Geometric Relation for Neutrino Mixing Angles and Theta(13)

E. M. Lipmanov
40 Wallingford Road # 272, Brighton MA 02135, USA

Abstract

Inspired by the recent T2K indication of a relatively large theta(13) angle in the neutrino mixing matrix we propose here a definite geometric relation between the three usually thought 'independent' neutrino mixing angles – solar θ_{12}, atmospheric θ_{23} and reactor θ_{13} ones: $\cos^2(2\theta_{sol}) + \cos^2(2\theta_{atm}) + \cos^2(2\theta_{13}) = 1$. Using the estimations for the two largest neutrino mixing angles from experimental data analyses in the literature, $\theta_{sol} \cong 34.4^o$, $\theta_{atm} \cong 42.8^o$, the reactor neutrino mixing angle is predicted $\theta_{13} \cong 10.8^o$. In case a little changed data, $\theta_{sol} \cong 34^o$ and $\theta_{atm} \cong 43^o$ the result will be $\theta_{13} \cong 11.2^o$, the θ_{13}-value is not very sensitive to the accurate values of the two largest mixing angles. The prediction for the 'small' neutrino mixing angle is compatible with the recent T2K experimental results with best fit values for the reactor angle $(\theta_{13})_{bf} = 9.7^o(11^o)$ for normal and inverted hierarchies respectively. It seems that the suggested geometric relation, which implies that neutrino mixing can be described by a 'unit mixing vector', may have profound physical meaning if confirmed by further experimental results.

We start with the benchmark flavor pattern in the form of bimaximal neutrino mixing matrix namely

$$\cos^2(2\theta_{12}) = 0,\ \cos^2(2\theta_{23}) = 0,\ \cos^2(2\theta_{13}) = 1, \tag{1}$$

$$\begin{pmatrix} 1/\surd 2 & 1/\surd 2 & 0 \\ -1/2 & 1/2 & 1/\surd 2 \\ 1/2 & -1/2 & 1/\surd 2 \end{pmatrix}_\nu . \tag{1'}$$

With two maximal neutrino mixing angles $\theta_{12} = 45^{o}$ and $\theta_{23} = 45^{o}$, the third independent neutrino mixing angle in (1) is uniquely determined $\theta_{13} = 0$ by the geometric relation that follows from definition (1),

$$\cos^2(2\theta_{sol}) + \cos^2(2\theta_{atm}) + \cos^2(2\theta_{13}) = 1. \quad (2)$$

It follows from this relation that neutrino mixing can be geometrically interpreted by a 'neutrino mixing vector' **N** of unit length,

$$\mathbf{N} = (N_x, N_y, N_z), \ |\mathbf{N}| = 1, \ N_x = \cos\alpha, \ N_y = \cos\beta, \ N_z = \cos\gamma,$$

$$\alpha = 2\theta_{sol}, \beta = 2\theta_{atm}, \gamma = 2\theta_{13}. \quad (3)$$

By natural suggestion, the neutrino mixing vector should remain a unit one after deviation from benchmark by a small parameter to reach realistic neutrino mixing angles; if this parameter approaches zero, neutrino mixing approaches benchmark bimaximal one with enhanced symmetry.

Considered geometric interpretation is unique for neutrino mixing, it has no analogy in the quark mixing pattern. The unit neutrino mixing vector is a specific neutrino quantity as exceptionally small neutrino mass and probably Majorana nature are.

The simple equation (2) is a suggestive zero approximation of neutrino mixing. It contains the seeds of realistic neutrino mixing. It tells that the three neutrino mixing angles are not independent. It should predict a realistic 'small' reactor neutrino angle since the known from oscillation data solar and atmospheric angles are large.

Estimations of the two largest neutrino mixing angles from analyses of neutrino oscillation data [1-3] are at 1σ

$$\theta_{sol} = (34.4 \pm 1.0)^{o}, \ \theta_{atm} = (42.8^{o} + 4.7 - 2.9)^{o}. \quad (4)$$

If the central values of the two large angles in (4) are used, the estimation of the realistic reactor mixing angle is uniquely predicted from the relation (2),

$$\theta_{13} \cong 10.8^{o}. \quad (5)$$

In a phenomenological model where the neutrino and quark mixing angles are united by one universal small ε-parameter [4] the solar and atmospheric angles are predicted

$$\theta_{sol} \cong 34.04^{o}, \ \theta_{atm} \cong 42.64^{o}. \quad (6)$$

In this case, from the geometric relation (2) follows[1]

$$\theta_{13} \cong 11.24^{o}. \qquad (7)$$

So, it seems from data indications that the θ_{13}-angle is not very sensitive to small corrections for the two large neutrino mixing angles.

The predictions (5) and (7) for the small reactor neutrino mixing angle are compatible with the recent T2K experimental results [5]:

$$0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34) \qquad (8)$$

for normal and inverted hierarchies respectively. The best fit experimental values for that mixing angle are

$$(\theta_{13})_{bf} \cong 9.7^{o}(11^{o}). \qquad (9)$$

The emerging hierarchies of neutrino and quark mixing angles may be outlined in terms commonly used for neutrino mass hierarchies – the neutrino mixing-angle hierarchy, e.g. (6) and (7), is like an 'inverted' one and the quark mixing-angle hierarchy is like 'normal' one (e.g. $\theta_c \cong 13^{o}$, $\theta_{23} \cong 2.4^{o}$, $\theta_{13} \cong 0.2^{o}$). With regard to particle mass spectra, the quark mass hierarchies are more like normal ones especially for the down quark masses (e.g. $m_d \cong$ 5-8, $m_s \cong 100$, $m_b \cong$ 5,000) MeV. Is the neutrino mass hierarchy of inverted type as probably indicated by comparison of (5), (7) and (9)? More confident experimental results are needed to reach a definite answer to this question.

The simple geometric suggestion that neutrino mixing can be described by a unit mixing vector (3) may have profound physical meaning if confirmed by further experimental results.

References

[1] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado, arXiv:1001.4524, v.4.

[2] L. Fogli, E. Lisi, A. Marone, A. Palazzo and A. M. Rotunno, arXiv:1106.6028.

[3] T. Schwetz, M.A. Tortola, J.W.F. Valle, arXiv:1103.0734.

[4] E. M. Lipmanov, arXiv:1101.4644.

[5] K. Abe et al., arXiv:1106.2822.

---

[1] From [4], $\cos^2\alpha = (2\varepsilon)\exp(-2\varepsilon)$, $\cos^2\beta = (\varepsilon^2)\exp(\varepsilon^2)$ and so from relation (2) follows $\cos\gamma = [1 - 2\varepsilon\exp(-2\varepsilon) - \varepsilon^2\exp(\varepsilon^2)]^{1/2}$, which determines the estimation (7).