

# Cross-sections of long and short baseline neutrino and antineutrino oscillations of which some change the flavor

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## Abstract

The Pontecorvo-Maki-Nakagava-Sakata (PMNS) modified electroweak Lagrangian yields, within the perturbative kinematical procedure in the massive neutrino Fock space, in addition to the Lorentz invariant standard model (SM) neutrino and antineutrino cross-sections, also the "infinitesimal" neutrino and antineutrino cross-sections some of which are either conserving or violating the Lorentz symmetry as well as also either conserving or violating the flavor symmetry. Some of these infinitesimal differential cross-sections can be extended into the space oscillation region beyond the collision point. The extension goes along the baseline defined by the flavor neutrino or antineutrino scattering angle. Each of these oscillation differential cross-sections, being sinusoidal, change sign along the baseline; some start positive and some negative at the collision point. For each of them one seeks the baseline distance to the first differential cross-section maximum. For the 10 MeV energy neutrino or antineutrino colliding with an electron at rest, the following processes are analyzed with the oscillation differential cross-sections:  $\nu_{(e)}, \bar{\nu}_{(e)} + e \rightarrow \nu_{(e)}, \bar{\nu}_{(e)} + e$ ;  $\nu_{(\mu)}, \bar{\nu}_{(\mu)} + e \rightarrow \nu_{(e)}, \bar{\nu}_{(e)} + e$ ;  $\nu_{(\mu)}, \bar{\nu}_{(\mu)} + e \rightarrow \nu_{(\tau)}, \bar{\nu}_{(\tau)} + e$ . While all six oscillation differential cross-sections, presented here, violate the Lorentz invariance only four of them violate the flavor conservation infinitesimally at the collision point and along the baseline from the collision point. The baseline distance  $L_M(\phi)$  to the first oscillation (production) maximum depends on the neutrino and antineutrino scattering angle  $\phi$ , where  $0 \leq \phi \leq \pi$ . It is interesting how strongly some of the baseline distances to production maxima (and also likely to absorption minima) depend on  $\phi$ , as some of the following examples show:

$$\begin{aligned} \nu_{(e)} + e \rightarrow \nu_{(e)} + e : sL_M(\phi = 0, \frac{\pi}{2}, \pi) &\approx 2326km, 111km, 57km \\ \bar{\nu}_{(\mu)} + e \rightarrow \bar{\nu}_{(\tau)} + e : sL_M(\phi = 0, \frac{\pi}{2}, \pi) &\approx 84km, 4km, 2km, etc. \end{aligned}$$

where  $s$  is a scaling parameter numerically expected to be close to one. These results suggest easy experimental verifications, particularly at larger scattering angles.

## 1. Introduction

The neutrino and antineutrino oscillations, among the same flavor and between different flavors, have been already established by experiments such as the Super-Kamiokande [1], SNO [2], KAMLAND [3] and Homestake [4] among others. Summaries of oscillatory and other neutrino properties can be already found in the books by Fukugita and Yanagida [5] and by Giunti and Kim [6] as well as in the articles, for example, by Bilenky et al. [7], Giunti and Laveder [8] and Kyser [9].

Pontecorvo [10] noticed that the Schroedinger equation is natural for discussing probabilities of neutrino oscillations. It has been shown, however, that the general probabilities of neutrino oscillations can be also formed from the extrapolated differential cross-sections [11, 12]. When dealing with massless flavor neutrino or antineutrino oscillations, one usually assumes that the left-handed (massless) flavor neutrino fields are unitary linear combination of the massive left-handed neutrino fields and analogously for the states ([5-10] and references therein). This unitary transformation defines the PMNS (Pontecorvo-Maki-Nakagita-Sakata) massive neutrino field Lagrangian density [5-10]. How the presence of neutrino masses affect the standard model (SM) was, among the first, addressed by Schrock [13]. The decays, such as the  $\Pi$ ,  $K$ , and the nuclear  $\beta$  decays, with appearance of neutrinos and antineutrinos in the final states, were mostly the Schrock's interests [13]. Namely, these decays are friendly to the use of the PMNS unitary transformations to the SM. Then the kinematics is that of the massive neutrinos augmented with the unitary PMNS matrix dependence. Schrock also proposed specific tests of the  $\Pi$  and  $K$  decays to determine neutrino masses and the lepton mixing angles. These tests, unfortunately, could detect the neutrino masses in the 1 - 400 MeV range and the tests in the nuclear  $\beta$  decays in the 1 keV - 5 MeV range. Today, however, all the neutrino masses are almost certainly below 1 eV, as seen from the analysis by Fritzsch [14].

The connection of massive neutrinos to the SM was done more recently by Li and Liu [15] by studying the inequivalent vacua model [16]; here, the transformation between the Fock space of neutrino mass states and the unitary inequivalent flavor states is a Bogoliubov transformation [15, 16]. This transformation yields the  $O(m^2)$  ( $m$  denoting generically any of three neutrino masses) correction to the Pontecorvo neutrino oscillation probability. But, it also yields that the branching ratio of  $W^+ \rightarrow e^+ + \nu_\mu$  to  $W^+ \rightarrow e^+ + \nu_e$  is of  $O(m^2)$  contradicting the Hamiltonian that one started from. Hence, in the inequivalent vacua model there is a flavor changing current such as  $W^+ \rightarrow e^+ + \nu_\mu$  with the branching ratio different from that of the SM when  $m = 0$ . However, Li and Liu [15] show that the neutrino oscillation effects are large enough to neglect the inequivalent vacua model effects and that the sum of all three decay widths of  $W^+ \rightarrow e^+ + \nu_{e,\mu,\tau}$  equals the width of  $W^+ \rightarrow e^+ + \nu_e$  in the SM [15].

The aim here is to show explicitly, that from the PMNS modified Lagrangian density with massive neutrinos, the calculated neutrino or antineutrino cross-section consists of the following parts:

1. The SM cross-section that is Lorentz and flavor invariant.

2. To the  $O(m^2)$  the "infinitesimal" positive (production) and negative (absorption) flavor neutrino or antineutrino cross-sections with the oscillatory potentials. Some of these cross-sections either conserve or violate the Lorentz symmetry and similarly either conserve or violate the flavor symmetry.

3. To the  $O(m^2)$  the "infinitesimal" non cross-sectional terms that either conserve or violate the Lorentz symmetry and similarly either conserve or violate the flavor symmetry. Since, presently there is no applicability potential for these terms, they will be neglected due to the smallness of neutrino masses.

The existence of the infinitesimal (positive) production and (negative) absorption cross-sections at the collision point is taken here as an indication that at later time each of them oscillates along the baseline; and as such each keeps changing the label from production (absorption) to absorption (production) and so on. The baseline, originating at the collision point, is at the angle  $\phi$  which is the neutrino or antineutrino scattering angle. Without the oscillations these cross-sections, being of  $O(m^2)$ , could be simply neglected. The form of these infinitesimal cross-sections indicates that these neutrino or antineutrino oscillations are sinusoidal in time  $t$  or distance  $L$  from the collision point.

In pursuing the oscillatory differential neutrino and antineutrino cross-sections, the plan is as follows. In the next section ( Section 2 ), summary of the formalism connecting the massive neutrinos with massless flavor neutrinos and antineutrinos through the perturbative kinematical procedure with ratios of the neutrino masses to the corresponding energies as the expansion parameters. Expositions of neutrino and antineutrino differential cross-sections, due to the  $W$  and  $Z$  vector bosons exchanges is done in Section 3. Here, also the cross-sections are separated into three parts, the SM part plus two parts proportional to  $O(m^2)$ , one with the neutrino and antineutrino oscillatory potentials and the other not. Section 4 is devoted to extending the  $O(m^2)$  values of differential cross-sections at the collision point to new, oscillatory, values at the finite baseline distance  $L(\phi)$  with  $\phi$ , the neutrino or antineutrino scattering angle. The angle  $\phi$  defines also the direction of the baseline  $L$  even when transitions such as  $\nu_\mu \rightarrow \nu_\tau$  and  $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$  occur. Discussion as to how to approach the measurement of first maxima of such flavor conserving and or violating oscillations is done in the Section 5. Here also the further neutrino and antineutrino oscillation possibilities through the scattering experiments are discussed.

## 2. Perturbative kinematics connecting massive with massless flavor neutrinos

According to Fritzsche [14] the neutrino masses are practically infinitesimal,  $m_i \leq 1eV, i = 1, 2, 3$ . Hence, one can build the massive neutrino four-momentum around the massless flavor neutrino four-momentum since, for any two neutrino masses  $m_{i1}$  and  $m_{i2}$  ( $i=1, 2, 3$ ), as noted in [17] and [18],  $\left| (m_{i1}^2 - m_{i2}^2) / q_{(\gamma)}^0 \right|$ , with  $q_{(\gamma)}^0$  as neutrino energy, is much smaller than the quantum-mechanical uncertainty in energy ( S. M. Bilenky et al. [17] and a more general discussion by the same authors in [19]). Hence, for the fixed neutrino flavor  $\gamma$  it is impossible to distinguish emission of neutrinos with different

masses in the neutrino processes [17]. ( In [11, 12] this is tied up to the perturbative kinematical procedure). Consequently, such massive neutrinos can be viewed as superposing themselves to form a flavor neutrino  $\nu_{(\gamma)}$  [17, 18] where the left-handed neutrino fields and states, through the PMNS transformations, satisfy [5 - 9]

$$\begin{aligned}
\nu_{\gamma L}(x) &= U_{\gamma i} \nu_{iL}(x), \bar{\nu}_{\gamma L}(x) = \bar{\nu}_{iL}(x) U_{i\gamma}^\dagger; \\
|\nu_\gamma(q(\gamma), s_\gamma)\rangle &= U_{i\gamma}^\dagger |\nu_i(q(i,\gamma), s_{i,\gamma})\rangle, \\
\langle\nu_\gamma(q(\gamma), s_\gamma)| &= \langle\nu_i(q(i,\gamma), s_{i,\gamma})| U_{\gamma i}, \\
|\bar{\nu}_\gamma(q(\gamma), s_\gamma)\rangle &= U_{\gamma i} |\bar{\nu}_i(q(i,\gamma), s_{i,\gamma})\rangle, \\
\langle\bar{\nu}_\gamma(q(\gamma), s_\gamma)| &= \langle\bar{\nu}_i(q(i,\gamma), s_{i,\gamma})| U_{i\gamma}^\dagger; \gamma = e, \mu, \tau, i = 1, 2, 3 \quad ((1))
\end{aligned}$$

As for fixed neutrino (antineutrino) flavor  $\gamma$  ( $\bar{\gamma}$ ) ( when appearing alone the antineutrino flavor index will have bar over it ) one cannot distinguish different massive neutrinos, hence one assumes that four-momenta of massive neutrinos with masses  $m_i, i = 1, 2, 3$ , are connected to four momenta of massless neutrinos as (for the sake of simplicity, only flavor neutrinos are discussed here)

$$\begin{aligned}
q^\mu(i, \gamma) &= \left( \vec{q}(i, \gamma), (\vec{q}^2(i, \gamma) + m_i^2)^{1/2} \right), \\
q^\mu(\gamma) &= (\vec{q}(\gamma), |\vec{q}(i, \gamma)|), q^2(\gamma) = 0, \gamma = e, \mu, \tau \quad ((2))
\end{aligned}$$

For  $q^\mu(i, \gamma)$  to be useful, one has to expand it in terms of  $m_i$  :

$$\begin{aligned}
\vec{q}(i, \gamma) &= \vec{q}(\gamma), \\
q^\mu(i, \gamma) &= q^\mu(\gamma) - g^{\mu 0} \left[ \frac{m_i^2}{2q(\gamma)^0} - \frac{m_i^4}{8q(\gamma)^{03}} + O(m_i^6) \right], \\
q^2(i, \gamma) &= -m_i^2 + O(m_i^4) \quad ((3))
\end{aligned}$$

In relation (3)  $\gamma$  is the flavor number of either neutrino or antineutrino. Relation (3) is the basis of the perturbative kinematics [11, 12] and will be utilized to  $O(m_i^2)$  and it shows that  $q^\mu(\gamma)$  was chosen to be Lorentz four-vector at the expense of  $q^\mu(i, \gamma)$ . However, since  $m_i$  is much smaller than any of the relevant energies, the main portions (the SM portions) of cross-sections are expected to be Lorentz invariant.

Because of the relation (3), the helicity  $s(i, \gamma)$  of each massive neutrino  $\nu_i$ , comprising the massless fixed flavor  $\gamma$  neutrino or antineutrino  $\nu_\gamma(\bar{\nu}_\gamma)$ , has the helicity  $s(\gamma)$  of  $\nu_\gamma(\bar{\nu}_\gamma)$ ;  $s(i, \gamma) = s(\gamma)$ , as can be seen directly from the helicity operator itself [11, 12],

$$\hat{s}(i, \gamma) = \vec{q}(i, \gamma) \cdot \vec{\sigma} / |\vec{q}(i, \gamma)| = \vec{q}(\gamma) \cdot \vec{\sigma} / |\vec{q}(\gamma)| = \hat{s}(\gamma) \quad ((4))$$

This relation holds for both, the initial and final states.

Next, one writes down the free massive spinor field operator with mass  $m_i$  accomodating relations (1) to (4):

$$\begin{aligned}
& \nu_{(i,\gamma)}(x) \\
&= \frac{1}{(2\pi)^{3/2}} \sum_{s(\gamma)} \int d^3q(i,\gamma) [e^{iq(i,\gamma)x} u(q(i,\gamma), s(\gamma)) a_i(q(i,\gamma), s(\gamma)) \\
&+ e^{-iq(i,\gamma)x} v(q(i,\gamma), s(\gamma)) b_i^\dagger(q(i,\gamma), s(\gamma))], \\
d^3q(i,\gamma) &= d^3q(\gamma)
\end{aligned} \tag{5}$$

where the spinors reflect boh the kinematical and helicity inter-relationships between massive neutrinos  $\nu_i$  and the flavor neutrino  $\nu_\gamma$  (antineutrino  $\bar{\nu}_\gamma$ ) :

$$\begin{aligned}
u(q(i,\gamma), s(\gamma)) &= \frac{m_i - \underline{q}(i,\gamma)}{\sqrt{2(m_i + q^0(i,\gamma))}} u(m_i, \vec{0}, s(\gamma)), \\
u(m_i, \vec{0}, s(\gamma) = \pm 1) &= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}; \\
v(q(i,\gamma), s(\gamma)) &= \frac{m_i + \underline{q}(i,\gamma)}{\sqrt{2(m_i + q^0(i,\gamma))}} v(m_i, \vec{0}, s(\gamma)), \\
v(m_i, \vec{0}, s(\gamma) = \pm 1) &= \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}
\end{aligned} \tag{6}$$

The different from zero, canonical anticommutation rules that connect the massive neutrinos  $\nu_{i,j}$ ,  $i, j = 1, 2, 3$  with respective flavor neutrinos  $\nu_{\gamma,\delta}$  are

$$\begin{aligned}
& \left\{ a_i(q(i,\gamma), s(\gamma)), a_j^\dagger(q(j,\delta), s(\delta)) \right\} \\
&= \left\{ b_i(q(i,\gamma), s(\gamma)), b_j^\dagger(q(j,\delta), s(\delta)) \right\} \\
&= \delta_{ij} \delta_\gamma \delta_\delta \delta_3 (\vec{q}(i,\gamma) - \vec{q}(j,\delta)) = \delta_{ij} \delta_\gamma \delta_\delta \delta_3 (\vec{q}(\gamma) - \vec{q}(\delta))
\end{aligned} \tag{7}$$

As compared to [11, 12] here, to avoid overcrowding, the anticommutation rules (7) are written non-covariantly. The differerntial cross-sections can be calculated either with covariant or non-covariant anticommutation rules yielding the same results.

The inter-relationship between massive and massless flavor neutrinos makes the coherent energy projection operators different from those in the eletro-weak theory. In fact, generalizing the results from [11, 12], the coherent (with equal helicities,  $s(\gamma)$ ) positive neutrino and negative antineutrino energy projection

operators  $[q(i, \gamma), q(k, \gamma); \pm, c]$ , respectively are:

$$\begin{aligned} [q(i, \gamma), q(k, \gamma); +, c] &= 2 \sum_{s(\gamma)} u(q(i, \gamma), s(\gamma)) \otimes \bar{u}(q(k, \gamma), s(\gamma)) \\ &= \frac{(m_i - \underline{q}(i, \gamma)) (1 + \gamma^0) (m_k - \underline{q}(k, \gamma))}{2 [(m_i + q^0(i, \gamma)) (m_k + q^0(k, \gamma))]^{(1/2)}} \quad ((8)) \end{aligned}$$

$$\begin{aligned} [q(i, \gamma), q(k, \gamma); -, c] &= -2 \sum_{s(\gamma)} v(q(i, \gamma), s(\gamma)) \otimes \bar{v}(q(k, \gamma), s(\gamma)) \\ &= \frac{(m_i + \underline{q}(i, \gamma)) (1 - \gamma^0) (m_k + \underline{q}(k, \gamma))}{2 [(m_i + q^0(i, \gamma)) (m_k + q^0(k, \gamma))]^{(1/2)}} \quad ((9)) \end{aligned}$$

Direct comparison of (8) with (9) shows that the positive neutrino and negative antineutrino energy coherent projection operators are related by the  $\gamma^5$  transform,

$$[q(i, \gamma), q(k, \gamma); \pm, c] = \gamma^5 [q(i, \gamma), q(k, \gamma); \mp, c] \gamma^5 \quad ((10))$$

This result is consistent with the momentum space spinor connections from their explicit expressions in (6) ( see also [20], page 55 ) from which one has,

$$\begin{aligned} u(q(i, \gamma), s(\gamma)) &= \gamma^5 v(q(i, \gamma), s(\gamma)), \quad \bar{u}(q(k, \gamma), s(\gamma)) \\ &= -\bar{v}(q(k, \gamma), s(\gamma)) \gamma^5 \quad ((11)) \end{aligned}$$

### 3. Neutrino and antineutrino differential cross-sections affected by the neutrino masses

Utilizing the PMNS substitution rules (1) the usual SM Lagrangian density with massless flavor neutrino fields transforms into the one with massive neutrino fields [11,12]:

$$\begin{aligned} \alpha, \beta, \dots, \varepsilon &= e, \mu, \tau; i, j, a, \dots, b = 1, 2, 3; \\ l_{\alpha L} &= \begin{pmatrix} U_{\alpha i} \nu_{iL} \\ \alpha_L \end{pmatrix}, \\ \epsilon_{L,R} &= P_{L,R} \epsilon, P_{L,R} = \frac{1}{2} (1 \mp \gamma^5), \end{aligned}$$

$$\begin{aligned} L_{W,int}^{Lepton} &= \frac{g}{\sqrt{2}} \sum_{\epsilon=e,\mu,\tau; i=1,2,3} [\bar{\nu}_{iL}(x) U_{i\epsilon}^\dagger \gamma^\mu \epsilon_L(x) W_\mu(x, +) \\ &\quad + \bar{\epsilon}_L(x) \gamma^\mu U_{\epsilon j} \nu_{jL}(x) W_\mu^\dagger(x, +)], \\ W^\mu(x, \pm) &= \frac{1}{\sqrt{2}} [W^\mu(x, 1) \mp iW^\mu(x, 2)], \end{aligned}$$

$$\begin{aligned}
L_{Z,int}^{Lepton} &= \frac{g}{c_W} Z_\mu(x) \sum_{\epsilon=e,\mu,\tau} [\bar{l}_{\epsilon L}(x) \frac{\tau_3}{2} \gamma^\mu l_{\epsilon L}(x) \\
&\quad - s_W^2 (-) \bar{\epsilon}(x) \gamma^\mu \epsilon(x)] \\
&= \frac{g}{4c_W} Z_\mu(x) \sum_{\epsilon=e,\mu,\tau; a,b=1,2,3} [\bar{\nu}_a(x) U_{a\epsilon}^\dagger \gamma^\mu (1 - \gamma^5) U_{\epsilon b} \nu_b(x) \\
&\quad + \bar{\epsilon}(x) \gamma^\mu [(4s_W^2 - 1) + \gamma^5] \epsilon(x)]; \\
s_W &= \sin \Theta_W, c_W = \cos \Theta_W
\end{aligned} \tag{12}$$

The neutrino and antineutrino scattering processes of interest here are

$$\nu(\gamma, i), \bar{\nu}(\gamma, i) + \alpha(P_{(1)}) \longrightarrow \nu(\delta, j), \bar{\nu}(\delta, j) + \beta(P_{(2)}) \tag{13}$$

where  $\gamma$  and  $\delta$  are fixed flavor values, respectively, in the initial and final state. They are opposite in signs for antineutrinos from those for neutrinos; when appearing alone they are denoted with  $\bar{\gamma}$  and  $\bar{\delta}$  to be distinguished from  $\gamma$  and  $\delta$  of neutrinos. The indices  $i$  and  $j$  go over values 1, 2, 3 but are contained in fixed flavor indices  $\gamma$  and  $\delta$  of initial and final state, respectively. In neutrino and antineutrino scattering cross-sections one needs the product of delta functions assuring the overall energy and momentum conservation. Remembering that the flavors  $\gamma$  and  $\delta$  are fixed and that massive neutrino indices  $i, j, k, l$  vary, then consistent with (3), as shown in [11, 12] one evaluates

$$\begin{aligned}
&\delta_4(q(i, \gamma) + P_{(1)} - q(j, \delta) - P_{(2)}) \delta_4(q(k, \gamma) + P_{(1)} - q(l, \delta) - P_{(2)}) \\
&= \delta_4^2(q(\gamma) + P_{(1)} - q(\delta) - P_{(2)}) + O(m^4), i, j, k, l = 1, 2, 3
\end{aligned} \tag{14}$$

Here,  $m^4$  symbolically denotes  $m_i^4, m_i^2 m_k^2, etc.$  As long as  $m^4$  terms can be ignored compared to other energy terms, the kinematics of the scattering process (14) is the same as of the massless flavor neutrinos or antineutrinos,

$$\nu(\gamma), \bar{\nu}(\gamma) + \alpha(P_{(1)}) \longrightarrow \nu(\delta), \bar{\nu}(\delta) + \beta(P_{(2)}) \tag{15}$$

The role of massive neutrinos is to adjust the flavor neutrino and antineutrino cross-sections so as to describe their oscillations when travelling beyond the collision point.

The aim here is to write down the differential neutrino (antineutrino) cross-section with  $\nu(\delta)$  ( $\bar{\nu}(\delta)$ ) emphasized in the final state. With the target lepton (electron) at rest,  $P_{(1)} = (\vec{0}, M)$ , one starts with the kinematics for neutrinos or antineutrinos described here simultaneously for either of them

$$\vec{q}(\gamma) \cdot \vec{q}(\delta) = q^0(\gamma) q^0(\delta) \cos \phi, \vec{q}(\gamma) \cdot \vec{P}_{(2)} = q^0(\gamma) |\vec{P}_{(2)}| \cos \theta \tag{16.1}$$

where, with some work, one establishes the connection between the scattering

angles

$$\begin{aligned}\epsilon &= \frac{M}{q^0(\gamma)}, \cos^2 \theta = \frac{(1+\epsilon)^2 (\cos \phi - 1)}{[(\cos \phi - 1)(1+2\epsilon) - 2\epsilon^2]}, \\ \cos \phi &= \frac{[\sin^2 \theta (1+\epsilon)^2 - \epsilon^2 \cos^2 \theta]}{[\sin^2 \theta (1+2\epsilon) + \epsilon^2]}\end{aligned}\quad ((16.2))$$

With relations (16), the normalized neutrino (antineutrino or charged lepton) energy transfer can be expressed in multitude of ways

$$\begin{aligned}y(\theta \text{ or } \phi) &= \frac{q^0(\gamma) - q^0(\delta)}{q^0(\gamma)} = \frac{P_{(2)}^0 - M}{q^0(\gamma)} \\ &= \frac{2\epsilon \cos^2 \theta}{(1+\epsilon)^2 - \cos^2 \theta} = \frac{1 - \cos \phi}{1 + \epsilon - \cos \phi}\end{aligned}\quad ((17))$$

$$\begin{aligned}q^0(\delta) (\theta \text{ or } \phi) &= \left( \frac{\sin^2 \theta (1+2\epsilon) + \epsilon^2}{\sin^2 \theta + 2\epsilon + \epsilon^2} \right) q^0(\gamma) \\ &= \left( \frac{\epsilon}{1 + \epsilon - \cos \phi} \right) q^0(\gamma)\end{aligned}\quad ((18))$$

Relations (17) and (18) are equally valid for neutrinos and antineutrinos.

The values for the neutrino masses, from the analysis by Fritzsche [14], are

$$m_1 = 0.004eV, m_2 = 0.01eV, m_3 = 0.05eV \quad ((19))$$

while the neutrino/antineutrino mixing matrix due to Harrison, Perkins and Scott [21], is

$$(U_{\alpha i}) = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \quad ((20))$$

From (19) and (20) one notices that  $m_i^* = m_i$  and  $U_{\alpha i}^* = U_{\alpha i}$  which allows one to deduce the following properties of the mass matrices,

$$\begin{aligned}m_{\nu(\alpha)\nu(\beta)} &= U_{\alpha i} m_i U_{i\beta}^\dagger = U_{\beta i} m_i U_{i\alpha}^\dagger = m_{\nu(\beta)\nu(\alpha)}, \\ m_{\nu(\alpha)\nu(\beta)}^2 &= U_{\alpha i} m_i^2 U_{i\beta}^\dagger = \sum_{\gamma} m_{\nu(\alpha)\nu(\gamma)} m_{\nu(\gamma)\nu(\beta)} = m_{\nu(\beta)\nu(\alpha)}^2\end{aligned}\quad ((21))$$

$$\begin{aligned}m_{\bar{\nu}(\alpha)\bar{\nu}(\beta)} &= U_{\alpha i}^* m_i U_{i\beta}^T = U_{\beta i} m_i U_{i\alpha}^\dagger = m_{\nu(\beta)\nu(\alpha)} = m_{\nu(\alpha)\nu(\beta)}, \\ m_{\bar{\nu}(\alpha)\bar{\nu}(\beta)}^2 &= U_{\alpha i}^* m_i^2 U_{i\beta}^T = U_{\beta i} m_i^2 U_{i\alpha}^\dagger \\ &= m_{\nu(\beta)\nu(\alpha)}^2 = m_{\nu(\alpha)\nu(\beta)}^2 = m_{\bar{\nu}(\beta)\bar{\nu}(\alpha)}^2\end{aligned}\quad ((22))$$



Basically, the right-and-left hand sides of relations (21) and (22) are c-number quantities, rather than matrices of the type (20), allowing within them the matrix transpositions at will which, in turn, results in no need to distinguish antineutrinos from neutrinos in the mass matrix elements.

Next one lists the numerical values of parameters needed for the evaluations of the oscillatory neutrino cross-sections beyond the collision point,

$$G = \frac{\sqrt{2}g^2}{8M_W^2} = 1.17 \times 10^{-5} GeV^{-2}, w_0 = s_W^2 = 0.23,$$

$$w_1 = 2w_0 - 1 = -0.54, z_1 = w_1 w_0 + \frac{1}{4} = 0.1258, \quad ((23.1))$$

$$z_2 = w_1 w_0 = -0.1242, z_3 = w_0 - \frac{1}{4} = -0.02, z_4 = w_0 (w_0 - 1)$$

$$+ \frac{1}{4} = 0.0729, z_1 + z_3 = 2w_0^2 = 0.1058,$$

$$z_1 - z_3 = w_0 (w_1 - 1) + \frac{1}{2} = 0.1458 \quad ((23.2))$$

$$eV = 0.5076 \times 10^{10} km^{-1};$$

$$m_{\nu(e)\nu(e)} = 6 \times 10^{-3} eV, m_{\nu(\mu)\nu(\mu)} = m_{\nu(\tau)\nu(\tau)} = 2.9 \times 10^{-2} eV,$$

$$m_{\nu(e)\nu(\mu)} = m_{\nu(\mu)\nu(e)} = m_{\nu(e)\nu(\tau)} = m_{\nu(\tau)\nu(e)} = 2 \times 10^{-3} eV,$$

$$m_{\nu(\mu)\nu(\tau)} = m_{\nu(\tau)\nu(\mu)} = -0.021 eV;$$

$$m_{\nu(e)\nu(e)}^2 = 4.4 \times 10^{-5} eV^2, m_{\nu(\mu)\nu(\mu)}^2$$

$$= m_{\nu(\tau)\nu(\tau)}^2 = 1.29 \times 10^{-3} eV^2 \quad ((24))$$

Now concentrating first on neutrinos, from the Lagrangian density (12), one writes down the total differential cross-sections from the contributions of  $W, Z$  and  $(W - Z)$  exchanges for

$$\nu(q(\gamma)) + \alpha(P_{(1)}) \rightarrow \nu(q(\delta)) + \beta(P_{(2)}) \quad ((25))$$

that evolved from (13). Derivations of these cross-sections have been done already in [11] and [12]. Here, one is interested in the total differential cross-section as a function of  $q^0(\delta)$  the energy of scattered or created neutrino  $\nu(\delta)$  from the charged lepton (electron)  $\beta(P_{(2)})$  at rest,  $P_{(2)} = (\vec{0}, M)$  and leaving the collision point at an angle  $\phi$ . Hence, from references [11, 12], taking into account relations (19) -(23), one can assemble for (25) the differential cross-

sections up to  $O(m^2)$  in the following format

$$\begin{aligned}
& \left[ \frac{d\sigma_W}{dy} + \frac{d\sigma_Z}{dy} + \frac{d\sigma_{W,Z}}{dy} \right] (\nu(q(\gamma)) + \alpha(P_{(1)}) \rightarrow \nu(q(\delta)) + \beta(P_{(2)})) \\
&= \frac{d\sigma_T(\nu(\gamma), \alpha; ; \beta, \nu(\delta))}{dy} = \frac{d\sigma(SM; \nu(\gamma), \alpha; ; \beta, \nu(\delta))}{dy} \\
& \quad + \frac{d\sigma(OSC; \nu(\gamma), \alpha; ; \beta, \nu(\delta))}{dy} + \frac{d\sigma(NOSC; \nu(\gamma), \alpha; ; \beta, \nu(\delta))}{dy} \quad (26)
\end{aligned}$$

Here  $SM$  refers to the usual standard model differential cross-section,  $OSC$  refers to the portions of the differential cross-section that depend quadratically on the inverse of the final state neutrino energy  $q^0(\delta)$  while  $NOSC$  refers to the portions that do not have inverse quadratic dependence on  $q^0(\delta)$ . The  $OSC$  portions of the differential cross-section can be extended beyond the collision point so as to show the oscillations of the final state neutrino  $\nu(\delta)$  away from the interaction region along the baseline at the angle  $\phi$  with respect to the incoming neutrino  $\nu(\gamma)$ . Next, one details the right hand side of (26).

$$\begin{aligned}
& \frac{d\sigma(SM; \nu(\gamma), \alpha; ; \beta, \nu(\delta))}{dy} \\
&= \frac{2G^2 M \delta_{\alpha\beta} \delta_{\gamma\delta}}{\pi} \\
& \quad \{ \delta_{\beta\delta} \left[ q^0(\gamma)(1+w_1) + Mw_0 \left( \frac{q^0(\delta)}{q^0(\gamma)} - 1 \right) \right] \right. \\
& \quad + \frac{1}{2} [z_2 M \left( \frac{q^0(\delta)}{q^0(\gamma)} - 1 \right) \\
& \quad \left. + (z_1 + z_3) \frac{q^{02}(\delta)}{q^0(\gamma)} + (z_1 - z_3) q^0(\gamma) \right] \} \quad (27)
\end{aligned}$$

$$\begin{aligned}
& \frac{d\sigma(OSC; \nu(\gamma), \alpha; ; \beta, \nu(\delta))}{dy} \\
&= \frac{G^2 M \delta_{\alpha\beta}}{\pi} \{ 2q^0(\gamma) \delta_{\alpha\gamma} \\
& \quad - z_2 M + (z_1 - z_3) q^0(\gamma) + 2 [w_1 q^0(\gamma) - w_0 M] \delta_{\beta\delta} \} \\
& \quad \times \left[ \delta_{\gamma\delta} \frac{m_{\nu(\gamma)\nu(\gamma)}^2}{4q^{02}(\delta)} - \frac{(m_{\nu(\gamma)\nu(\delta)})^2}{4q^{02}(\delta)} \right] \quad (28)
\end{aligned}$$

Relation (27) is the superposition of the production (positive) and absorption (negative) differential cross-sections for  $\nu(\delta)$  at the collision point. Each of them is poised to oscillate and change signs along the baseline at the angle  $\phi$ , the task that will be dealt with shortly. Next, one writes down the details of the last,

the third, term in (26).

$$\begin{aligned}
& \frac{d\sigma(NOSC; \nu(\gamma), \alpha; \beta, \nu(\delta))}{dy} \\
= & \frac{G^2 M \delta_{\alpha\beta}}{\pi} \left\{ \frac{m_{\nu(\gamma)\nu(\delta)}^2 \delta_{\gamma\delta}}{2q^0(\gamma)} [\delta_{\alpha\gamma} [1 + w_1 + \frac{M}{q^0(\gamma)} \left( \frac{q^0(\gamma)}{q^0(\delta)} + \frac{q^0(\delta)}{q^0(\gamma)} - 1 \right)]] \right. \\
& + \frac{1}{2} [ \frac{z_2 M}{q^0(\gamma)} \left( \frac{q^0(\gamma)}{q^0(\delta)} + \frac{q^0(\delta)}{q^0(\gamma)} - 1 \right) + (z_1 + z_3) \left( 1 + \frac{q^{02}(\delta)}{q^{02}(\gamma)} \right) + (z_1 - z_3)] \\
& + (m_{\nu(\gamma)\nu(\delta)})^2 [\delta_{\alpha\gamma} [ \frac{1}{M} \left( \frac{M}{q^0(\delta)} + \frac{q^0(\delta)}{q^0(\gamma)} - 1 \right) + \frac{M}{2q^{02}(\gamma)} \left( 1 - \frac{q^0(\delta)}{q^0(\gamma)} \right) \\
& + \frac{w_1}{q^0(\gamma)} \left( -\frac{1}{2} + \frac{q^0(\gamma)}{q^0(\delta)} + \frac{q^{02}(\gamma)}{Mq^0(\delta)} - \frac{q^0(\gamma)}{M} \right) - \frac{w_0}{q^0(\delta)}] \\
& + \frac{\delta_{\beta\delta}}{2q^0(\gamma)} [1 + w_1 - \frac{M}{q^0(\delta)} - \frac{2w_0 q^0(\delta)}{q^0(\gamma)}]] \\
& - \frac{(m_{\nu(\gamma)\nu(\delta)})^2}{4q^0(\gamma)} [z_2 [2 \left( \frac{q^0(\gamma)}{q^0(\delta)} + \frac{q^0(\delta)}{q^0(\gamma)} \right) + M \left( \frac{q^0(\delta)}{q^{02}(\gamma)} + \frac{1}{q^0(\delta)} - \frac{1}{q^0(\gamma)} \right)] \\
& + (z_1 + z_3) \left( -3 + \frac{2q^{02}(\delta)}{Mq^0(\gamma)} + \frac{q^{02}(\delta)}{q^{02}(\gamma)} - \frac{2q^0(\delta)}{M} \right) \\
& \left. - 2(z_1 - z_3) \left( -\frac{1}{2} + \frac{q^0(\gamma)}{q^0(\delta)} + \frac{q^{02}(\gamma)}{Mq^0(\delta)} - \frac{q^0(\gamma)}{M} \right) \right\}. \tag{29}
\end{aligned}$$

The standard model portion (27) is in its usual finite form. The portion (28), although being proportional to  $m^2$ 's, can be amplified in oscillatory forms for  $\nu(\delta)$  away from the collision point. The portion (29), although rather complex, does not presently suggest itself for a useful amplification beyond the collision point and because of  $m^2$  dependences it will be neglected in this presentation.

Now, one needs to figure out the differential cross-section for the process with antineutrinos that evolve from (13)

$$\bar{\nu}(q(\gamma)) + \alpha(P_{(1)}) \rightarrow \bar{\nu}(q(\delta)) + \beta(P_{(2)}) \tag{30}$$

where one should take into account that antineutrinos have opposite flavor quantum numbers from those of neutrinos. In order to make sure of this fact, the flavor quantum numbers appearing alone are denoted as  $\bar{\gamma}$ ,  $\bar{\delta}$ , ...for  $\bar{\nu}(\gamma)$ ,  $\bar{\nu}(\delta)$ , ...etc. in both the initial and final states. As shown in [11,12] when evaluating the differential cross-section for the neutrino process (25) one utilizes the trace with positive neutrino energy coherent projection operators (8). Similarly, when evaluating the differential cross-section for the antineutrino process (30) one utilizes the trace with negative antineutrino energy

coherent projection operators (9) which, however, in view of the  $\gamma^5$  transform (10), reduces to the trace with positive neutrino coherent projection operators

$$\begin{aligned} & Tr [q(i, \bar{\gamma}), q(j, \bar{\gamma}); -, c] [q(k, \bar{\delta}), q(l, \bar{\delta}); -, c] \\ &= Tr [q(i, \bar{\gamma}), q(j, \bar{\gamma}); +, c] [q(k, \bar{\delta}), q(l, \bar{\delta}); +, c] \end{aligned} \quad ((31))$$

Relation (31) shows that in form the differential cross-sections for (30) and (25) are the same providing that, except in the normalization factor, the interchange  $P_{(1)} \longleftrightarrow P_{(2)}$  is carried out. However, the flavor conservations,  $\delta_{\beta\bar{\gamma}} = 0$ , etc. set to zeros the contributions from  $W-$  and  $(W, Z)$ -exchanges so that only the contributions from the  $Z$ -exchange remains. Hence, with the help from [11, 12] and the fact that  $\frac{d\sigma_W}{dy} = \frac{d\sigma_{W,Z}}{dy} = 0$ , one writes,

$$\begin{aligned} & \frac{d\sigma_Z(\bar{\nu}(q(\gamma)) + \alpha(P_{(1)}) \rightarrow \bar{\nu}(q(\delta)) + \beta(P_{(2)}))}{d\gamma} \\ &= \frac{d\sigma_T(\bar{\nu}(\gamma), \alpha; ; \beta, \bar{\nu}(\delta))}{dy} = \frac{d\sigma(SM; \bar{\nu}(\gamma), \alpha; ; \beta, \bar{\nu}(\delta))}{dy} \\ &+ \frac{d\sigma(OSC; \bar{\nu}(\gamma), \alpha; ; \beta, \bar{\nu}(\delta))}{dy} + \frac{d\sigma(NOSC; \bar{\nu}(\gamma), \alpha; ; \beta, \bar{\nu}(\delta))}{dy} \end{aligned} \quad ((32))$$

$$\begin{aligned} & \frac{d\sigma(SM; \bar{\nu}(\gamma), \alpha; ; \beta, \bar{\nu}(\delta))}{dy} \\ &= \frac{G^2 M \delta_{\alpha\beta} \delta_{\bar{\gamma}\bar{\delta}}}{\pi} \left[ \left( \frac{q^0(\bar{\delta})}{q^0(\bar{\gamma})} - 1 \right) z_2 M \right. \\ & \left. + (z_1 + z_3) q^0(\bar{\gamma}) + (z_1 - z_3) \frac{q^{02}(\bar{\delta})}{q^0(\bar{\gamma})} \right]. \end{aligned} \quad ((33))$$

$$\begin{aligned} & \frac{d\sigma(OSC; \bar{\nu}(\gamma), \alpha; ; \beta, \bar{\nu}(\delta))}{dy} \\ &= \frac{G^2 M \delta_{\alpha\beta}}{\pi} \left[ -M z_2 + (z_1 + z_3) q^0(\bar{\gamma}) \right] \\ & \times \left( \delta_{\bar{\gamma}\bar{\delta}} \frac{m_{\bar{\nu}(\gamma)}^2}{4q^{02}(\bar{\delta})} - \frac{(m_{\bar{\nu}(\gamma)} \bar{\nu}(\delta))^2}{4q^{02}(\bar{\delta})} \right). \end{aligned} \quad ((34))$$

$$\begin{aligned}
& \frac{d\sigma (NOSC; \bar{\nu}(\gamma), \alpha; ; \beta, \bar{\nu}(\delta))}{dy} \\
&= \frac{G^2 M \delta_{\alpha\beta}}{4\pi q^0(\bar{\gamma})} \{ \delta_{\bar{\gamma}\bar{\delta}} m_{\bar{\nu}(\gamma)\bar{\nu}(\delta)}^2 [M z_2 \left( \frac{1}{q^0(\bar{\delta})} + \frac{q^0(\bar{\delta})}{q^{02}(\bar{\gamma})} - \frac{1}{q^0(\bar{\gamma})} \right) \right. \\
& \quad + (z_1 - z_3) \left( 1 + \frac{q^{02}(\bar{\delta})}{q^{02}(\bar{\gamma})} \right) + (z_1 + z_3)] \\
& \quad - (m_{\bar{\nu}(\gamma)\bar{\nu}(\delta)})^2 z_2 M \left( \frac{1}{q^0(\bar{\delta})} + \frac{q^0(\bar{\delta})}{q^{02}(\bar{\gamma})} - \frac{1}{q^0(\bar{\gamma})} \right) \\
& \quad - (m_{\bar{\nu}(\gamma)\bar{\nu}(\delta)})^2 [2z_2 \left( \frac{q^0(\bar{\gamma})}{q^0(\bar{\delta})} + \frac{q^0(\bar{\delta})}{q^0(\bar{\gamma})} \right) \\
& \quad + (z_1 - z_3) \left( \frac{q^{02}(\bar{\delta})}{q^{02}(\bar{\gamma})} + 2q^0(\bar{\delta}) \left( \frac{q^0(\bar{\delta})}{M q^0(\bar{\gamma})} + \frac{1}{q^0(\bar{\gamma})} - \frac{1}{M} \right) - 1 \right) \\
& \quad \left. + (z_1 + z_3) \left( 2q^0(\bar{\gamma}) \left( \frac{1}{M} - \frac{1}{q^0(\bar{\delta})} - \frac{q^0(\bar{\gamma})}{M q^0(\bar{\delta})} \right) - 1 \right) \right] \}. \quad (35)
\end{aligned}$$

Except for the SM cross-sections (27) and (33) the other cross-sections denoted with *OSC* and *NOSC*, point to the infinitesimal space quantum structure (ISQS) through their dependences on  $m^{2'}$ s and the fact that the *OSC* might be observed through the space oscillation.

#### 4. Oscillatory differential cross-section at the baseline beyond the collision point

The portion of the cross-section denoted with *OSC*, (28) and (34), can be extended to the finite time  $t$  or baseline distance  $L$  with the characteristic Pontecorvo dimensionless argument in the same maner as Dvornikov [22] within the classical field theoretical model with the result for the neutrino oscillation transition probability [22] as

$$P(t) = \sin^2(2\Theta_{vac}) \sin\left(\frac{\Delta m^2 t}{4E}\right). \quad (36)$$

where  $\Theta_{vac}$  is the vacuum mixing angle,  $\Delta m^2 = m_1^2 - m_2^2$  is the mss squared difference,  $E$  is the enegy of the system, with other details in [22].

Similar to [11, 12] here also it is reasonable to assume that (28) and (34) have origins in sinusoidal like forms. The difference here from [11, 12] is that in the sinusoidal form the dimensionless scale factor  $s$  is introduced,

$$\frac{\Delta m^2}{4E^2} = \frac{\Delta m^2 L}{4E} \rightarrow \frac{1}{s} \sin\left(\frac{\Delta m^2 s L}{4E}\right). \quad (37)$$

where  $L$  is the baseline distance from the collision point. The new scale factors  $u$  and  $s$ , because the sin function is bounded, cannot be too much different from unity but still, they might be useful for fitting the data.

Next, one extends (28) and (34) respectively, into the neutrino and antineutrino oscillating differential cross-sections:

$$\begin{aligned}
& \frac{d\sigma(O\text{SC}; \nu(\gamma), \alpha; ; \beta, \nu(\delta); \phi, L)}{dy} \\
&= \frac{G^2 M \delta_{\alpha\beta}}{\pi} \{2q^0(\gamma) \delta_{\alpha\gamma} + [-z_2 M + (z_1 - z_3) q^0(\gamma)] \\
&+ 2(w_1 q^0(\gamma) - w_0 M) \delta_{\beta\delta}\} [\delta_{\gamma\delta} \frac{1}{u} \sin \frac{m_{\nu(\gamma)\nu(\gamma)}^2 u L}{4q^0(\delta)} \\
&- \frac{1}{s} \sin \frac{(m_{\nu(\gamma)\nu(\delta)})^2 s L}{4q^0(\delta)}]. \tag{38}
\end{aligned}$$

$$\begin{aligned}
& \frac{d\sigma(O\text{SC}; \bar{\nu}(\gamma), \alpha; ; \beta, \bar{\nu}(\delta); \phi, L)}{dy} \\
&= \frac{G^2 M \delta_{\alpha\beta}}{\pi} [-M z_2 + (z_1 + z_3) q^0(\bar{\gamma})] \\
&\times \left( \delta_{\bar{\gamma}\bar{\delta}} \frac{1}{u} \sin \frac{m_{\bar{\nu}(\gamma)\bar{\nu}(\gamma)}^2 u L}{4q^0(\bar{\delta})} - \frac{1}{s} \sin \frac{(m_{\bar{\nu}(\gamma)\bar{\nu}(\delta)})^2 s L}{4q^0(\bar{\delta})} \right). \tag{39}
\end{aligned}$$

In both differential cross-sections (38) and (39) the violation of flavor may occur in their second terms when respectively,  $\gamma \neq \delta$  and  $\bar{\gamma} \neq \bar{\delta}$ ; while the Kronecker deltas conserve the flavor in their first terms with respectively,  $\delta_{\gamma\delta}$  and  $\delta_{\bar{\gamma}\bar{\delta}}$ .

The direction of the baseline  $L$  in (38) and (39) is given by the angle  $\phi$  which kinematically is connected to the recoil angle  $\theta$  through relation (17) even when there are flavor nonconservations, that is, when  $\gamma \neq \delta$  and  $\bar{\gamma} \neq \bar{\delta}$ . Now, according to (17)  $q^0(\delta, \bar{\delta}; \phi) = (1 - y(\phi)) q^0(\gamma, \bar{\gamma})$  where it is worth noticing that  $q^0(\delta, \bar{\delta}; 0) = q^0(\gamma, \bar{\gamma})$ . Furthermore, since the extrema of trigonometric functions are fixed, then from (38) and (29) the baseline length  $L_M$  at the extremum satisfy

$$L_M(\phi) = (1 - y(\phi)) L_M(0) \tag{40}$$

Thus, it is sufficient to find  $L_M$  at  $\phi = 0$  since at any other  $\phi$  it is derived from (40).

## 5. Analysis of the oscillation differential cross-sections

This analysis, for both the neutrino and antineutrino oscillation differential cross-sections, will have examples of flavor conserving and flavor violating cases by the final state neutrino or antineutrino. The example parameters for this

analysis are: the electron as a target, initial neutrino energy together with the range of the scattering angle,

$$\begin{aligned}
1 - y(\phi) &= \frac{M}{M + q^0(\gamma, \bar{\gamma})(1 - \cos \phi)}; \\
M &= 0.5 \text{ MeV}; \quad q^0(\gamma, \bar{\gamma}) = 10 \text{ MeV}; \\
(1 - y(0)) &= 1; \quad \left(1 - y\left(\frac{\pi}{2}\right)\right) = 0.0476; \\
(1 - y(\pi)) &= 0.0244.
\end{aligned} \tag{41}$$

In order to get a general idea and feel for the oscillation differential cross-sections, consisting of negative absorption and positive production parts, the analysis will be kept as simple as possible, particularly with respect to the scaling parameters  $u$  and  $s$ . The baseline lengths at maxima (production) of oscillating differential cross-sections will be known as a function of the scattering angle  $\phi$  and specified just for  $\phi = 0, \frac{\pi}{2}, \pi$ .

Consistent with (38) and parameters (15)-(24) one starts with the flavor conserving neutrino oscillation scattering,

$$\begin{aligned}
\nu(e) + e &\rightarrow \nu(e) + e, \quad q^0(\gamma = e) = 10 \text{ MeV}; \\
&\frac{d\sigma(\text{OSC}; \nu(e), e; e, \nu(e); \phi, L)}{dy} \\
&= 8.92 \times 10^{-44} \text{ cm}^2 \left[ \frac{1}{u} \sin \frac{m_{\nu(e)\nu(e)}^2 u L}{4(1 - y(\phi) q^0(e))} \right. \\
&\quad \left. - \frac{1}{s} \sin \frac{(m_{\nu(e)\nu(e)})^2 s L}{4(1 - y(\phi) q^0(e))} \right] \\
&= 8.92 \times 10^{-44} \text{ cm}^2 \left[ \frac{1}{u} \sin \frac{5.5 \times 10^{-3} u L}{(1 - y(\phi) km)} - \frac{1}{s} \sin \frac{4.6 \times 10^{-3} s L}{(1 - y(\phi) km)} \right]. \tag{42}
\end{aligned}$$

Because the production and absorption parts of the cross-section are comparable in strength, one rewrites it as

$$\begin{aligned}
&\frac{d\sigma(\text{OSC}; \nu(e), e; e, \nu(e); \phi, L)}{dy} \\
&= 8.92 \times 10^{-44} \text{ cm}^2 \\
&\quad \times \left[ \left( \frac{1}{u} + \frac{1}{s} \right) \sin \frac{L 10^{-3} (5.5u - 4.6s)}{2(1 - y(\phi) km)} \right. \\
&\quad \times \cos \frac{L 10^{-3} (5.5u + 4.6s)}{2(1 - y(\phi) km)} \\
&\quad \left. + \left( \frac{1}{u} - \frac{1}{s} \right) \sin \frac{L 10^{-3} (5.5u + 4.6s)}{2(1 - y(\phi) km)} \right. \\
&\quad \left. \times \cos \frac{L 10^{-3} (5.5u - 4.6s)}{2(1 - y(\phi) km)} \right]. \tag{42.1}
\end{aligned}$$

To continue, it is worthwhile to assume, at least as an exercise, that the scaling parameters are approximately equal, yielding

$$\begin{aligned}
u &\approx s : \frac{d\sigma(O\mathcal{S}\mathcal{C}; \nu(e), e; ; e, \nu(e); \phi, L)}{dy} \\
&= 17.84 \times 10^{-44} \text{cm}^2 \times \\
&\quad \frac{1}{u} \sin \frac{0.9uL10^{-3}}{2(1-y(\phi) km)} \cos \frac{10.1uL10^{-3}}{2(1-y(\phi) km)} \quad ((42.2))
\end{aligned}$$

The first (production) maximum of the oscillatory cross-section in (42.2) is approximately at  $\frac{0.9uL(\phi)10^{-3}}{2(1-y(\phi)km)} \approx \frac{\pi}{3}$  giving for the scaled baseline distances at this (production) maximum,

$$\begin{aligned}
uL_M(\phi = 0) &\approx 2326 \text{ km}, \\
uL_M\left(\phi = \frac{\pi}{2}\right) &\approx 111 \text{ km}, uL_M(\phi = \pi) \approx 57 \text{ km}. \quad ((43))
\end{aligned}$$

These results show that the baseline length gets shorter with increasing scattering angle  $\phi$ . Consistent with relation (40) the production differential cross-section has the same value for each of the  $\phi$ 's ,

$$(42.2) (0 \leq \phi \leq \pi, L_M(\phi)) \approx \frac{1}{u} 10.5 \times 10^{-44} \text{cm}^2. \quad ((44))$$

where, as already mentioned,  $u$  should not differ much from unity. Experimentally, fitting relations (43) and (44), one should be able to determine  $u$  numerically.

Next, utilizing (15)-(24) from (38) one addresses the differential cross-section for the flavor changing neutrino oscillation scattering,

$$\nu(\mu) + e \rightarrow \nu(e) + e, q^0(\gamma = \mu) = 10 \text{MeV} :$$

$$\begin{aligned}
&\frac{d\sigma(O\mathcal{S}\mathcal{C}; \nu(\mu), e; ; e, \nu(e); \phi, L)}{dy} \\
&= \frac{1}{s} 0.85 \times 10^{-44} \text{cm}^2 \times 9.51 \sin \frac{(m_{\nu(\mu)\nu(e)})^2 sL}{4(1-y(\phi) q^0(\mu))} \\
&= \frac{1}{s} 8.08 \times 10^{-44} \text{cm}^2 \sin \frac{5.1 \times 10^{-4} sL}{(1-y(\phi) km)}. \quad ((45))
\end{aligned}$$

The first (production) maximum of this oscillation differential cross-section yields, with the help of (40) and (41), the scaled baseline distances at three scattering angles,

$$\begin{aligned}
sL_M(\phi = 0) &\approx 3078 \text{ km}, sL_M\left(\phi = \frac{\pi}{2}\right) \approx 140.5 \text{ km}, \\
sL_M(\phi = \pi) &\approx 75.1 \text{ km}. \quad ((46))
\end{aligned}$$



Relation (40) again shows the decrease of  $L_M$  with increase in  $\phi$  as well as the common value of the differential cross-section

$$(45) \quad (0 \leq \phi \leq \pi, L_M(\phi)) \approx \frac{1}{s} 8.08 \times 10^{-44} \text{ cm}^2 \quad ((46.1))$$

A case without the electron neutrino in the flavor changing neutrino oscillatory scattering, consistent with (38) and (15)-(24), is

$$\begin{aligned} \nu(\mu) + e &\rightarrow \nu(\tau) + e, q^0(\gamma = \mu) = 10 \text{ MeV} : \\ &\frac{d\sigma(\text{OSC}; \nu(\mu), e; ; e, \nu(\tau); \phi, L)}{dy} \\ &= \frac{1}{s} 0.85 \times 10^{-44} \text{ cm}^2 \times (-) 1.52 \times \sin \frac{(m_{\nu(\mu)\nu(\tau)})^2 sL}{4(1-y(\phi)) q^0(\mu)} \\ &= -\frac{1}{s} 1.3 \times 10^{-44} \text{ cm}^2 \sin \frac{0.56 \times 10^{-1} sL}{(1-y(\phi)) km} \end{aligned} \quad ((47))$$

At different scattering angles the baseline distances at the first (production) maximum,  $-\sin 3\pi/2$ , are

$$sL_M(\phi = 0) \approx 84 \text{ km}, sL_M\left(\phi = \frac{\pi}{2}\right) \approx 4 \text{ km}, sL_M(\phi = \pi) \approx 2 \text{ km} \quad ((48))$$

and, as before, the oscillating differential cross-section, which could be practically measured at the collision point, has the same value at these scattering angles,

$$(47) \quad (0 \leq \phi \leq \pi, L_M(\phi)) \approx \frac{1}{s} 1.3 \times 10^{-44} \text{ cm}^2 \quad ((48.1))$$

The time has come to deal with antineutrino oscillation differential cross-sections. The first process is (see note after (30))

$$\begin{aligned} \bar{\nu}(e) + e &\rightarrow \bar{\nu}(e) + e, q^0(\bar{\gamma} = \bar{e}) = 10 \text{ MeV} : \\ &\frac{d\sigma(\text{OSC}; \bar{\nu}(e), e; ; e, \bar{\nu}(e); \phi, L)}{dy} \\ &= 0.95 \times 10^{-44} \text{ cm}^2 \left[ \frac{1}{u} \sin \frac{m_{\bar{\nu}(e)\bar{\nu}(e)u}^2 L}{4(1-y(\phi)) q^0(\bar{e})} \right. \\ &\quad \left. - \frac{1}{s} \sin \frac{(m_{\bar{\nu}(e)\bar{\nu}(e)})^2 sL}{4(1-y(\phi)) q^0(\bar{e})} \right] \\ &= 0.95 \times 10^{-44} \text{ cm}^2 \left[ \frac{1}{u} \sin \frac{5.6 \times 10^{-3} uL}{(1-y(\phi)) km} \right. \\ &\quad \left. - \frac{1}{s} \sin \frac{4.6 \times 10^{-3} sL}{(1-y(\phi)) km} \right] \end{aligned} \quad ((49))$$

One rewrites (49) as

$$\begin{aligned}
& \frac{d\sigma(OSC; \bar{\nu}(e), e; e, \bar{\nu}(e); \phi, L)}{dy} \\
= & 0.95 \times 10^{-44} \text{cm}^2 \left[ \left( \frac{1}{u} + \frac{1}{s} \right) \sin \frac{10^{-3}L(5.6u - 4.6s)}{2(1-y(\phi))km} \right. \\
& \times \cos \frac{10^{-3}L(5.6u + 4.6s)}{2(1-y(\phi))km} \\
& + \left( \frac{1}{u} - \frac{1}{s} \right) \sin \frac{10^{-3}L(5.6u + 4.6s)}{2(1-y(\phi))km} \\
& \left. \times \cos \frac{10^{-3}L(5.6u - 4.6s)}{2(1-y(\phi))km} \right] \quad (50)
\end{aligned}$$

Again, at least as an exercise, it is worthwhile to assume that the scaling parameters are approximately equal,  $u \approx s$ , yielding

$$\begin{aligned}
& \frac{d\sigma(OSC; \bar{\nu}(e), e; e, \bar{\nu}(e); \phi, L)}{dy} \\
= & 1.9 \times 10^{-44} \text{cm}^2 \frac{1}{u} \sin \frac{10^{-3}Lu}{2(1-y(\phi))km} \cos \frac{10^{-3}Lu \times 10.2}{2(1-y(\phi))km} \quad (50.1)
\end{aligned}$$

Numerically one finds that (50.1) has first (production) maximum at approximately  $\frac{10^{-3}Lu}{2(1-y(\phi))km} = \frac{\pi}{5}$  yielding for the scaled baseline distances at this approximate maximum,

$$\begin{aligned}
uL_M(\phi = 0) & \approx 1256 \text{km}, uL_M\left(\phi = \frac{\pi}{2}\right) \approx 60 \text{km}, \\
uL_M(\phi = \pi) & \approx 51 \text{km} \quad (50.2)
\end{aligned}$$

Finally, of course, the differential cross-section has the same value at these scattering angles,

$$(50.1) (0 \leq \phi \leq \pi, L_M(\phi)) \approx \frac{1}{u} 1.1 \times 10^{-44} \text{cm}^2 \quad (50.3)$$

With the help from (39) and (15)-(24) one looks at

$$\begin{aligned}
\bar{\nu}(\mu) + e & \rightarrow \bar{\nu}(e) + e, q^0(\bar{\gamma} = \bar{\mu}) = 10 \text{MeV} : \\
& \frac{d\sigma(OSC; \bar{\nu}(\mu), e; e, \bar{\nu}(e); \phi, L)}{dy} \\
= & 0.952 \times 10^{-44} \text{cm}^2 \times (-) \frac{1}{s} \sin \frac{(m_{\bar{\nu}(\mu)\bar{\nu}(e)})^2 sL}{4(1-y(\phi))q^0(\bar{\mu})} \\
= & -0.952 \times 10^{-44} \text{cm}^2 \frac{1}{s} \sin \frac{0.51 \times 10^{-3}Ls}{(1-y(\phi))km} \quad (51)
\end{aligned}$$

The first positive (antineutrino production) maximum of this oscillation differential cross-section yields, with the help of (40) and (41), the scaled baseline

distances at three scattering angles,

$$\begin{aligned} sL_M(\phi = 0) &\approx 9235km, sL_M\left(\phi = \frac{\pi}{2}\right) \approx 440km, \\ sL_M(\phi = \pi) &\approx 225km \end{aligned} \quad ((52))$$

with the differential cross-section being the same at all the three scattering angles,

$$(51) \quad (0 \leq \phi \leq \pi, L_M(\phi)) \approx \frac{1}{s} 0.952 \times 10^{-44} cm^2 \quad ((52.1))$$

From 39) and (15)-(24), the last process with antineutrinos for the oscillatory differential cross-section is

$$\begin{aligned} \bar{\nu}(\mu) + e &\rightarrow \bar{\nu}(\tau) + e, q^0(\bar{\gamma} = \bar{\mu}) = 10MeV : \\ &\frac{d\sigma(Osc; \bar{\nu}(\mu), e; ; e, \bar{\nu}(\tau); \phi, L)}{dy} \\ &= 0.952 \times 10^{-44} cm^2 \times (-) \frac{1}{s} \sin \frac{(m_{\bar{\nu}(\mu)\bar{\nu}(\tau)})^2 sL}{4(1-y(\phi)) q^0(\bar{\mu})} \\ &= -0.952 \times 10^{-44} cm^2 \frac{1}{s} \sin \frac{0.56 \times 10^{-1} Ls}{(1-y(\phi)) km} \end{aligned} \quad ((53))$$

The scaled baseline distances at the first (production) positive differential cross-section maximum from (53) are

$$\begin{aligned} sL_M(\phi = 0) &\approx 84km, sL_M\left(\phi = \frac{\pi}{2}\right) \approx 4km, \\ sL_M(\phi = \pi) &\approx 2km \end{aligned} \quad ((54))$$

which is similar to the neutrino case and could be measured practically at the place of scattering. Finally, also the differential cross-section does not change with  $\phi$  with  $L_M(\phi)$ ,

$$(53) \quad (0 \leq \phi \leq \pi, L_M(\phi)) \approx \frac{1}{s} 0.952 \times 10^{-44} cm^2 \quad ((54.1))$$

## 6. Discussion

Providing that the data from the literature (19)-(24) still hold, the derived scaled baseline lengths for larger scattering angles, by and large, should be possible to measure experimentally practically in the laboratories where the collisions occur. The results of very short baseline lengths (48) and (54) are particularly inviting for their verifications.

The advantage and the power of quantum field theoretical approach to neutrino and antineutrino oscillations is in the fact that it leaves the SM Lorentz invariant (LI) and the Lorentz invariance violation (LIV) is associated with some of the terms that are proportional to  $O(m^2)$  and, as such, are negligible in the ordinary non-oscillatory laboratory experiments where the SM dominates.

However, the extrapolated neutrino and antineutrino oscillation differential cross-sections, although still proportional to  $O(m^2)$ , are very rich in content. These baseline oscillatory differential cross-sections are both flavor conserving,  $\nu(e), \bar{\nu}(e) \rightarrow \nu(e), \bar{\nu}(e)$ , but also flavor violating,  $\nu(\mu), \bar{\nu}(\mu) \rightarrow \nu(e, \tau), \bar{\nu}(e, \tau)$ . Length of the baseline (of production maximum or even absorption minimum) depends on the scattering angle  $\phi$ ; at  $\phi = 0$  they are the longest while at  $\phi = \pi$  they are the shortest. For the exemplary initial  $10\text{Mev}$  neutrino and antineutrino energy the scaled baseline at  $\phi = 0$  for production maxima stretches from  $84\text{km}$  to  $9235\text{km}$  while at  $\phi = \pi$  from  $2\text{km}$  to  $225\text{km}$ . These oscillation differential cross-sections at the lower end baseline lengths of  $L_M(\phi = \pi) \approx 2\text{km}$  for production maxima, fall into the category of oscillations at short distances and should be practically measurable in the same way as the ordinary scattering differential cross-sections. In this connection, it is worthwhile to mention that recently G. Mention et al. [23] have discussed, in reactor neutrino experiments, the appearance of  $\bar{\nu}_e$  at distances  $\leq 100\text{m}$  from the reactor core for which the non-oscillation explanation is disfavored.

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