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Comparative Study on the Complex Eigenvalue Prediction of Brake Squeal by Two Infinite Element Modeling Approaches

CHEN Guangxiong*, WU Pingbo, DAI Huanyun, and ZHOU Zhongrong

State Key Laboratory of Traction Power, Southwest Jiaotong University, Chengdu 610031, China

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Abstract: The complex eigenvalue analysis is currently a common approach to predict squealing vibration and noise. There are two methods for modeling friction contact in the complex eigenvalue analysis of friction systems. In one method, contact springs are used to simulate friction contact. In another method, no contact spring is used. However, it has been uncertain whether these two modeling methods can predict approximately identical results. In order to clarify the uncertainty, two finite element models of the same brake system for the brake squeal prediction are established and simulated by using ABAQUS and NASTRAN software tools, respectively. In the ABAQUS model, friction coupling is applied to determine normal contact force and no contact spring is assumed. Whilst in the NASTRAN model, the contact spring is assumed by the penalty method to simulate contact connection. Through the numerical simulations, it is recognized that even if the same mesh geometry is applied, generally, these two finite element approaches are not capable of predicting approximately identical unstable frequencies. The ABAQUS approach can predict instabilities of high frequency up to 20 kHz or more, while the NASTRAN approach can only predict some instabilities of high frequency, not all. Moreover, the simulation results also show that both the contact spring stiffness and mesh size have influences to some extent on the prediction results of squeal. The present comparative work illuminates that the modeling method without contact springs is more suitable to predict squealing vibration and noise, comparing to the modeling method with contact springs. It is proposed that one should prefer using the modeling method without contact springs to predict squealing vibration and noise. The proposed study provides the reference for predicting squealing vibration and noise.

Key words: friction, brake, squeal, noise, finite element

1 Introduction

Investigations into brake squeal may be traced back to 1930s. However, there are not any technology and method available to completely eliminate brake squeal up to now[1]. The effective remedy of brake squeal is dependent on the understanding of the generation mechanisms of brake squeal. Among many squeal generation mechanisms published in the literature, several theories, such as stick-slip, negative friction-velocity slope, sprag-slip, mode coupling, modal splitting and hammering mechanisms, are generally accepted as potential mechanisms[1–5]. Most recently, the authors proposed a new mechanism for brake squeal, which was based on the time delay between the varying normal force and consequent friction[6]. The new mechanism may explain some phenomena associated with squeal[7]. However, there is still a long way to go for a complete understanding of squeal generation mechanisms.

According to CROLLA, et al[3] and OUYANG, et al[8], the finite element complex eigenvalue analysis of brake squeal has become one of main research and application activities in the brake squeal research community currently. OUYANG, et al [8], published a comprehensive review on numerical analysis of automotive disc brake squeal. In OUYANG's paper, the states of the art of the finite element analysis of brake squeal are introduced and commented in detail. Historically, LILES[9] presented a finite element complex eigenvalue analysis of a disc brake system in 1989, which appeared to be the first paper by using the finite element method to analyze the vibration stability of brake systems. He applied an approximate method, which was called the component modal synthesis method, to build the finite element model of a disc brake system. Friction was incorporated into the model as a geometric coupling. The eigenvalue equations of the finite element model of a brake system are solved and the complex eigenvalues are obtained. If there is one or more eigenvalues with positive real parts, the system is considered to be unstable. The corresponding imaginary part is thought to be a possible squeal frequency. One advantage of applying the component modal synthesis method is that the freedom

* Corresponding author. E-mail: chen_guangx@163.com
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degrees of the finite element model may be largely reduced, which was very important for investigators in 1980s and the early days of 1990s when computation techniques were limited. Another advantage is that the influential component modes on squeal can be found and the squeal can be suppressed through modifying corresponding component structures. It was reported that only modifying several special points of the components of a brake system can remedy some squeals[10]. The drawbacks are that the method is an approximate method and is not easy to include real boundary conditions. NACK[11] conducted a new finite element complex eigenvalue analysis of a disc brake system in 2000, in which the stiffness matrix of the finite element system was directly modified to include friction coupling. NACK's direct stiffness matrix modification method is easy to include real boundary conditions and easier to establish the finite element model. It should be noted that both LILES and NACK's methods require connection of coincident nodes on the contact surfaces with massless springs. The assumption of massless contact springs results in several problems. YUAN[12] revealed that massless springs might not be suitable for representing rotor/pads contact at high squeal frequencies. In terms of contact modeling, the contact conditions are exactly applied when the spring stiffness k_c is infinitely large. In the actual numerical analysis, however, k_c has to be finite for a stable numerical solution and thus leads to concerns on accuracy. YUAN[12] proposed an innovative method to tackle this issue. In YUAN's method, contact constraints in the normal direction are mathematically imposed. Recently, ABAQUS version 6.4 and the higher versions provide a complex eigenvalue solution to the disc brake squeal problem. This new capability uses direct contact coupling at the friction sliding interface described by YUAN and there is no need to introduce contact springs at the interface[8]. Many useful results have been produced by using the new capability of ABAQUS[13–14]. Other investigators also made effective endeavors in modeling of brake squeal and application of finite element approaches to solution of brake squeal. OUYANG, et al[15–16], formally proposed to treat the disc brake vibration and squeal as a moving load problem and put forward an analytical-numerical combined method. LEE, et al[17], proposed a new model philosophy of brake squeal, in which the contact stiffness was variable. It was dependent on local contact pressure distribution, which was determined by nonlinear contact analysis. HUANG, et al[18], made a modification of the component modal synthesis method so that this method can also predict the squeal propensity of brake systems without the need for connection of coincident nodes on the contact surfaces.

So far, the finite element analysis methods published in the literature may be classed into two categories. One is based on the contact spring assumption at the interface, which includes the component modal synthesis method and NACK's direct stiffness matrix modification method. Another is based on YUAN's method, whose characteristic is without contact spring assumption at the interface. It is well known that the reliability of the finite element analysis is strongly dependent on the finite element modeling method. Naturally, these factors also affect the accuracy of the finite element complex eigenvalue analysis of brake squeal. However, it appears that there are very limited publications on the reliability of the modeling methods with and without the contact spring assumption in the literature. OUYANG, et al[8], cited YUAN's orally published conclusion that when k_c was in the order of 1 GN/m. The results obtained by YUAN's method are the same as by the modal synthesis method. Recently, LOUA, et al[19], applied a modeling method similar to YUAN's method to perform a finite element analysis of brake squeal. They reported that the results obtained by these two modeling approaches were in good agreement. LOUA's results show that the modal synthesis method could predict high frequency instabilities of 22.98 kHz, which is not consistent with YUAN's result[12]. YUAN's result clearly concluded that massless springs might not be suitable for representing rotor/pads contact at high squeal frequencies[12]. However, the material and modeling parameters used in LOUA's paper were not given. It appears that OUYANG, et al do not comment which one of these two modeling approaches is more appropriate for prediction of squeal[8].

If different finite element modeling approaches predict different results of squeal, these approaches can not be considered to be reliable. The purpose of the paper is to study the influence of the finite element modeling approaches on prediction results of brake squeal. The present results show that the prediction results of squeal are dependent on modeling approaches to some extent. The results obtained by the method with contact spring assumption are generally different from those obtained by YUAN's approach without contact spring assumption.

2 Modeling of a Brake System for Squeal and Model Details

2.1 Modeling of a brake system based on the contact spring assumption

Fig. 1 shows a simple brake system model, which consists of one disc and two pads. In the model, subscripts d and p stand for one node at the contact surface on the disc and corresponding coincident node on the pad, respectively. Pure sliding friction between the disc and pads is assumed. A static preload is assumed to be large enough to maintain full contact on the pad surface. The frictional coefficient is assumed to be constant. Frictional forces on the pad and sliding surface are proportional to the normal contact forces, which in turn may vary with dynamic response. The normal force may be written as follows[17, 19]:

$$F_{nd} = -F_{np} = -K_f(u_d - u_p), \tag{1}$$

where F_{nd} and F_{np} represent the normal forces acting on the disc node and the corresponding pad node, respectively. K_f stands for the stiffness of the massless contact spring. u_d and u_p represent the normal displacements of the disc node and the corresponding coincident pad node, respectively. The friction force is expressed as

$$F_{d} = -F_{p} = -\mu K_{f}(u_{d} - u_{p}), \tag{2}$$

where F_d and F_p stand for the friction forces acting on the disc node and the corresponding pad node, respectively. The direction of friction force is consistent with the relative sliding direction of the contact node pair. The components of the friction force in Cartesian system of coordinates are shown in Fig. 1. They may be expressed as follows:

$$F_{dy} = F_{d}\sin\theta = -\mu K_{f}(u_{d} - u_{p})\sin\theta, \tag{3}$$

$$F_{dz} = F_{d}\cos\theta = -\mu K_{f}(u_{d} - u_{p})\cos\theta, \tag{4}$$

$$F_{py} = F_{p}\sin\theta = -F_{d}\sin\theta = -F_{dy} = \mu K_{f}(u_{d} - u_{p})\sin\theta, \tag{5}$$

$$F_{pz} = F_{p}\cos\theta = -F_{d}\cos\theta = -F_{dz} = \mu K_{f}(u_{d} - u_{p})\cos\theta. \tag{6}$$

Where F_{dy} and F_{dz} represent the components in y and z directions of the friction force acting on disc node, respectively. F_{py} and F_{pz} stand for the components in y and z directions of the friction force acting on pad node, respectively. The matrix of friction force components may be written as follows:

$$\begin{pmatrix} F_{dy} \\ F_{dz} \\ F_{py} \\ F_{pz} \end{pmatrix} = \mu K_{f} \begin{pmatrix} -\sin\theta & \sin\theta \\ -\cos\theta & \cos\theta \\ \sin\theta & -\sin\theta \\ \cos\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} u_{d} \\ u_{p} \end{pmatrix} = \boldsymbol{K}_{ff}\boldsymbol{u}, \tag{7}$$

where $\boldsymbol{K}_{ff}$ and $\boldsymbol{u}$ are friction stiffness matrix and displacement vector of all nodes, respectively.

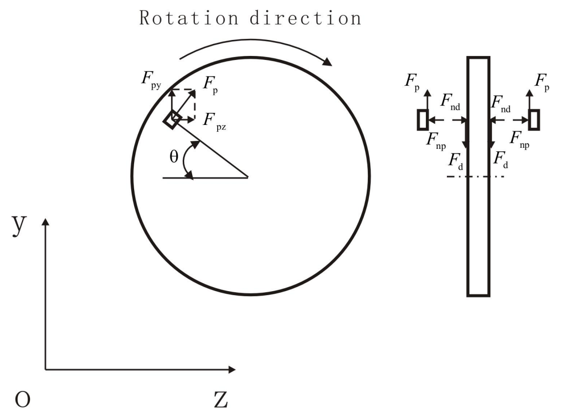


Fig. 1. Friction force and its components

When all components of brake system are meshed, the equations of motion without including friction are established as

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{0}, \tag{8}$$

where $\boldsymbol{M}$ and $\boldsymbol{K}$ are mass matrix and stiffness matrix, respectively. $\ddot{\boldsymbol{u}}$ is an acceleration vector. Further, the equations of motion including friction are established as

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + (\boldsymbol{K} - \boldsymbol{K}_{ff})\boldsymbol{u} = \boldsymbol{0}. \tag{9}$$

And corresponding eigenvalue equations is expressed as

$$(\boldsymbol{K} - \boldsymbol{K}_{ff} - \boldsymbol{\lambda}\boldsymbol{M})\boldsymbol{\varphi} = \boldsymbol{0}, \tag{10}$$

where $\boldsymbol{\lambda}$ is an eigenvalue vector, $\boldsymbol{\varphi}$ is an eigenvector. Assuming $\lambda_n = \alpha_n + \mathrm{i}\omega_n$, we can express the displacement of a node as

$$u(t) = \mathrm{Re}\sum \boldsymbol{\varphi}_{n}\exp((\alpha_{n} + \mathrm{i}\omega_{n})t). \tag{11}$$

From Eq. (11), it can be seen that if a real part of the eigenvalues is positive, the displacement of a node will increase with time, e.g., the vibration system is unstable.

In the present paper, NASTRAN software is applied to implement corresponding analysis. The new stiffness matrix $\boldsymbol{K}-\boldsymbol{K}_{ff}$, which includes the friction stiffness matrix, in Eq. (9), may be generated easily by using the DMIG card of NASTRAN. This procedure is called Nack's direct stiffness matrix modification in this paper.

2.2 ABAQUS modeling method for brake squeal

Although the details of modeling brake squeal are not presented in the ABAQUS manual documentations, it is known that the modeling method uses direct contact coupling at the disc/pads interface described by YUAN and there is no need to introduce contact springs at the interface[8]. The main procedures for applying ABAQUS to perform the complex eigenvalue analysis of brake squeal are given as follows.

(1) Nonlinear static analysis for applying brake-line pressure.

(2) Nonlinear static analysis to impose the rotational speed on the disc.

(3) Normal mode analysis to extract natural frequencies of undamped system.

(4) Complex eigenvalue analysis that incorporates the effect of friction coupling.

2.3 Model details

The brake system used in the present work is shown in Fig. 2. The system consists of a disc, two pads, two backplates, a caliper, and a carrier bracket. The disc is 260 mm in diameter. The contact area between each pad and the disc is 4 071 mm^2. In order to compare the results obtained by these two finite element modeling approaches, the identical mesh geometry is applied to establish an ABAQUS model without the contact spring assumption and a NASTRAN model with the contact spring assumption, respectively. It needs to be noted that the nodes on the disc at the sliding contact interface are coincident with those on the pads in Fig. 2. In the ABAQUS model, the elements of the disc, pads and backplates are chosen as modified 10-node tetrahedral element C3D10M. In the NASTRAN model, the elements of the disc, pads and backplates are set as 10-node tetrahedral element CTETRA. The same constraints are imposed for both models. The brake pressure is acted on the corresponding positions of a backplate and a caliper in the ABAQUS model, while no brake pressure is applied in the NASTRAN model. In the present work, two models are established which correspond to two different mesh sizes. These two models are called the rough model and the fine model, respectively. In the rough model, there are nodes of 62 804, elements of about 30 893 and coincident contact node pairs of 576 between the disc and each pad. In the fine model, there are nodes of 126 645, elements of about 94 165 and coincident contact node pairs of 1 272 between the disc and each pad.

Fig. 2. Model of a brake system

Nominal values of material and simulation parameters are presented in Table.

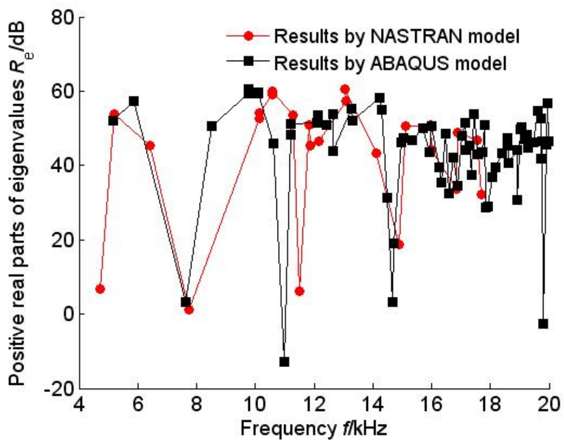
Table. Nominal values of material and simulation parameters

Material and simulation parameter	Value
Density of disc material ρ_d/(kg • m^{-3})	7 600
Young's modulus of disc material E_d/(GN • m^{-2})	180
Poisson's ratio of disc material ν_d	0.29
Density of backplate material ρ_b/(kg • m^{-3})	7 800
Young's modulus of backplate material E_b/(GN • m^{-2})	210
Poisson's ratio of backplate material ν_b	0.3
Density of lining material ρ_{lin}/(kg • m^{-3})	2 800
Young's modulus of lining material E_{lin}/(MN • m^{-2})	800
Poisson's ratio of lining material ν_{lin}	0.3
Friction coefficient μ	0.4
Pressure acting on the backplate F_{np}/kN	1
Rotational velocity of the disc ω/(rad • s^{-1})	5

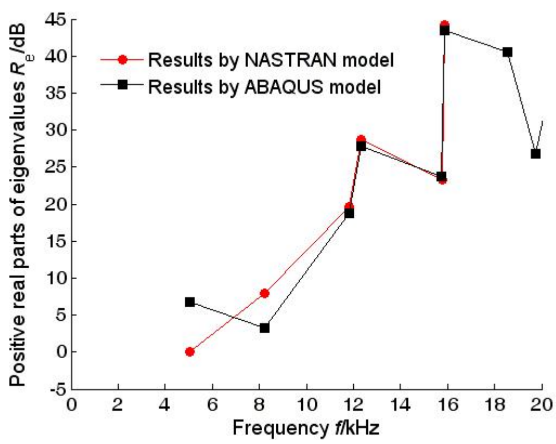
2.4 Validity of NASTRAN and ABAQUS modeling approaches

Before the present work proceeds, the validity of both NASTRAN and ABAQUS modeling approaches needs to be verified. Since the NASTRAN modeling approach is different from the ABAQUS modeling method, we consider that the NASTRAN modeling approach and the ABAQUS modeling approach are valid if they can predict approximately identical results in some conditions. Fig. 3 shows a simple brake system and a comparison between the results obtained by the NASTRAN and ABAQUS modeling approaches.

(a) Model of a simple brake system



(b) Comparison between the results from two models



(c) Comparison between the results from two models for special material parameters

Fig. 3. Comparison between the results from two

modeling approaches

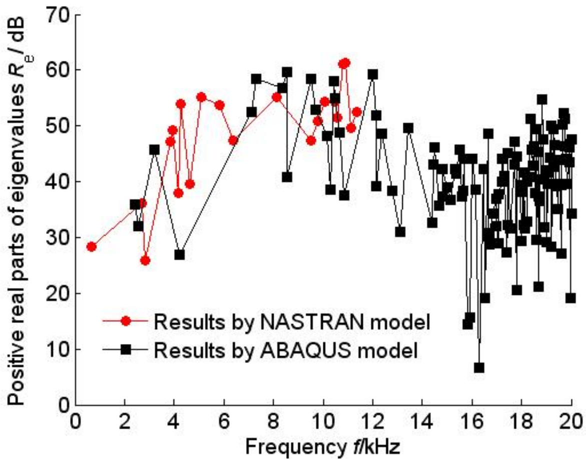
In Fig. 3, since maximum and minimum positive real parts are largely different, all positive real parts are alternatively expressed in Decibel. The Decibel value of a positive real part in the figures is equal to $20\lg\alpha$, where α is the value of positive real part. From Fig. 3(b), it can be seen that applying nominal material parameters theses two modeling approaches predict different unstable frequencies. From Fig. 3(c), it can be seen that the unstable frequencies predicted by the NASTRAN model are close to those predicted by the ABAQUS model when only material parameters of lining are changed to those of the disc. On the basis of the approximately identical results, it can be concluded that the NASRTRAN and ABAQUS modeling approaches used in the present work are valid.

3 Results and Discussion

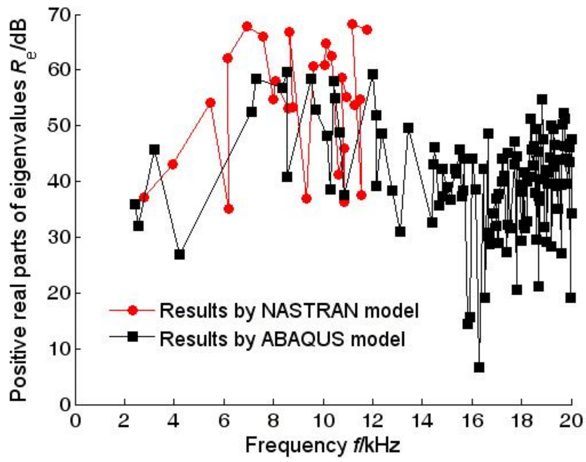
3.1 Comparison between the results from two modeling approaches

Fig. 4 shows a comparison between the results obtained by using these two modeling approaches. From Figs. 4(a)–4(b), it can be seen that the unstable frequencies predicted by ABAQUS model are much more than those predicted by NASTRAN model, especially for those in the high frequency range. It is also found that the values of the positive real parts of the eigenvalues obtained by these two approaches are different in most cases. If the material parameters of lining are changed to those of the disc whilst other parameters are kept constant, the conclusion is also true as shown in Fig. 4(c). However, it is found that some unstable frequencies predicted by these two approaches are approximately identical. For example, the NASTRAN model predicts an unstable frequency of 1 011.1 Hz whilst the ABAQUS model predicts an unstable frequency of 1 081.5 Hz. From Fig. 4, it can be concluded that the results predicted by the NASTRAN modeling approach are generally different from those obtained by the ABAQUS modeling approach. It can be also seen that the NASTRAN model can only predict the maximum unstable frequency of 11 967.1 Hz while the ABAQUS model can predict the maximum unstable frequency of 20 kHz or more. The present conclusions appear not to be consistent with YUAN's and LUO's conclusions. This suggests that there is some work needed to be done in order to obtain an optimal finite element modeling approach of brake squeal.

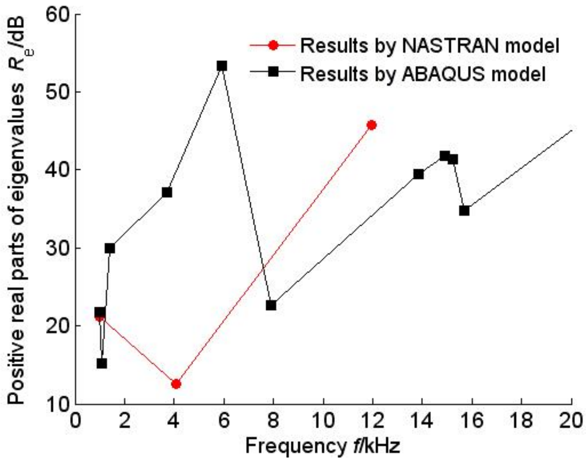
In addition, the calculation costs also need to be taken into account. The times consumed to implement these two approaches are largely different. Building the ABAQUS model and implementing its simulation spent 30 min and 120 min, respectively. While using the same computer, we spent 30 min and 15 min on building the NASTRAN model and implementing its simulation, respectively. Therefore, applying NASTRAN to analyze complex eigenvalus of brake systems is more efficient than applying ABAQUS to do.



(a) Contact spring k_c=320 GN/m



(b) Contact spring k_c =0.62 MN/m

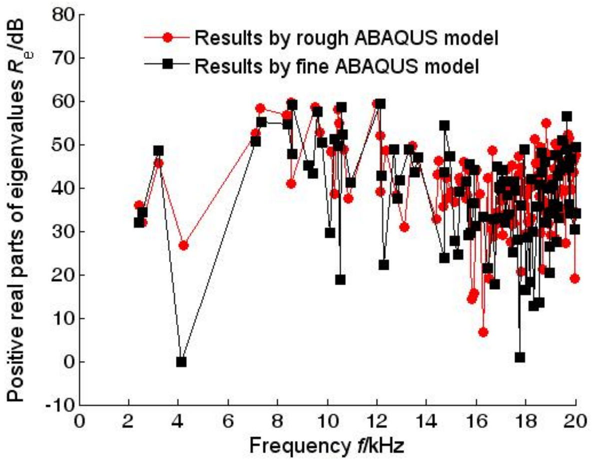


(c) Material parameters of lining are changed to those of the disc

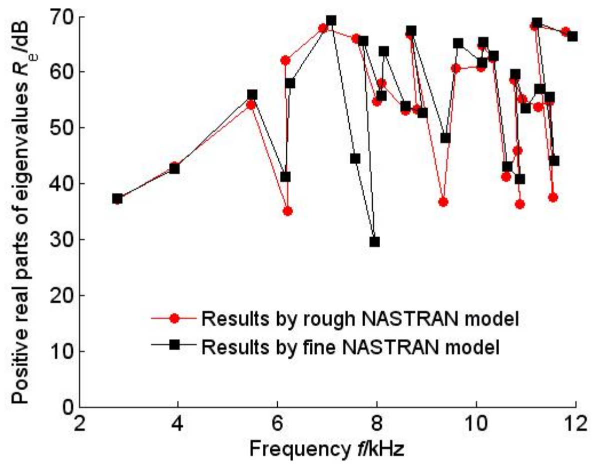
Fig. 4. Comparison between the results obtained from two modeling approaches by using the rough model

3.2 Effect of the mesh size on prediction results of squeal

It is well known that the accuracy of the results of finite element analyses is strongly dependent on the mesh size of models. Generally, the smaller the mesh size is, the higher the accuracy of the results of finite element analyses is. Fig. 5 shows two results of finite element complex eigenvalue analyses when the mesh sizes are different. From Fig. 5(a), it can be seen that when the mesh sizes are different, the results of finite element complex eigenvalue analyses obtained by using the ABAQUS modeling approach are also somewhat different. The time consumed to implement the fine ABAQUS model is very expensive, about 840 min. From Fig. 5(b), it is found that for different mesh sizes, the results of finite element complex eigenvalue analyses obtained by using the NASTRAN modeling approach are roughly alike. The time consumed to implement the fine NASTRAN model is about 40 min. Comparing Fig. 5(a) with Fig. 5(b), it is found that the mesh size has a smaller influence on prediction results from NASTRAN model than on those from ABAQUS model.



(a) Comparison between the results from the fine ABAQUS model and the rough ABAQUS model



(b) Comparison between the results from the fine NASTRAN model and the rough NASTRAN model

Fig. 5. Effect of the mesh size on prediction results of squeal

3.3 Influence of the contact spring stiffness on prediction results of squeal

In Refs. [17–20], there are two different considerations in determining the value of the contact spring stiffness. One consideration is based on the penalty method, in which the spring stiffness k_c is required to be infinitely large in order to exactly simulate the contact conditions[11, 17]. LEE, et al[17], recommended the contact spring stiffness to be in the order of 1–100 GN/m. Another consideration is based on the actual contact stiffness between the disc and pad, in which the spring stiffness k_c is determined by experiments or theoretical calculations. The formula of contact spring stiffness is given as follows[18, 20]:

$$k_c = \frac{E_{lin} A_{lin}}{n h_{lin}}, \tag{12}$$

Where E_{lin} is the Young's modulus of lining material, A_{lin} is the contact area between the disc and each lining, h_{lin} is the thickness of each lining and n is the total number of contact springs between the disc and each lining. In the present work, the contact spring stiffness is calculated as 620 kN/m for the NASTRAN model. Obviously, these two different considerations may lead to different values of contact spring stiffness. It is a concern to researchers in which one of these two considerations is more appropriate for predicting squeal propensity. Fig. 6 shows the variation of the predicted results for several stiffness values of a contact spring. From Fig. 6, it can be seen that when the contact spring stiffness k_c=3.2–320 GN/m, the unstable frequencies are very close to each other and the positive real parts of eigenvalues are approximately identical. It can be also seen that when the contact spring stiffness k_c is below 32 MN/m the unstable frequencies are different from those when the contact spring stiffness k_c is above 3 200 MN/m. Especially when the contact spring stiffness is determined to k_c=0.62 MN/m based on Eq. (12), the predicted results is clearly different from those when the contact spring stiffness k_c is above 3 200 MN/m.

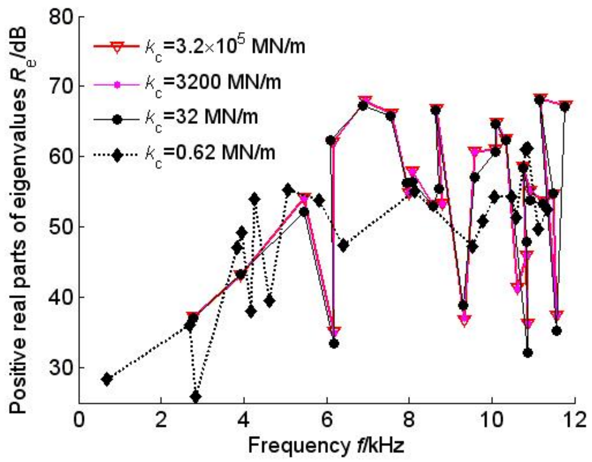


Fig. 6. Influence of contact spring stiffness on prediction results of squeal using the rough model

3.4 Influence of negative friction-velocity slope on prediction results of squeal

YUAN[12] indicated that the influence of negative friction–velocity gradient should not be ignored. KUNG, et al[13], indicated that the real parts increased with the negative slope of the friction–velocity curve. In the present work, the influence of negative friction–velocity gradient is also studied. The test data exponential model provided by ABAQUS packages is applied, which represents a stribeck curve. In the test data exponential model, the static friction coefficient μ_s=0.6, the dynamic friction coefficient measured at the wheel rotational speed 5 rad/s μ_d=0.4, the asymptotic value of the friction coefficient at infinite slip rate μ_∞=0.32. Fig. 7 shows a comparison between the results from the constant friction and the friction with negative friction-velocity slope. From Fig. 7, it can be seen that the positive part values of eigenvalues with a negative friction–velocity slope are generally larger than those with a constant friction. The result suggests that the propensity of squeal occurrence increases in the presence of a negative friction-velocity slope.

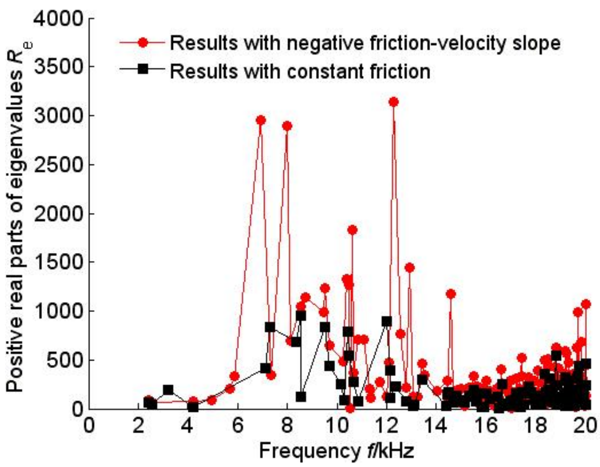


Fig. 7. Influence of friction-velocity slope on squeal propensity by using the rough model

3.5 Influence of the connection of coincident and non-coincident nodes on prediction results of squeal

The modeling method based on the contact spring assumption requires connection of coincident nodes at the interface with massless springs. The requirement is to enforce the condition that the friction interfaces do not separate in the normal contact direction during squeal. The ABAQUS modeling method does not require the connection of coincident nodes. In the present work, the ABAQUS modeling method is applied to study the influence of the connections of coincident and non-coincident nodes at the interface on prediction results of squeal. The non-coincident node model has modified 10-node tetrahedral elements of 32 841 with nodes of 62 632. The results are shown in Fig. 8. From Fig. 8, it is found that there is not a distinct difference between the results obtained by using the connections of coincident and non-coincident nodes except that there is an additional frequency of 4 204.2 Hz for the results with the connection of non-coincident nodes.

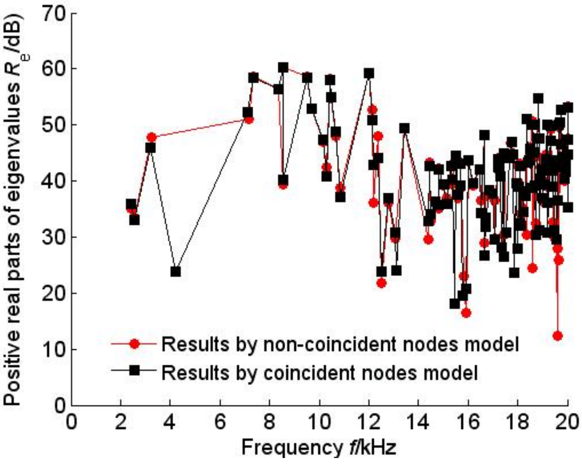


Fig. 8. Comparison between the results obtained by using the connections of coincident and non-coincident nodes

4 Conclusions

The complex eigenvalue analysis of brake systems is a very useful tool to predict generation tendency of brake squeal nowadays. In this work, the comparison between the results from these two finite element modeling approaches is studied. The following conclusions can be drawn.

(1) In generally, the unstable frequencies predicted by the ABAQUS method without contact spring assumption are different from those predicted by NASTRAN model based on the contact spring assumption.

(2) The modeling method based on the contact spring assumption can only predict some instability at high frequency, not all. The modeling method without contact spring assumption can predict unstable frequencies up to 20 kHz. Therefore, it is recommended that one should prefer applying ABAQUS modeling approach for predicting brake squeal.

(3) When contact spring stiffness is more than 3 200 MN/m, the unstable frequencies predicted by the modeling method based on the contact spring assumption are very close to each other and the positive real parts of their eigenvalues are approximately identical.

(4) For the ABAQUS modeling approach, when the mesh sizes are equivalent the unstable frequencies obtained by using the connections of coincident are close to those obtained by using the connections of non-coincident nodes.

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Biographical notes

CHEN Guangxiong, born in 1962, is currently a professor in State Key Laboratory of Traction Power, Southwest Jiaotong University, China. He received his PhD degree from Southwest Jiaotong University, China, in 2003. His research interests include friction-induced vibration and noise, friction and wear of materails and vehicle system dynamics.
Tel: +86-28-87634122; E-mail: chen_guangx@163.com

WU Pingbo, born in 1968, is currently a professor in State Key Laboratory of Traction Power, Southwest Jiaotong University, China. He received his PhD degree from Southwest Jiaotong University, China, in 1991. His research interests include vehicle system dynamics and control, and vehicle structural strength.
Tel: +86-28-87600882; E-mail: wupingbo@163.net

DAI Huanyun, born in 1966, is currently a professor in State Key Laboratory of Traction Power, Southwest Jiaotong University, China. He received his PhD degree from Southwest Jiaotong University, China, in 2001. His research interests include vehicle system dynamics and control, and vehicle structural strength.
Tel: +86-28-86466616; E-mail: daihuanyun@163.com

ZHOU Zhongrong, born in 1965, is currently a professor in State Key Laboratory of Traction Power, Southwest Jiaotong University, China. He received his PhD degree from the Central School of Lyon, France, in 1995. His research interests include friction and wear of materials and surface engineering.
Tel: +86-28-87600971; E-mail: zrzhou@swjtu.edu.cn