s resulting in the American ‘Great Inflation’. Therefore, it is not clear whether this evidence reduces, or increases, concerns about attempts to pursue the natural rate(s) in the conduct of monetary policy.

Another argument in the same vein will be discussed at the end of this section.

Poor stabilization performance, whether it be towards excess inflation or deflation, may arise not because of the lack of the 'right' rule but because of the lack of the 'right' information for that rule. This conclusion is shared by the literature on monetary policy under imperfect information on 'natural rates' (of output, unemployment etc.: e.g. Orphanides and Williams, 2002, 2006, 2008; Primiceri, 2006). Concern about risks involved in monetary policy rules with mismeasurement of the natural interest rate largely prevailed in the past, starting from Wicksell himself, to whom one may add Keynes, Friedman, Greenspan (Orphanides, 2006). Scepticism concerning the practical use of the natural interest rate for monetary policy is now mounting again (see, for example, Laubach and Williams, 2003; Caresma *et al.*, 2005; D'Amato, 2005; Garnier and Wilhelmssen, 2005).

### 3.2 Adaptive Taylor rules

The theoretical as well as operational problems surfacing around the informational requirements of optimal Taylor rules have paved the way for research of more parsimonious, 'robust' rules that dispense with 'natural rates' altogether; these may not match theoretical criteria of optimality but allow for reliable stabilization policy (e.g. Orphanides and Williams, 2002, 2006, 2008).

In this perspective, a hint may be provided by an interpretation of Wicksell's thought according to which he saw the natural rate, not as a variable observable by anyone in the system, but as a 'hidden attractor' of the system where the latter is driven by agents reacting to observable market signals. Wicksell also thought that cumulative processes might be *self-correcting*, since banks would simply raise their lending rates in accordance with the acceleration of prices if only they and their client entrepreneurs had expectations anchored to the 'normal' price level.<sup>20</sup> If one conceives the Taylor rule as a reduced form of the behaviour of the banking system as a whole anchored to the target inflation rate  $\pi^*$ , one may examine this

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<sup>20</sup> At the time when Wicksell was writing, there was already clear evidence that nominal interest rates would tend to move together with the general price level (see, for example, the diagrams in Wicksell, 1898a)—a phenomenon that was later labelled 'Gibson paradox' by Keynes. Wicksell argued that this phenomenon would not contradict his theory, but that it could instead be explained as the ongoing adjustment process of nominal interest rates towards a new level consistent with the steady-state level of prices.



mechanism by replacing the NAIRI with the interest rate from the previous period, and use the current inflation rate as informational input. One thus obtains an ‘adaptive’ Taylor rule, meaning that the central bank simply lowers or raises the nominal interest rate according to the output and inflation gaps, with no explicit reference to the natural interest rate.

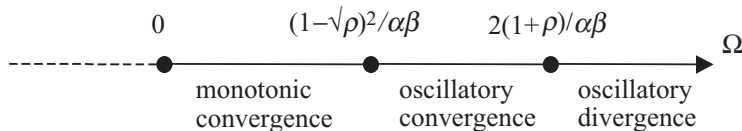
$$i_t = i_{t-1} + \phi(y_t - y^*) + \gamma(\pi_t - \pi^*) \tag{13}$$

This specification may be further specialized in various directions. Setting  $\phi = 0$  captures Wicksell’s own view of the monetary policy rule with no reference to output at all. This would also be the simplest possible format for a pure inflation-targeting rule. Setting  $y^* = y_{t-1}$ , one obtains the ‘first difference’ specification put forward by Orphanides and Williams (2002), which retains the signal provided by output dynamics but dispenses with information about the ‘natural rate of output’. The change in the nominal interest rate dictated by the output and inflation gaps may also be ‘smoothed’, in which case a less than unit parameter should be appended to the lagged term  $i_{t-1}$ . These variations on the theme will not be developed here, given that they (qualitatively) share the same dynamic properties that we are going to study for our simpler adaptive Taylor rule (13).

Subtracting  $i^*$  from both sides of equation (13), writing it one period forward and adding it to system (9)–(10), we again obtain a homogeneous system of three first-order difference equations in the three gaps  $[\hat{y}_{t+1}, \hat{\pi}_{t+1}, \hat{i}_{t+1}]$ . Therefore, an adaptive Taylor rule may well support a determinate rational-expectations steady state with zero gaps.

As to convergence and stability, the system now displays a richer dynamic structure, and a different parametric choice set-up, with respect to the one with the optimal Taylor rule. As before, let us first consider the main properties of this system.

*Proposition 3:* Dynamic regimes: The system presents different dynamic regimes according to the critical values of the compound coefficient  $\Omega \equiv \phi/\beta + \gamma$  displayed below:



*Proposition 4:* Boundedness of the Taylor-rule coefficients: The output and inflation coefficients of the Taylor rule cannot be chosen independently. The scope for the choice of the two policy parameters is bounded by the requirements of system’s stability and regime choice.

*Proof:* Appendix B.

Proposition 3 highlights the key role played by the compound coefficient  $\Omega$ , which measures the compound response of the interest rate to one unit of inflation gap. The compound parameter  $\Omega$ , and *not its composition*, matters because the interest rate reacts to both inflation and output gaps that are interdependent via the AS function, and because, in this case of an ISI that perturbs only aggregate demand, the output and inflation gaps are positively correlated. Hence, stabilizing inflation is also to stabilize output—or, for that matter, the other way round—and the mechanical application of the Taylor rule implies that the interest rate reacts twice to the same signal.<sup>21</sup> That is why, as  $\Omega$  increases, the system switches from monotonic to oscillatory stability, and from stability to instability.

This phenomenon immediately leads to Proposition 4. The boundedness of the Taylor-rule coefficients, and hence of the interest-rate responsiveness to inflation gaps, is an important characteristic of the ISI model that deserves careful consideration. In fact, this issue involves the so-called ‘Taylor principle’, which plays a key role in the NNS. Woodford’s (2003) version of the Taylor principle refers precisely to the compound parameter  $\Omega$  (pp. 253–254), and he establishes that  $\Omega > 1$  must hold. Hence, Woodford sets a *lower* bound, whereas we have an *upper* bound for stability. The latter may admit or not admit  $\Omega > 1$  depending on the values of  $\rho$ ,  $\alpha$ ,  $\beta$ .

This major difference is closely related to the structural difference in the ISI equations pointed out in section 2, namely the co-movement of future with present output gaps in response to present interest-rate gaps as long as the economy faces imbalances in the capital market. The, apparently sound, popular argument that in order to curb inflation the central bank should overreact on the nominal interest rate to the extent necessary for the market real interest rate to rise does not take into account that the future inflation gaps will be affected directly. By overreacting to current inflation gaps, the interest rate may overshoot and push the economy onto a divergent path. It is therefore intuitive that higher  $\alpha$  and  $\beta$  call for slighter interest-rate adjustments; for these parameters transmit interest-rate impulses to output and eventually inflation over time. Note in particular the role played by the AS parameter  $\beta$ . When this parameter is large, small corrections of output gaps induce large effects on inflation gaps; hence the system’s stability requires gentle interventions on the interest rate.

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<sup>21</sup> Indeed, this is also the case with the standard NNS model when only aggregate demand shocks are considered.

Therefore, as far as stability analysis of monetary policy under ISI is concerned, two conclusions can be drawn at this stage. First, once the role of the Taylor-rule coefficients has been properly understood and controlled, adaptive Taylor rules may have desirable out-of-equilibrium dynamic properties with relatively limited information (namely no direct reference to the NAIRI). Second, in this perspective, the Taylor principle is not necessary in general for stability, and it may be harmful in specific conditions. High sensitivity by the central bank towards inflation gaps is only justified to the extent that inflation is *not* highly responsive to output gaps because in this case disequilibrium gaps (albeit small) would persist for a long time (see also section 3.4 below).

### 3.3 *Small gaps versus smooth paths*

A subsequent interesting issue concerns the fact that, as argued above, the processes under consideration do not generate any inflation/output trade-off for the central bank, so that one wonders whether there is any other scope for the choice of the Taylor-rule coefficients. The possibility of different dynamic regimes raises the choice problem of parameters in a way that is seldom scrutinized in the empirical literature on the Taylor rule.

Let us retain the sum of square gaps as a measure of losses in the central bank's objective function. Then, two properties of the adaptive rule (13) should be noted. First, the dynamic paths of output and inflation, and the ensuing respective total losses, are univocally determined by the value of  $\Omega$ , so that they cannot be traded-off by shifting the weights of the respective coefficients. Second, the total loss of each variable decreases as  $\Omega$  increases.<sup>22</sup> Therefore, an adaptive central banker may wish to set  $\Omega$  as *large* as possible regardless of the relative weights of the two coefficients. However, as we have seen, the largest value of  $\Omega$  consistent with stability generates oscillatory convergence. Hence if any trade-off arises in this set-up, it concerns the choice of *small gaps versus smooth paths* in the process as a whole.<sup>23</sup>

Although it is reasonable to presume that central bankers may not be willing to set the economy on an oscillatory path, albeit convergent, the linear-quadratic format of the standard loss function prevents any treatment of the preference for monotonic versus oscillatory paths. To gauge the practical importance of this issue, it is convenient to look at some empirical

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<sup>22</sup> Proof available on request.

<sup>23</sup> 'Fast' and 'small' oscillatory convergence may yield a lower value of the loss function than does 'slow' and 'wide' monotonic convergence.

estimates of the relevant structural parameters  $\rho$ ,  $\alpha$ ,  $\beta$ . Table 1 reports possible values of these parameters taken from the empirical literature.

The second to fourth columns report values obtained by means of direct estimates of IS–AS equations such as (9) and (10). The fifth column is borrowed from Rotemberg and Woodford's (1997) calibration of their own NNS model for the US economy. The fifth makes reference to the ISI model and it is added solely for the sake of comparison between possibly different economic structures. Considering the model specification obtained in Appendix A, given that we can take a parameter  $\alpha$  of, say, 0.15 as an empirical regularity, and  $a = 0.4$ ,  $b = 0.6$  as representative figures of income shares in industrialized countries, we can exploit the model's restrictions on the parameters to 'calibrate' the others, namely  $\rho = a - \alpha b = 0.31$ , and  $\beta = a/b = 0.67$ .

In the first place, I have used the values in the table to compute the stability upper bound of the Taylor rule (the maximum value of  $\Omega$ ), taking monotonic convergence as the reference case. The figures exemplify that, under all parameter sets, the central banker can comply with the Taylor principle ( $\Omega > 1$ ), if he or she so wishes, but the ISI model sets an upper bound to  $\Omega$  substantially more binding than the others. In the second place, I have also plotted in figure 1 the stability frontiers of the Taylor-rule coefficients, i.e. the relationship between the coefficients  $\phi$  and  $\gamma$  consistent with the stability upper bound of  $\Omega$ . The lines represent the maximum output-gap coefficient as a function of the inflation-gap coefficient,<sup>24</sup> and the dots identify some credited estimates of the Taylor-rule coefficients. Clearly, the output/inflation frontier imposed by the ISI model is much steeper than the others.

A glance at the table suggests that these differences between our model economy and the 'real' economies should ultimately be due to the single parameter whose value differs the most, namely  $\beta$ . As a matter of fact, the ISI model portrays a competitive, flex-price economy, whereas the others in the table capture various goods market imperfections reflected in the low AS parameter. As explained above, high inflation-to-output response requires only slight corrections in the interest rate and hence less global sensitivity of the Taylor rule. In contrast, small values of  $\beta$  inject substantial slackness into the system and justify stronger sensitivity of the nominal interest rate to the gaps. Price stickiness is not the cornerstone of the ISI approach, but this phenomenon obviously matters in practice in order to design an efficient interest-rate rule.

Nevertheless, the existence of an upper bound to the Taylor-rule coefficients (instead of a lower bound) remains a key implication of the ISI model of general value. Consider, for instance, the simulation of the ISI model with

<sup>24</sup> That is to say,  $\phi = (1 - \sqrt{\rho})^2 / \alpha - \beta\gamma$ .

Table 1. Sets of parameters for the stability upper bound of the Taylor rule (monotonic convergence)

	Laubach–Williams (2003) US	Garnier–Wilhelmsen (2005) EU	Garnier–Wilhelmsen (2005) US	Rotemberg–Woodford (1997) US	ISI model
$\rho$	0.45 <sup>a</sup>	0.42 <sup>b</sup>	0.45 <sup>b</sup>	n.a. <sup>c</sup>	0.31
$\alpha$	0.10 <sup>d</sup>	0.06	0.18	0.16	0.15
$\beta$	0.05 <sup>d</sup>	0.05	0.10	0.02	0.67
Upper bound <sup>e</sup>	20.82	41.28	6.02	33.86	1.96

<sup>a</sup>Average value of two-year lags across different estimation techniques.

<sup>b</sup>Average value of two-year lags.

<sup>c</sup>Set equal to 0.40 in subsequent computations.

<sup>d</sup>Average value across different estimation techniques.

<sup>e</sup>Maximum compound response  $\Omega$  of the Taylor rule to 1 point of inflation gap  $(1 - \sqrt{\rho})^2 / (\alpha\beta)$ .

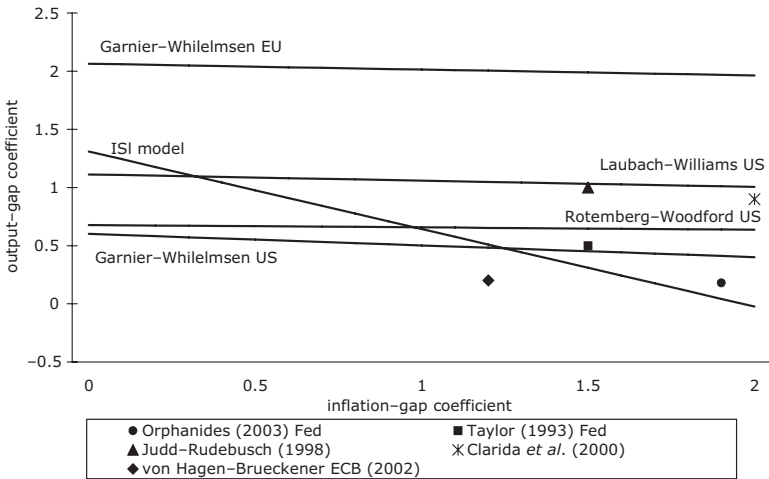


Figure 1. Monotonic stability frontiers of the Taylor rule.

the adaptive Taylor rule reproduced in figure 2. For convenience, the variables (output, inflation and nominal interest rate) are represented in levels (not in gaps) and are normalized to 100 basis points. The simulation consists of an unobserved initial negative interest-rate gap (10 basis points, the true NAIRI is 110). The parameter values are those given in table 1. At outset, the consequence is excess output and inflation.

The figure reproduces the dynamic paths of the economy with two specifications of the adaptive Taylor rule. In case A the rule's coefficients have been chosen beyond the monotonic stability frontier, and they satisfy the Taylor principle, i.e.  $\phi = 1.0$ ,  $\gamma = 2.0$  ( $\Omega = 3.5$ ). The system converges to the zero-gaps steady state with (slight) oscillations. In case B the rule's coefficients have been set below the stability frontier and do not comply with the Taylor principle, i.e.  $\phi = 0.2$ ,  $\gamma = 0.6$  ( $\Omega = 0.9$ ). Also this rule (even with no Taylor principle) drives all the variables on their respective targets along a smooth path. The interesting difference between the two cases lies in the respective total losses (the square sums of the gaps that appear in parentheses for each variable). As explained above, case B (low  $\Omega$ ) delivers larger gaps with smoother paths than does case A (high  $\Omega$ ), which delivers the reverse. On the other hand, any other combination of  $\phi$  and  $\gamma$  yielding the same values of  $\Omega$  would also yield exactly the same dynamic profiles in the two respective cases.

Going back to real data, it should be noted that not all estimated Taylor rules in figure 1 satisfy all estimated 'realistic' monotonic stability frontiers. For instance, Taylor's (1993) original estimate for the Fed would (slightly)

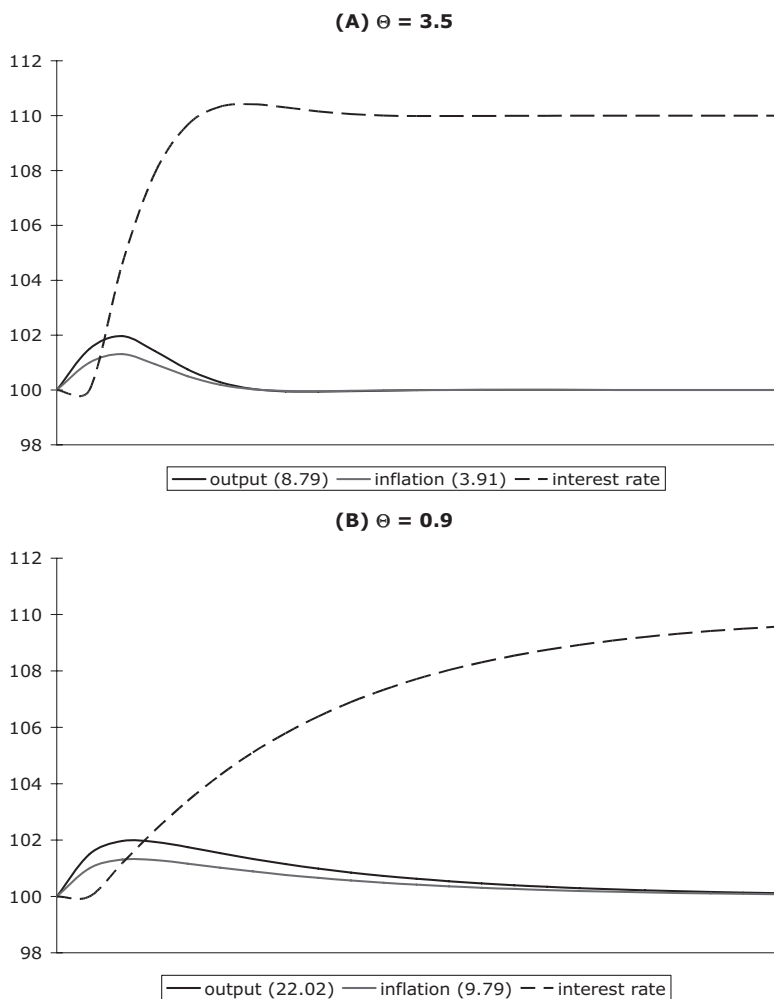


Figure 2. Simulation of a permanent increase in the natural interest rate (10 basis points) with adaptive Taylor rule.

violate the frontier with the US parameters found by Garnier and Wilhelmsen (2005), while subsequent estimates by Judd and Rudebusch (1998) and Clarida *et al.* (2000) would also violate the frontier set by the Rotemberg and Woodford (1997) data. A glance at the marked oscillation in the federal funds rate between 2001 and 2008 (see figure 3) suggests that the question of the extent of the small gaps/smooth paths alternative perceived by central banks warrants further investigation.

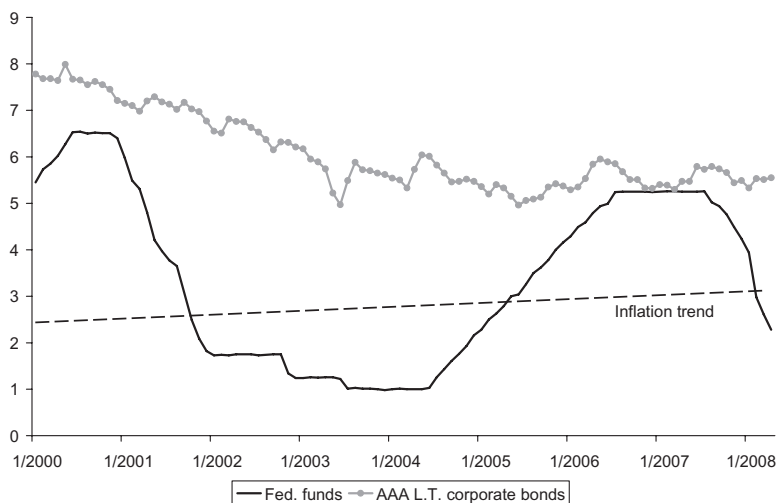


Figure 3. US federal funds rate and yields on AAA long-term (L.T.) corporate bonds, 2000:1–2008:4 (monthly data).

Source: FRED online database, Federal Reserve of St. Louis.

### 3.4 The 'missing inflation' puzzle

It is fair to conclude with the limitations of adaptive rules as well. The most interesting one concerns the key signalling role of excess inflation. It is clear that the entire adjustment process hinges on the fact that inflation (typically, consumer price inflation) does respond to output gaps (excess demand) to an extent that should be deemed significant by the central bank. This fact entails that the combination of preference for smooth monetary policy (small  $\Omega$ ) and low cyclical sensitivity of inflation (small  $\beta$ ) may determine an extremely slow, if not flat, adjustment path of the economy *vis-à-vis* the development of ISI.<sup>25</sup> In this context—to borrow from Borio and Lowe (2002)—monetary policy may let financial imbalances mount up, which remain disguised in a benign economic environment. 'Benign' means low inflation and sustained economic activity, which is precisely the scenario along the adjustment path

<sup>25</sup> Recent analyses of these phenomena may provide further arguments. First, the 'flattening' of the Phillips curve (i.e. a fall in our parameter  $\beta$ ) has been largely documented, although not conclusively explained (see, for example, Mishkin, 2008). Second, Borio and Lowe (2002) put forward some important reasons why larger financial imbalances, sustained economic activity and moderate inflation may reinforce each other endogenously. The basic model presented in this paper does not take these complex feedback effects into account, but it does imply that as long as firms are allowed to invest in excess of saving, the production capacity in the economy increases, and a stronger activity level may be sustained with less inflationary pressure.



of the economy indicated by the ISI model when the parameter  $\beta$  is as low as can be seen from estimates in table 1.

As a simple experiment, let us consider the latest estimates of parameters for the US economy in table 1 (Garnier and Wilhelmsen, 2005), and those of the Fed's Taylor rule in figure 1 (Orphanides, 2003:  $\phi = 0.18$ ,  $\gamma = 1.9$ ). As shown by the figure, these parameter values are consistent with monotonic stability: the implied value of  $\Omega$  is 3.7 *vis-à-vis* an upper bound of 6.02. Note that this value of  $\Omega$  is of the same order of magnitude as the one used in our previous simulation of the ISI model, case A. Yet in that context (large value of  $\beta$ )  $\Omega$  resulted larger than the upper bound. As a consequence, if we now repeat the same simulation as before (a negative 10-basis-point interest-rate gap) with this new set of parameters, we obtain that convergence is much slower than in the previous case A. A measure of this is given by the square sum of gaps, which is now 51.2 (8.8 in the previous case A) for output, and 0.5 (3.9 in the previous case A) for inflation. The extraordinarily good performance in terms of price stability is entirely due to the structural low sensitivity of inflation. The other side of the coin is that output gaps, *which indicate the ongoing ISI process*, persist for much longer and reach a much larger cumulated value.

This seemingly golden age may mislead the monetary authority (and the public opinion at large) if it is not tested against the ISI hypothesis and is instead mistaken as a sustainable intertemporal equilibrium. This result may also lead to the more radical criticism that (consumer price) inflation targeting *per se*, or the use of this measure of inflation as a 'catch-all' cyclical indicator *à la* Bernanke and Gertler (2001), may be misleading in the event of ISI processes, so that broader measures or alternative indicators for monetary policy are advocated (e.g. Cecchetti *et al.*, 2000a; Borio and Lowe, 2002; Crockett, 2003; Leijonhufvud, 2008).<sup>26</sup>

If the problem is *limited* to the structural low sensitivity of inflation, an immediate alternative suggested by the ISI model itself is that output gaps, instead of inflation gaps, convey stronger signals that a disequilibrium

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<sup>26</sup> In contrast, Bean (2003) has argued that 'flexible', forward-looking, inflation targets are enough to control for the development of financial imbalances. However, Bean's point rests on the traditional assumption that 'significant financial instability will also have a significant impact on activity and inflation' (p. 18). Hence it is not clear how longer-run forecasts of the future developments of economic activity may overcome the 'missing inflation' problem if this is due to a low  $\beta$  parameter. For instance, if in the simulation of the ISI process with the Garnier–Wilhelmsen–Orphanides US parameter values, we introduce the one period baseline expected inflation rate, instead of the current rate, into our adaptive Taylor rule, the cumulative square sum of output gaps reduces from 51.2 to 44.4, which however remains a remarkably large figure compared with the simulation with a high  $\beta$  value.

process is under way. Adding more weight to output gaps, *to the extent that it raises the value of  $\Omega$* , may lead to a faster correction of the problem. Yet this simplistic solution cannot be taken at face value. First, the small-gaps/smooth-paths trade-off created by ISI processes is still there. Second, detecting output gaps correctly is by no means simpler than detecting where the true NAIRI is; measurement errors may equally destabilize the system.<sup>27</sup> Relatedly, it may be difficult for a central bank to explain that a tight monetary restriction is necessary when economic activity is high and inflation is low. Therefore, from the point of view of the ISI approach, the search for a broader set of direct indicators of financial imbalances seems necessary.

#### 4. CONCLUSIONS

Recurrent episodes of financial instability in most developed countries have forcefully put the case for extending macroeconomics and monetary-policy theory beyond the DSGE methodology of the NNS, within which these phenomena are hardly conceivable. The aim of this paper has been to take a first step forward in the analysis of monetary policy in the context of ISI, that is to say, the gene of intertemporal disequilibrium processes and of various manifestations of financial instability.

Interest-rate management has placed monetary policy at the very core of the investment–saving regulating mechanism, yet central banks can hardly collect all the relevant information about the determinants of investment–saving equilibrium. Thus, misalignment of the nominal interest rate with its intertemporal equilibrium value (NAIRI) is a likely phenomenon giving rise to ISI. The thrust of the model presented here is that ISI trigger *disequilibrium* business cycles with endogenous real as well as nominal effects. These processes persist as long as the original misalignment of the nominal interest rate persists. Wage–price stickiness is not the only problem, wage–price flexibility is not the only solution. ISI, by way of forward-looking agents' allocations, transmit the effects of present interest-rate gaps to present as well as future output gaps and hence inflation gaps. As a result, these variables display endogenously the autocorrelated dynamic structure that is typically observed in the data, while the forward spillover effect of interest-rate gaps considerably modifies the dynamic properties of the system with respect to the NNS standard model.

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<sup>27</sup> On the other hand, as recalled at the beginning, the output gap with respect to potential output can be replaced by the observed growth rate. The ensuing dynamic properties of the system are closer to those presented here the smaller is the product of parameters  $\phi\alpha$ .

As for monetary policy, main implications can be summarized as follows.

- Interest-rate feedback rules may operate to correct ISI. The critical element that eventually determines whether a rule is good or bad is information. Rules relying upon an explicit interest-rate target and requiring timely and precise information about the natural interest rate are hazardous in that wrong information may seriously destabilize the system.
- In general, adaptive rules, using step-by-step adjustments of the interest rate *vis-à-vis* observable conditions in the economy with no explicit interest-rate target, are preferable in that they produce adjustment paths that are generally slower but safer.
- Under ISI no trade-off arises between stabilizing output or inflation. The emphasis placed on rules that optimize the output–inflation trade-off may overlook more compelling requirements of (possibly monotonic) convergence and stability.
- In this perspective, the choice of the Taylor-rule coefficients also needs careful scrutiny. The Taylor principle is not necessary, and may be harmful, for stability. Contrary to the NNS intertemporal equilibrium dynamics, in the course of out-of-equilibrium dynamics monetary policy should have a bounded reaction to inflation and output gaps because of the inbuilt reactivity of the present *and future state* of the economy to interest-rate gaps.
- In this context, however, central banks may face a trade-off, not between inflation and output control, but between small gaps and smooth paths in the adjustment process. The preference for smooth paths entails a bounded reaction of the central bank to inflation and output gaps, and longer persistence of imbalances.

These findings suggest that an apparently consistent, orthodox monetary policy may be undermined by two major faults in the face of ISI. The first possibility is that an intertemporal disequilibrium process is mistaken for an equilibrium one (little is done because everything looks fine). The second is that inflation targeting *per se* may be ineffective. The ISI model indicates that a critical role is played by the cyclical sensitivity of inflation. If this parameter is very low—as indicated by many empirical studies—identifying and correcting ISI via inflation gaps may become quite difficult, and the ongoing disequilibrium process may persist for a long time in a seemingly golden age of low inflation and high economic activity. As far as the open debate on alternative indicators is concerned, the ISI model suggests that output gaps, instead of inflation gaps, may provide stronger appropriate signals. Yet this simplistic alternative may also have serious limits, so that the search for direct indicators of financial imbalances is to be pursued more forcefully.

APPENDIX A: DETERMINATION OF THE RELATIONSHIP BETWEEN INTEREST-RATE GAPS, OUTPUT GAPS AND INFLATION GAPS

This appendix expounds the procedure to obtain the output-gap and inflation-gap equations (9) and (10) in the main text. The model consists of the assumptions in section 2

*Natural rate of interest and intertemporal equilibrium*

In the first place, I examine households' consumption–saving choices, i.e.

$$\max: V_t = \ln C_t + E_t \left( \sum_{s=1}^{\infty} \Theta^{-s} \ln C_{t+s} \right) \quad (\text{A1})$$

$$\text{s.t. } C_t + E_t \sum_{s=1}^{\infty} (C_{t+s} + B_{t+s+1}) R(t, s) = H_t + E_t \sum_{s=1}^{\infty} H_{t+s} R(t, s) + R_t B_t$$

where the budget constraint is obtained by iteration of (6),  $R(t, s) = \prod_{s=1}^{\infty} R^{-(t+s)}$  is the compound discount factor, and the transversality condition imposes that  $\lim_{s \rightarrow \infty} B_{t+s} R(t, s) = 0$ .

The first-order condition for a maximum yields

$$C_t = E_t (C_{t+1} \Theta / R_{t+1}) \quad (\text{A2})$$

This admits of a steady-state solution where the employment of labour is normalized to 1,  $L^* = 1$  and  $R_{t+1} = \Theta \equiv R^*$ , so that  $C_t = C^*$  for all  $t$ . For the given Cobb–Douglas production function, the constant real interest rate also yields a constant capital stock such that  $F'(K^*) = R^*$  and hence constant output and factor incomes. As long as optimal saving is equal to optimal investment, the real value of outstanding bonds coincide with the operating capital stock,  $B_{t+1} = B_t = K^*$ . Consequently, the resource constraint is satisfied for

$$C^* \left( 1 + \sum_{s=1}^{\infty} R^{*-s} \right) = H^* \left( 1 + \sum_{s=1}^{\infty} R^{*-s} \right) + R^* K^* \quad (\text{A3})$$

Since  $\lim_{s \rightarrow \infty} \sum_s R^{*-s} = 1/r^*$ , the following values result

$$C^* + K^* = Y^* \quad (\text{A4})$$

$$K^* (Aa/R^*)^{1/b} \quad (\text{A5})$$

$$Y^* = AK^{*a} \tag{A6}$$

The real interest rate  $R^*$  associated with intertemporal equilibrium is the natural rate. Note, also, that  $S'_t = I'_t = K^*$ , i.e. net saving and investment are nil in all  $t$ , and the economy only replaces the optimal stock of capital  $K^*$ . Finally, it should be that  $(1 + i_t) = R^*(1 + \pi^*)$ , or  $R_{t+1} \equiv (1 + i_t)/(1 + \pi^*) = R^*$ , for all  $t$

*Interest-rate gaps and output gaps*

I now examine the allocations that result if, starting in the steady state, at time  $t$  the market real interest rate differs from the natural rate. As explained in the main text, if saving exceeds investment the central bank sells extra bonds, if investment exceeds saving the central bank buys extra bonds. As explained in the text, any  $R_{t+1} \neq R^*$  is assumed to be constant.

To begin with, let us examine the plan of households. Given  $\Theta = R^*$ ,  $B_t = K^*$ ,  $R_{t+1} \neq R^*$ , their optimal consumption path (A2) would be

$$C_t = E_t(C_{t+1}R^*/R_{t+1}) \tag{A7}$$

Therefore

$$S'_t = H_t + R_tK^* - C_t \tag{A8}$$

*Ceteris paribus*, with respect to the steady state,  $R_{t+1} \neq R^*$  shifts consumption to the present (if  $R_{t+1} < R^*$ ) or to the future (if  $R_{t+1} > R^*$ ). As a result, saving is decreased or increased, respectively.

Now let us see investment of firms, i.e.

$$I'_t = K_{t+1} = (Aa/R_{t+1})^{1/b}$$

Hence investment is increased (if  $R_{t+1} < R^*$ ) or decreased (if  $R_{t+1} > R^*$ ).

Consequently, there is a unique relationship between interest-rate gaps and saving–investment gaps, namely

- if  $R_{t+1} > R^*$ :  $\Rightarrow S'_t > I'_t$ ;
- if  $R_{t+1} < R^*$ :  $\Rightarrow S'_t < I'_t$ .

As long as the central bank pegs  $R_{t+1} \neq R^*$ , it allows households and firms to finance their saving and investment plans. However, these plans are not mutually consistent in the capital market, and therefore, by Walras Law, they are not consistent in the output market either. In fact,

- if  $R_{t+1} > R^*$ , consumption is shifted from  $t$  to  $t + 1$ , while investment in  $t$ , and the capital stock available in  $t + 1$ , are reduced: there is excess supply in  $t$  and excess demand in  $t + 1$ ;
- if  $R_{t+1} < R^*$ , the excesses are reversed.

How can these inconsistent plans be transformed into mutually consistent actual plans?

To address this point, I follow the same procedure as in the NNS model, namely I plug each period budget constraint (B1) directly into households' Euler equation (A2):

$$(H_t + R_t K^* - B_{t+1}) = E_t[(H_{t+1} + R_{t+1} B_{t+1} - B_{t+2}) R^* / R_{t+1}] \quad (\text{A9})$$

Here the saving–investment inconsistency can also be seen in the fact that  $B_{t+s} \neq K_{t+s}$  for all  $s > 1$ : the real value of the stock of bonds owned by households differs from the actual capital stock of firms at each point in time, and wrong resource accounting results. As long as  $R_{t+1} \neq R^*$ , the actual consumption path consistent with  $B_{t+1} = K_{t+1}$  should satisfy

$$(Y_t - K_{t+1}) = (Y_{t+1} - K_{t+2}) R^* / R_{t+1} \quad (\text{A10})$$

where  $Y_t = H_t + R_t K^*$ ,  $Y_{t+1} = H_{t+1} + R_{t+1} K_{t+1}$ . This reformulation of households' consumption path leads to the following propositions.

*Proposition A1:* Given the capital stock chosen by firms for  $R_{t+1} \neq R^*$ , there exists a unique intertemporal vector of output realizations associated with consistent *ex post* output market clearing.

*Proposition A2:* These output realizations correspond to non-zero gaps with respect to the level of 'potential output' given by the capital stock that would obtain with the natural rate of interest  $R^*$ .

The proof goes as follows. First, since, for  $R_{t+1}$  constant,  $K_{t+2} = K_{t+1}$ , (A10) can be rewritten as

$$Y_t = Y_{t+1} R^* / R_{t+1} + K_{t+1} (1 - R^* / R_{t+1})$$

Now divide both sides by  $Y^*$  to obtain the intertemporal relationship between output gaps:

$$\hat{Y}_t = \hat{Y}_{t+1} R^* / R_{t+1} + (K_{t+1} / Y^*) (1 - R^* / R_{t+1})$$

Upon recollecting the following relationships:

$$Y_{t+1} = AK_{t+1}^a, \quad K_{t+1} = (aA/R_{t+1})^{1/b}, \quad Y^* = AK^{*a}, \quad K^* = (aA/R^*)^{1/b}$$

the two output gaps result:

$$\hat{Y}_t \approx (R_{t+1}/R^*)^{-1/b} \tag{A11}$$

$$\hat{Y}_{t+1} = (R_{t+1}/R)^{-a/b} \tag{A12}$$

where the approximation concerns the multiplicative term  $(1 - a(1/R_{t+1} - 1/R^*))$ , which, for sufficiently small rates, is close to 1.

Therefore, (A11) and (A12) show that the main implication of the market real interest rate being set above (below) the natural rate is a *sequence of intertemporal negative (positive) output gaps* each depending on the *current* interest-rate gap  $(R_{t+1}/R^*)$ .

Furthermore, since  $\hat{Y}_t$  and  $\hat{Y}_{t+1}$  share the common factor  $(R_{t+1}/R^*)$  it is, in general, possible to express them in a single reduced form displaying autocorrelation. In fact, let  $\hat{Y}_t = Z_t^m$  and  $\hat{Y}_{t+1} = Z_t^n$ . Then it is always possible to write  $\hat{Y}_{t+1} = \hat{Y}_t \rho Z_t^a$  for linear combinations of the parameters  $\rho$  and  $\alpha$ , namely  $\rho m + \alpha = n$ . Note that the autocorrelation between  $\hat{Y}_{t+1}$  and  $\hat{Y}_t$  should be understood as a *spurious* correlation due to the common factor  $Z_t$ . The log of the previous expression yields the dynamic equation in the main text.

### *Inflation gaps*

As to price determination in relation to output gaps, let us assume that all nominal prices and wages are fully indexed to the inflation rate  $\pi$ . Given the general-equilibrium real wage rate  $w^*$  and capital stock  $K^*$ , potential output at any time  $t$  can also be expressed as

$$Y^* = K^*(b/w^*)^{b/a} \tag{A13}$$

Let the nominal wage rate for  $t$  be given by indexing  $w^*$  with the expected inflation rate  $\pi^*$ , i.e.  $W_t = w^* P_{t-1} (1 + \pi^*)$ . Therefore, firms can still adjust output for  $t$  by choosing the labour input upon observing the current real wage rate  $w_t \equiv W_t/P_t$ , where  $P_t \equiv P_{t-1} (1 + \pi_t)$ . As a result,

$$Y_t = K^* \left( \frac{b}{w^*} \frac{1 + \pi_t}{1 + \pi^*} \right)^{b/a} \quad (\text{A14})$$

*Ceteris paribus*, profit-maximizing firms are ready to expand (contract) output as long as  $\pi_t$ , being greater (smaller) than  $\pi^*$ , increases (reduces) the current nominal value of the marginal product of labour *vis-à-vis*  $W_t$ . Conversely, we can derive the Marshallian supply curve of firms, i.e. the inflation gap  $\hat{\Pi}_t \equiv (1 + \pi_t)/(1 + \pi^*)$ , which supports a given output gap. Using (A14) in (A13) we obtain

$$\hat{\Pi}_t = (\hat{Y}_t)^{a/b} \quad (\text{A15})$$

This expression generates the log-linear equation in the main text.

## APPENDIX B: ANALYTICAL SOLUTIONS OF THE SYSTEMS OF LINEAR DYNAMIC EQUATIONS

### *Exogenous interest-rate gap*

Let us start with the initial structural model (9)–(10), which is reproduced here for convenience:

$$\hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{i}_t \quad (\text{B1})$$

$$\hat{\pi}_{t+1} = \beta \hat{y}_{t+1} \quad (\text{B2})$$

I first introduce the general solution method used in all subsequent cases. To begin with, the above system can be written in the general matrix form for non-homogenous first-order difference systems, i.e.

$$\mathbf{g}_{t+1} = \mathbf{A} \mathbf{g}_t + \mathbf{B} \mathbf{x} \quad (\text{B3})$$

where  $\mathbf{g}$  is the vector of endogenous variables,  $\mathbf{x}$  is a vector of exogenous variables, and  $\mathbf{A}$  and  $\mathbf{B}$  are matrices of coefficients. If the non-zero matrix  $(\mathbf{I} - \mathbf{A})^{-1}$  exists, system (B3) has a steady-state solution  $\mathbf{g}^*$  given by

$$\mathbf{g}^* = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{B} \mathbf{x} \quad (\text{B4})$$

In the case under consideration, with an initial value  $\hat{i}_0$  taken as a constant exogenous variable, we have  $\mathbf{g}' \equiv [\hat{y}, \hat{\pi}]$   $\mathbf{x}' \equiv [\hat{i}_0, 0]$ ,

$$\mathbf{A} \equiv \begin{bmatrix} \rho & 0 \\ \beta \rho & 0 \end{bmatrix}, \quad \mathbf{B} \equiv \begin{bmatrix} -\alpha & 0 \\ -\beta \alpha & 0 \end{bmatrix}$$



and

$$\mathbf{g}^* = \begin{bmatrix} -\frac{\alpha}{1-\rho} & 0 \\ \frac{\beta\alpha}{1-\rho} & 0 \end{bmatrix} \begin{bmatrix} \hat{i}_0 \\ 0 \end{bmatrix} \tag{B5}$$

Therefore, for any  $\hat{i}_0 \neq 0$ ,  $\mathbf{g}^* \neq 0$ , and a steady-state solution with zero endogenous gaps only exists if  $\hat{i}_0 = 0$ .

To study the dynamic properties of the system, let us examine the eigenvalues of matrix  $\mathbf{A}$ . These are the roots of its characteristic polynomial, which in the  $2 \times 2$  case is a quadratic equation of general form:

$$k^2 - \text{tr}(\mathbf{A})k + \det(\mathbf{A}) = 0$$

In the case under consideration we have  $\text{tr}(\mathbf{A}) = \rho$  and  $\det(\mathbf{A}) = 0$ . Therefore, the characteristic equation has two real roots:  $k_1 = \rho$ ,  $k_2 = 0$ . Hence, for stability it is necessary and sufficient that  $\rho \in [-1, 1]$ , whereas  $\rho \in [0, 1]$  ensures monotonic stability.

*Endogenous interest-rate gap*

Formally, the introduction of an ‘interest-rate rule’, i.e. an equation that determines the time path of the nominal interest rate in relation to output and inflation gaps, amounts to endogenizing the interest-rate gap with respect to the previous system. In general, it is obtained a new first-order difference system with the matrix form (B3), where now  $\mathbf{g}' \equiv [\hat{y}, \hat{\pi}, \hat{i}]$ . Therefore, for this class of rules to admit a zero-gaps steady-state solution  $\mathbf{g}^* = 0$ , it is first necessary that  $\mathbf{x} = 0$ , i.e. a zero vector of exogenous gaps. In other words, the interest-rate rule should provide a homogeneous transformation of the system:

$$\mathbf{g}_{t+1} = \mathbf{A}\mathbf{g}_t \tag{B6}$$

Subsequently, the conditions of stability and convergence only depend on the coefficient matrix  $\mathbf{A}$ , and specifically on the parameters of the interest-rate rule.

Let us begin with the optimal of Taylor rule considered in the text:

$$i_t = i^* + \phi(y_t - y^*) + \gamma(E_t[\pi_{t+1}|i_{t-1}] - \pi^*) \tag{B7}$$

where  $i^*$  is the NAIRI,  $E_t[\pi_{t+1}|i_{t-1}]$  indicates the inflation forecast for time  $t + 1$ , elaborated at time  $t$ , conditional upon *not* intervening on the nominal interest

rate set at time  $t - 1$ , and  $\phi = \rho/\alpha$ ,  $\gamma = \eta\beta/\alpha$ . Let us now consider the system (B1)–(B2)–(B7). Using the structural model to obtain  $E_t[\pi_{t+1}|i_{t-1}]$ , and upon algebraic manipulations, it is possible to obtain a homogenous system such as (B6). As recalled above, a homogenous system generally admits a zero-gaps steady-state solution. It is immediate to see that as long as  $i_t = i^*$ , or  $\hat{i}_t = 0$ , for all  $t$ , the system is constantly in, or jumps immediately to, the zero-gaps steady state.

The main text also introduces the case where the central bank follows a rule such as (B7), but with no direct information about the true NAIRI  $i^*$ . To this effect, let  $\tilde{i}$  be the target interest rate adopted by the central bank; the new rule is

$$i_t = \tilde{i} + \phi(y_t - y^*) + \gamma(E_t[\pi_{t+1}|i_{t-1}] - \pi^*) \quad (\text{B7}')$$

and  $\varepsilon = \tilde{i} - i^*$  the relevant informational error. As a result, the new system (B1)–(B2)–(B7') is again a non-homogenous system with  $\mathbf{x}' \equiv [0, 0, \varepsilon]$ . Consequently, the system cannot achieve a zero-gaps steady state as long as  $\varepsilon \neq 0$ .

The second specification of the interest-rate rule is an 'adaptive' Taylor rule in the form:

$$i_t = i_{t-1} + \phi(y_t - y^*) + \gamma(\pi_t - \pi^*) \quad (\text{B8})$$

Subtracting  $i^*$  from both sides, and taking one period forward, we re-obtain a homogenous system in the matrix form (B10). Hence, the adaptive Taylor rule is consistent with a zero-gaps steady state of the economy.

As to convergence and stability analysis, first note that upon substituting the inflation-gap equation into the interest-rate rule, the system can be decomposed into a block with only two endogenous gaps  $\mathbf{g}' \equiv [\hat{y}_t, \hat{i}_t]$ , the inflation gap being fully determined by the path of  $\hat{y}_t$  given by the previous block. The coefficient matrix of this reduced-form system is

$$\mathbf{A} = \begin{bmatrix} \rho & -\alpha \\ \rho\beta\Omega & 1 - \alpha\beta\Omega \end{bmatrix}$$

where  $\Omega \equiv \phi/\beta + \gamma$ . Hence,  $\text{tr}(\mathbf{A}) = 1 + \rho - \alpha\beta\Omega$  and  $\det(\mathbf{A}) = \rho$ . Consequently, the two roots of the characteristic equation of  $\mathbf{A}$  may be complex or real. The roots are complex if

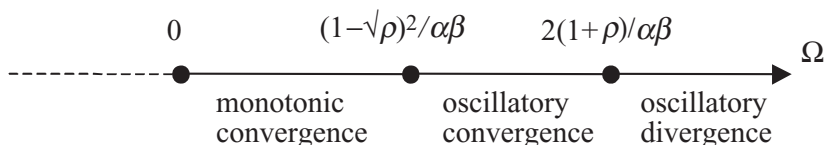
$$(1 - \sqrt{\rho})^2 / \alpha\beta < \Omega < (1 + \sqrt{\rho})^2 / \alpha\beta, \quad (\text{B9})$$

and the system can only achieve oscillatory convergence provided that  $\rho < 1$ .

The roots are real if

$$\Omega < (1 - \sqrt{\rho})^2 / \alpha\beta, \quad (1 + \sqrt{\rho})^2 / \alpha\beta < \Omega \tag{B10}$$

In this case, since  $\det(\mathbf{A}) > 0$ , there may be either two negative roots (if  $\text{tr}(\mathbf{A}) < 0$ ) or two positive roots (if  $\text{tr}(\mathbf{A}) > 0$ ). The system is convergent if the roots lie inside the unit circle. If the roots are negative, the system displays oscillatory convergence. If the roots are positive, the system displays monotonic convergence. Therefore, ordering the foregoing conditions yields the following dynamic regimes depending on the value of  $\Omega$ :



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Roberto Tamborini  
University of Trento  
Department of Economics,  
Via Inama 5,  
I-38100 Trento  
Italy  
E-mail: roberto.tamborini@economia.unitn.it

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