# Analytical approach to model of scientific revolutions

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The model of scientific paradigms spreading throughout the community of agents with memory is analyzed. The case of two competing ideas is analyzed for various networks of interactions. The pace of adopting a new idea by a community is considered, along with the distribution of time after which the new idea replaces the old one. The results are then extended onto the more general case when more than two ideas compete.

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## I. INTRODUCTION

There is a tendency to separate certain periods in the history of civilizations, such as Renaissance or Enlightenment, which qualitatively differ from each other by dominating trends in science, art or customs. In the human population innovations constantly emerge and it is not likely that it will ever reach some kind of equilibrium — "end of history" [1]. The changes (evolutionary and revolutionary ones) happen due to the interactions and exchange of innovative ideas [2–4] at the level of individuals, communities or even civilizations. Eventually, ideas spread throughout the communities [5, 6]. Some of the ideas gain broad (even global) acceptance and popularity, replacing old ones [7].

The process of adoption of an innovative technology [8] or a new scientific concept by individuals and communities differs from adoption of, for example, a new trend in arts. Obsolete technologies and discarded scientific theories, once abandoned, are not likely to be adopted by individuals again. To model such a process, agents should be given some kind of memory. Another important fact is that the will of individuals to adopt a new scientific concept depends on its global popularity. For example, spreading of technological innovations is often slowed down by incompatibility with existing standards. In the field of arts, the situation is different. Old ideas can reemerge and become popular again, such as Renaissance artists were inspired by Antique philosophy or architecture.

Recently a model was introduced by Bornholdt et al. which attempts to describe scientific revolutions [9]. The model combines interactions between individuals with influence of the whole community. Despite its simplicity, it managed to reconstruct key features of the dynamics of scientific paradigms spreading, including asymmetry between the speed of adopting a new idea by the community and the speed of its decline when new rival ideas appear. The presented results [9] were purely numerical — the model lacked analytical calculations. In this paper, the Markov processes theory [10] was applied to analyze the dynamics of the system in the case of small level of creativity of the agents, for various networks of interactions: chain, square lattice and complete graph.

## II. THE MODEL OF SPREADING OF IDEAS

The rules of the model [9] are very simple. $N$ agents occupy nodes of a network. Every agent has some opinion (idea), denoted by a natural number. In each time step a random agent $i$ (with opinion $s_i$) is selected, along with one of its neighbors $j$ (with opinion $s_j$). If agent $i$ has never had $j$'s current opinion, it adopts it with probability $N_{s_j}/N$, where $N_{s_j}$ denotes the number of agents sharing opinion $s_j$. Additionally, new opinions, which have never been present in the community, can appear: with probability $\alpha$ a random agent is selected, which changes its opinion into such an innovative one.

The most important feature of the model is the *memory* of the agents, who do not adopt the same opinion twice. One can find analogy between this model and evolutionary dynamics models: innovations can be regarded as mutations which allow the affected individuals outperform their rivals. Lack of any evident *fitness* parameter, which would describe how well a specie is adapted to the environment, is not necessarily a drawback of such an interpretation, as the *fitness* of a specie is alway *a posteriori* knowledge [11].

There are some general features of the evolution of the system, independent on the interactions network topology. For very small $\alpha$ at most two opinions exist simultaneously (other cases are neglectable due to their much smaller probability). This case was analyzed for various networks in sec. III.

For higher values of $\alpha$, other effects have to be considered. More than two opinions coexist, which "compete" with each other. However, one may suppose that within a relatively wide range of $\alpha$ still two opinions can be separated at every moment: the "old" opinion which is the most popular, but currently at the decline and the "new" opinion, the second most popular, which will prevail after some time (and then enter the stage of decline). This case was analyzed in sec. IV.

Throughout the further analysis, the number of agents in the network is denoted by $N$, the $i$th agent's opinion by $s_i$ and the network of interactions is described by the adjacency matrix $[a_{ij}]$. $P(A)$ denotes probability of $A$ and $P(A|B)$ — conditional probability.

## III. EVOLUTION OF THE SYSTEM FOR $\alpha < \langle T\rangle^{-1}$

### A. General case

When $\alpha$ is small enough, at most 2 opinions coexist, referred to as opinion 0 (at the stage of decline) and opinion 1 (at the stage of expansion). The evolution consists of a few distinct periods.

1. All the agents share the same opinion 0. The length of this *stage of stagnation* is a random variable of the distribution

$$P(T_{stag}) = (1-\alpha)^{T_{stag}}\alpha \tag{1}$$

and the mean value

$$\begin{aligned}\langle T_{stag}\rangle &= \sum_{T_{stag}=0}^{\infty} T_{stag}(1-\alpha)^{T_{stag}}\alpha \\ &= \frac{\alpha-1}{\alpha} \approx \frac{1}{\alpha}.\end{aligned} \tag{2}$$

2. An innovative opinion 1 appears.
3. Opinion 1 spreads across the community. The *time of expansion* of opinion 1 will be denoted $T$ and is a random variable distribution dependent on the interactions network topology.
4. When all the agents share opinion 1, the state of the system is equivalent to the initial one.

The state of the system is characterized by one variable — number $n$ of agents sharing opinion 1. The problem reduces to the problem of expansion of opinion 1 throughout the community, starting from one agent with opinion 1 at time $t = 0$. The generic master equation has only two terms:

$$\frac{\partial}{\partial t}P(n,t) = P(n-1,t)W_{n,n-1} - P(n,t)W_{n+1,n}, \tag{3}$$

where transition rates from state $n$ to $n+1$ (for $n \in [1, N-1]$) are equal to

$$\begin{aligned}W_{n+1,n} &\equiv W_n = \frac{n}{N}\sum_{k=1}^{N-1} P(k_i = k)P(i \in \partial S_1 | k_i = k) \\ &\cdot\ P(s_j = 1 | a_{ij} = 1 \wedge i \in \partial S_1 \wedge k_i = k),\end{aligned} \tag{4}$$

where $k_i$ is the degree of node $i$ and $\partial S_1 \equiv \partial\{i : s_i = 1\} \equiv \{i : s_i = 0 \wedge \exists_{a_{ij}=1} s_j = 1\}$ denotes the set of agents sharing opinion 0, which have at least one neighbor with the opposite opinion. For $n = N$ so defined transition rates are automatically equal to 0, since $\partial S_1 = \emptyset$.

It can be proved that if all the transition rates are different ($k \neq j \Rightarrow W_k \neq W_j$), the solution of Eq.(3) is

$$P(n,t) = \sum_{k=1}^{n} C_k^n e^{-W_k t}, \tag{5}$$

where

$$C_k^n \equiv \prod_{i=1}^{n-1} W_i \prod_{\substack{j=1 \\ j\neq k}}^{n} \frac{1}{W_j - W_k}. \tag{6}$$

Let us note that if $W_N = 0$ and $\forall_{1\leq n<N} W_n > 0$, the distribution evolves into

$$\lim_{t\to\infty} P(n,t) = \delta_{nN}, \tag{7}$$

(all the agents share opinion 1), which is an expected result.

The system evolves from state $n = 1$ to $n = N$ during *expansion time* $T$, which is a random variable whose distribution depends on the network topology and the initial condition. Thus, for each type of network the range of $\alpha$ has to be estimated for which the approximation of only 2 competing opinions makes sense: $\alpha \in (0, \langle T\rangle^{-1})$. In general

$$P(T = t) = W_{N-1}P(N-1, t-1) \approx \frac{1}{N}P(N-1,t), \tag{8}$$

and

$$\langle T\rangle \approx \int_0^\infty tP(T=t)dt \approx \frac{1}{N}\sum_{k=1}^{N-1}\frac{C_k^{N-1}}{W_k^2}. \tag{9}$$

### B. Chain topology

Let us consider the case when the agents occupy nodes of a chain. For simplicity, periodic boundary conditions will be assumed.

This specific topology makes the problem quite simple and in the first approximation it is not necessary to analyze the master equation (3). The average number of agents sharing opinion 1 can be derived from the following recursive equation:

$$\begin{cases} \langle n(0)\rangle = n(0) = 1 \\ \langle n(t+1)\rangle = \langle n(t)\rangle + \frac{2}{N}\frac{1}{2}\frac{\langle n(t)\rangle}{N} = \langle n(t)\rangle\left(1+\frac{1}{N^2}\right), \end{cases} \tag{10}$$

which has the solution

$$\langle n(t)\rangle = \left(1+\frac{1}{N^2}\right)^t \approx e^{t/N^2}. \tag{11}$$

On the average, after time

$$\langle T\rangle = N^2 \log N \tag{12}$$

opinion 1 will stop spreading as it will be shared by the whole community. The situation will be stable until another innovation appears. From this condition the range of $\alpha$ can be estimated for which this approximation makes sense: the average time between new innovations, $\alpha^{-1}$, should be higher than $\langle T\rangle$:

$$\alpha < \frac{1}{\langle T\rangle} = \frac{1}{N^2\log N}. \qquad (13)$$

For the more exact analysis of the problem one has to consider the master equation (3). In the case of chain topology

$$\begin{cases} P(k_i = k) = \delta_{k2} \\ P(i \in \partial S_1 | k_i = 2) = \frac{2}{N} \\ P(s_j = 1 | a_{ij} = 1 \wedge i \in \partial S_1 \wedge k_i = 2) = \frac{1}{2}. \end{cases}$$

Eventually, the transition rates from state $n$ to $n+1$ are equal to $W_n = \frac{n}{N^2}$ and the master equation has the form

$$\frac{\partial}{\partial t}P(n,t) = P(n-1,t)\frac{n-1}{N^2} - P(n,t)\frac{n}{N^2}. \qquad (14)$$

Let us stress that this equation does not take into consideration the limitation on the $n$ variable, which cannot be grater than $N$. This will be discussed further.

Due to the simple form of the transition rates, the equation (14) can be solved using the method of characteristic function $G$:

$$G(s,t) \equiv \langle e^{ins}\rangle. \qquad (15)$$

This approach has such an advantage over using (5), that the solutions are automatically in a compact form. The master equation (14) with the initial condition $P(n,0) = \delta_{n0}$ leads to the partial differential equation with the initial condition

$$\begin{cases} \frac{\partial}{\partial t}G(s,t) + \frac{1}{iN^2}\left(e^{is}-1\right)\frac{\partial}{\partial s}G(s,t) = 0 \\ G(s,0) = e^{is}, \end{cases} \qquad (16)$$

which has the solution

$$G(s,t) = \frac{1}{1 - e^{t/N^2}(1-e^{-is})}. \qquad (17)$$

After short algebra it can be proved that

$$G(s,t) = \sum_{n=1}^{\infty} \frac{1}{e^{t/N^2}-1}\left(1-e^{-t/N^2}\right)^n e^{isn}, \qquad (18)$$

so

$$P(n,t) = e^{-t/N^2}\left(1-e^{-t/N^2}\right)^{n-1}. \qquad (19)$$

This is valid for $n < N$. In order to take into consideration the limitation on the $n$ variable, one has to consider the accumulation of probability at point $n = N$:

$$\begin{aligned} P(n=N,t) &= \sum_{m=N}^{\infty} e^{-t/N^2}\left(1-e^{-t/N^2}\right)^{m-1} \\ &= \left(1-e^{-t/N^2}\right)^{N-1}. \end{aligned} \qquad (20)$$

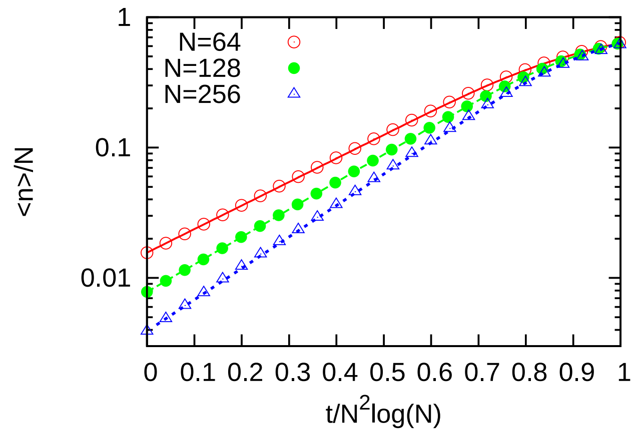



FIG. 1: (Color online) Chain graph topology, $N = 64, 128, 256$ nodes, $\alpha \ll 1/\langle T\rangle$. Evolution of the system starting from $n = 1$ innovative agent: $\langle n\rangle$ versus time. Points — simulated data. Lines — analytical predictions (Eq.(22)).

Eventually,

$$P(n,t) = \begin{cases} e^{-t/N^2}\left(1-e^{-t/N^2}\right)^{n-1} & ,1 \le n < N \\ \left(1-e^{-t/N^2}\right)^{N-1} & ,n = N. \end{cases} \qquad (21)$$

The mean value of $n$ resulting from the distribution (21) is equal to

$$\langle n\rangle = e^{t/N^2}\left(1 - \left(1-e^{-t/N^2}\right)^N\right), \qquad (22)$$

which for small $t$ reduces to (11).

From the distribution $P(n,t)$, a more exact approximation of $\langle T\rangle$ than Eq.(12) can be obtained. According to Eq.(8),

$$P(T=t) \approx \frac{1}{N}e^{-t/N^2}\left(1-e^{-t/N^2}\right)^{N-2}, \qquad (23)$$

and

$$\begin{aligned} \langle T\rangle &= \sum_{t=0}^{\infty} tP(T=t) \\ &\approx \frac{1}{N}\int_0^\infty te^{-t/N^2}\left(1-e^{-t/N^2}\right)^{N-2}dt \\ &= N^2 H_{N-1} \approx N^2(\log(N)+\gamma), \end{aligned} \qquad (24)$$

where $H_n$ is the $n$th harmonic number and $\gamma$ denotes the Euler-Mascheroni constant. Harmonic numbers grow approximately as fast as the natural logarithm, so the approximated solution (12) is actually very close to (24).

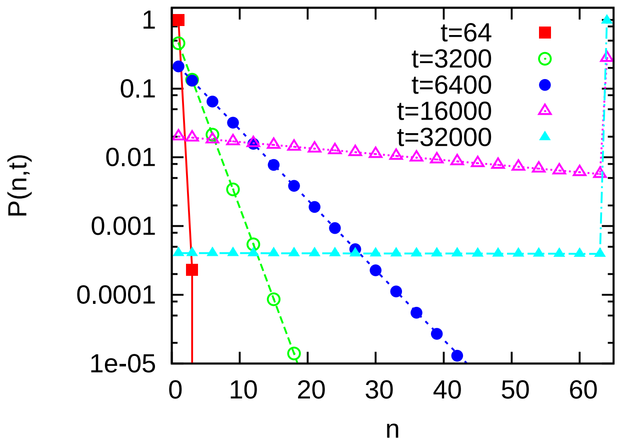



FIG. 2: (Color online) Chain graph topology, $N = 64$ nodes, $\alpha \ll 1/\langle T\rangle$. Evolution of the system starting from $P(n, t = 0) = \delta_{n1}$. Points are obtained from the numerical solution of the master equation (14). Lines — analytical predictions (Eq.(21)).

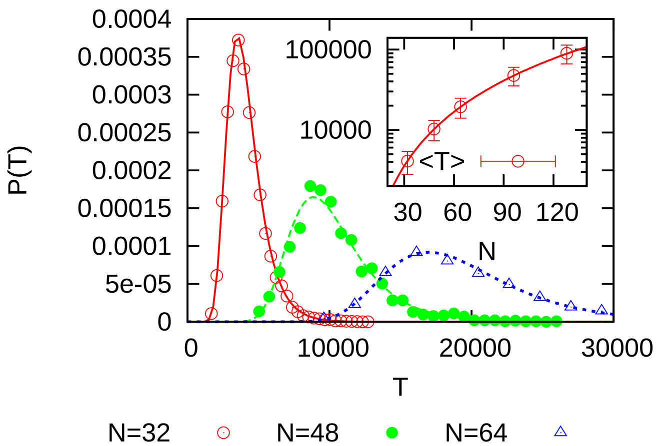



FIG. 3: (Color online) Chain graph topology, $\alpha \ll 1/\langle T\rangle$. Distribution of the time of expansion $T$ for different system sizes $N$. Points — simulations, lines — analytical predictions (Eq.(23)). (Inset) Mean time of expansion $\langle T\rangle$ versus system size $N$: simulations (points with error bars) compared with the analytical predictions (line) (Eq.(24)).

### C. Complete graph topology

Let us consider the situation when interaction is possible between every pair of agents, i.e. $\forall_{(i,j)} a_{ij} = 1$. Referring to the generic master equation (3),

$$\begin{cases} P(k_i = k) = \delta_{k,N-1} \\ P(i \in \partial S_1 | k_i = N-1) = \frac{N-n}{N} \\ P(s_j = 1 | a_{ij} = 1 \wedge i \in \partial S_1 \wedge k_i = N-1) = \frac{n}{N-1} \approx \frac{n}{N} \end{cases}$$

and the transition rates in the master equation are equal to

$$W_{n+1,n} \equiv W_n = \frac{N-n}{N} \cdot \frac{n}{N-1} \cdot \frac{n}{N} \approx \frac{n^2(N-n)}{N^3}. \quad (25)$$

The boundary conditions are automatically considered, since $P(0,t) = 0$ and $W_{N+1,N} = 0$. Eventually, the master equation has the form

$$\frac{\partial}{\partial t} P(n,t) = P(n-1,t)\frac{(n-1)^2(N-n+1)}{N^3} - P(n,t)\frac{n^2(N-n)}{N^3}. \quad (26)$$

If all the transition rates are different ($j \neq k \Rightarrow W_j \neq W_k$, which is satisfied if equation $N = a(1+b+b^2)$ does not have solutions $a, b$ among natural numbers), the solution can be written in the form of the sum (5):

$$P(n,t) = \sum_{k=1}^{n} C_k^n e^{-k^2(N-k)t/N^3}, \quad (27)$$

where

$$C_k^n \equiv \prod_{i=1}^{n-1} i^2(N-i) \prod_{\substack{j=1 \\ j\neq k}}^{n} \frac{1}{j^2(N-j) - k^2(N-k)} = (n-1)!^2 \frac{(N-1)!}{(N-n)!} \prod_{\substack{j=1 \\ j\neq k}}^{n} \frac{1}{j^2(N-j) - k^2(N-k)} \quad (28)$$

The expansion time $T$ distribution $P(T)$ can be derived from the $P(n,t)$ distribution:

$$P(T=t) \approx \frac{1}{N} P(N-1,t) = \frac{1}{N} \sum_{k=1}^{N-1} C_k^{N-1} e^{-k^2(N-k)t/N^3}. \quad (29)$$

The analytical predictions are in agreement with the simulations (Fig. 4–6).

### D. Square lattice topology

Let us consider the square lattice topology. Periodic boundary conditions will be assumed, so each agent has 4 neighbors. The first approximation would be to assume that the cluster of agents sharing opinion 1 grows uniformly in each direction, so at each moment it is circle-shaped, with radius of the circle equal to $r = \sqrt{n/\pi}$. Referring to the generic master equation (3),

$$\begin{cases} P(k_i = k) = \delta_{k,4} \\ P(i \in \partial S_1 | k_i = 4) = \begin{cases} \frac{2\sqrt{\pi n}}{N} & \text{if } 1 \leq n < N \\ 0 & \text{if } n \geq N \end{cases} \\ P(s_j = 1 | a_{ij} = 1 \wedge i \in \partial S_1 \wedge k_i = 4) = \frac{1}{4}, \end{cases}$$

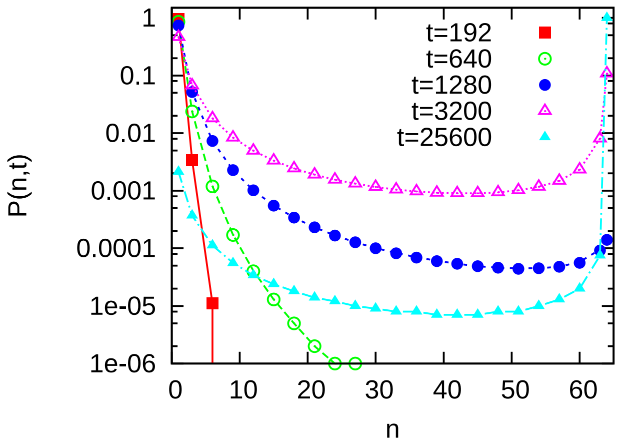



FIG. 4: (Color online) Complete graph topology, $N = 64$ nodes, $\alpha \ll 1/\langle T \rangle$. Evolution of the system starting from $P(n, t = 0) = \delta_{n1}$. Points are obtained from the numerical solution of the master equation (27). Lines — analytical predictions (Eq.(27)).

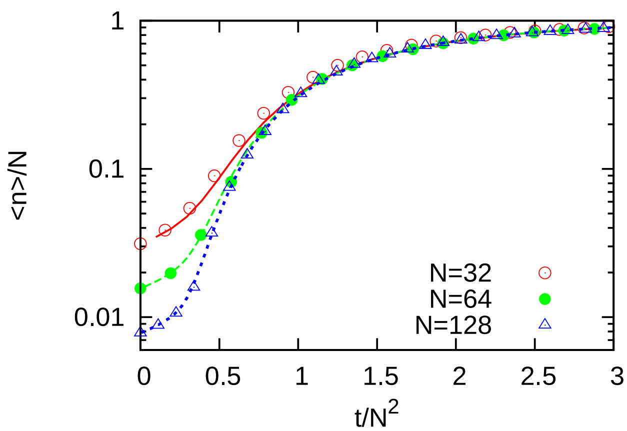



FIG. 5: (Color online) Complete graph topology, $N = 32, 64, 128$ nodes, $\alpha \ll 1/\langle T \rangle$. Evolution of the system starting from $n = 1$ innovative agent: $\langle n \rangle$ versus time. Points — simulated data. Lines — analytical predictions (obtained from Eq.(27)).

and the transition rates in the master equation are equal to

$$W_{n+1,n} \equiv W_n = \frac{\sqrt{\pi} n^{3/2}}{2N^2}(1 - \delta_{nN}). \qquad (30)$$

Similarly as in the case of complete graph topology, the solution of the master equation

$$\begin{aligned} \frac{\partial}{\partial t} P(n,t) &= P(n-1,t)\frac{\sqrt{\pi}(n-1)^{3/2}}{2N^2}(1 - \delta_{n-1,N}) \\ &- P(n,t)\frac{\sqrt{\pi} n^{3/2}}{2N^2}(1 - \delta_{nN}). \end{aligned} \qquad (31)$$

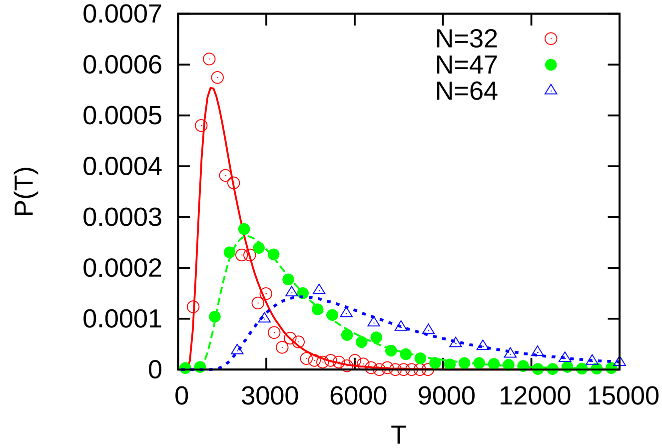



FIG. 6: (Color online) Complete graph topology, $\alpha \ll 1/\langle T \rangle$. Distribution of the time of expansion $T$ for different system sizes $N$. Points — simulations, lines — analytical predictions (Eq.(29)).

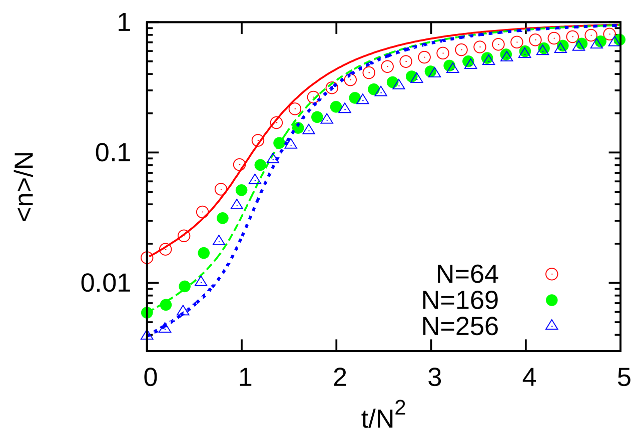



FIG. 7: (Color online) Square lattice topology, $N = 64, 169, 256$ nodes, $\alpha \ll 1/\langle T \rangle$. Evolution of the system starting from $n = 1$ innovative agent: $\langle n \rangle$ versus time. Points — simulated data. Lines — analytical predictions (obtained from Eq.(5) with transition rates (30)).

can be expressed in the form the sum of products (5). Having compared the results of such an approximation with the simulations (Fig. 7), one has to say that this approach significantly overestimates the pace of the growth of the new opinion cluster.

## IV. EVOLUTION OF THE SYSTEM FOR $\alpha \gtrsim \langle T\rangle^{-1}$

### A. Chain topology

For $\alpha$ higher than $1/N^2 \log N$ Eq.(10) has to be extended by terms describing the appearing of new clusters of ideas, which would slower the process of expansion of opinion 1. In the first approximation it will be assumed that the only important new clusters are those appearing inside the cluster of opinion 1 and that they do not overlap with each other. Their growth is described by Eq.(11). The recursive equation for $\langle n(t)\rangle$ is now

$$\begin{aligned}\langle n(t+1)\rangle &= \langle n(t)\rangle\left(1+\frac{1}{N^2}\right) \\ &- \sum_{\tau=0}^{t}\alpha\frac{\langle n(\tau)\rangle}{N}\langle\Delta n_{new}(t-\tau)\rangle \\ &= \langle n(t)\rangle\left(1+\frac{1}{N^2}\right) \\ &- \frac{\alpha}{N^3}\sum_{\tau=0}^{t}\frac{\langle n(\tau)\rangle}{N}\exp\left(\frac{t-\tau}{N^2}\right).\end{aligned} \tag{32}$$

Substituting the sum with the integral and stating that $\langle n(t)\rangle = \exp\left(t/N^2\right) f(t)$ leads to the following equation for $f(t)$:

$$f'(t) + \frac{\alpha}{N^3}\int_0^t f(\tau)d\tau = 0, \tag{33}$$

which, assuming the same initial conditions as in Eq.(10) (single innovation at time $t = 0$), has the solution

$$f(t) = \cos\left(\lambda t\right), \tag{34}$$

where $\lambda \equiv \sqrt{\alpha/N^3}$. The complete formula for the first approximation of $\langle n(t)\rangle$ is therefore

$$\langle n(t)\rangle = \exp\left(\frac{t}{N^2}\right)\cos\left(\lambda t\right). \tag{35}$$

As it should have been expected, for $\alpha \to 0$ this approximation converges to the previous one (Eq.(11)).

A better approximation can be obtained by substituting the term $\exp\left(\frac{t-\tau}{N^2}\right)$ by $\langle n(t-\tau)\rangle$ in Eq.(33), as new opinions can also be "attacked" by opinions appearing after them. The equation

$$\begin{aligned}\langle n(t+1)\rangle &= \langle n(t)\rangle\left(1+\frac{1}{N^2}\right) \\ &- \frac{\alpha}{N^3}\sum_{\tau=0}^{t}\frac{\langle n(\tau)\rangle}{N}\langle n(t-\tau)\rangle\end{aligned} \tag{36}$$

is not analytically solvable, but by substituting the sum with the integral and stating $\langle n(t)\rangle = \exp\left(t/N^2\right)\cos(\lambda t)(1+g(t))$, where $g(t) \ll 1$, an integral equation can be obtained,

$$\begin{aligned}0 &= -\lambda\sin(\lambda t) + g'(t)\cos(\lambda t) \\ &+ \lambda^2\int_0^t \cos(\lambda\tau)(1+g(\tau))\cos(\lambda(t-\tau))(1+g(t-\tau))d\tau,\end{aligned} \tag{37}$$

which can be is solvable provided all the terms integral apart from $\cos(\lambda\tau)\cos(\lambda(t-\tau))$ are neglected. Eventually, the second approximation of $\langle n(t)\rangle$ obtains the form

$$\langle n(t)\rangle = \exp\left(\frac{t}{N^2}\right)\cos(\lambda t)(1-\log|\cos(\lambda t)| - \frac{1}{4}\lambda^2 t^2). \tag{38}$$

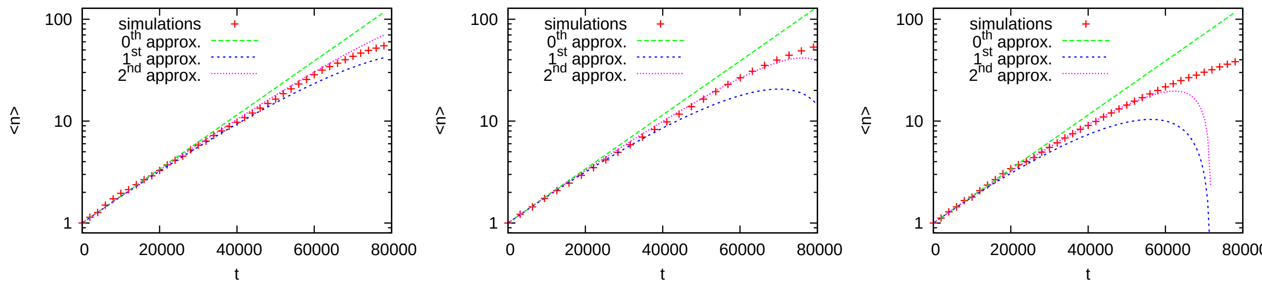
The comparison with the simulations (Fig. 8) shows that the last approximation (Eq.(38)) is better than the previous ones (Eq.(11), Eq.(35)).

## V. CONCLUSIONS

We have analyzed our analytical results and confronted them with the results obtained from simulations by Bornholdt et al. [9]. Our research suggests that the asymmetry between the paces of growth and decline of a dominant idea, observed in [9], is a generic property of the model and should be observed for any topology of interactions.

The crucial parameter of the dynamics is the $\alpha$ parameter, which describes the creativity of the agents. We have introduced the analytical methodology which can be used to analyze the system dynamics for various networks of interactions in the case when agents are almost non-innovative ($\alpha \gtrsim 0$) and estimated the range of $\alpha$ for which the proposed approach works properly ($\alpha < \langle T\rangle^{-1}$, where *the expansion time* $T$ is a function of the system size, depending on the interaction network topology). For every interactions topology the evolution consists of subsequent *stages of stagnation* (only one opinion/idea present among the agents) and *expansion* of a newly created opinion. The length of the *periods of stagnation* $T_{stag}$ is a random variable with the distribution Eq.(1) and is independent from the network topology, whereas the distribution of the lengths of the *periods of expansion* $P(T)$ strongly depends on the network topology and the system size. If $\alpha \ll \langle T\rangle^{-1}$, the mean time between shifts of the dominant paradigms can be approximated by $\langle T_{shift}\rangle \approx \langle T_{stag}\rangle = 1/\alpha$, which is a scaling observed in the simulated data by Bornholdt et al. [9] (note that the different time scale was used in [9]).

Three possible networks of interactions were taken into consideration — chain, square lattice and complete graph. For chain topology it was possible to find compact forms of the analytical solutions. It was found that $\langle T\rangle$ scales with the system size $N$ like $N^2 \log N$ and the at the *stage of expansion*, the cluster of the new idea grows like a damped exponential function Eq.(22). The analytical results are in the agreement with the simulated data.




(a) $\alpha = 5 \cdot 10^{-4}$ (b) $\alpha = 7 \cdot 10^{-4}$ (c) $\alpha = 10^{-3}$

FIG. 8: (Color online) Chain topology, $N = 128$, various levels of creativity $\alpha$. Comparison of approximations Eq.(11), Eq.(35) and Eq.(38). The red crosses refer to the simulated data.

For complete graph topology the proposed approach also results in the good agreement with the simulations, but the compact form of the solutions Eq.(27), Eq.(29) probably does not exist. In the case of the square lattice, the method failed to reproduce the results of the simulations. The problem probably lies in the evidently too rough estimation of the shape of the cluster of the new idea as a circle.

For higher level of creativity $\alpha$, when most of the time more than two ideas coexist, the dynamics of the system can be found starting from the results obtained for the near-zero-$\alpha$ case and using the method similar to the perturbation method. This approach proved to be useful in the simplest case — the chain topology. The resulting Eq.(38) is a scaled exponent and in the limit of $\alpha \to 0$ the solution, as expected, converges to the near-zero-$\alpha$ result Eq.(11).

Our analytical approach allowed for better understanding of the dynamics of the system described by the model and interpret some of relationships previously observed in the simulated data [9]. The proposed methodology can be used to analyze the dynamics of paradigms spreading in other networks [12]. It is especially interesting since the real networks of human contacts (including scientific collaboration networks) exhibit some nontrivial properties, such as being scale-free [13].

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