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Practical Method of Conical Cam Outline Expansion

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Abstract: Conical cam mechanism has been widely used in modern machinery and equipment. However, the commonly used planar expansion methods for the design of spatial cam contour produce significant errors, because these methods incorrectly use the distance from the axis of the follower to the main conical cam to replace the corresponding arc length on the conical cam. HSIEH, et al, used analytical methods to achieve higher accuracy, but these analytical methods have their own drawbacks since they are too complicated for practical use. Through the analysis of the errors created during the generation of conical cam contour using the existing expansion methods, this paper proposes to include diverge angle in the calculation of conical cam rotation angle in the equation of conical cam contour expansion. This correction eliminates the error generated by the commonly used methods. Based on the expression of the follower's 3D trajectory and the spatial geometry of conical cam, this paper has deduced the planar polar curve equation for determining polar coordinates for the curve of planar expansion outline. Furthermore, this paper provides an example of conical cam contour design based on sinusoidal acceleration variation. According to polar coordinates and the movement of curve equation function expression, this paper applies MATLAB software to solve coordinates for the cam expansion curve and use AutoCAD software to generate conical cam expansion contour that meets the requirement of the law of motion. The proposed method provides a design process that is simple, intuitive and easy to master and implement. It also avoids the design error in the traditional methods for generating contour of conical cam with oscillating follower that requires high precision.
Key words: oscillating follower, conical cam, 3D expansion, polar coordinates, curve equation

1 Introduction

Spatial cam mechanisms are extensively used in modern machinery and equipment, since they have the advantages of accuracy, reliability and adaptability to meet the requirement for complex motions, such as for manipulating and feeding mechanisms. In recent years, with the rapid development of automatic and high-speed machining tools, the demand for spatial cam mechanism is increasing rapidly and the requirements for designing cam-follower's motion curve become more complex. Therefore, the method for creating spatial cam contour has become a hot problem in recent years.

Usually there are two methods for obtaining the cam outline expansion, i.e., graphical method and analytical method. The former is simple, intuitive and easy to master with the help of graph, but the shortcoming of this method is its low accuracy, especially for conical cam with oscillating follower. The later method uses multiple parameters, complex equations and requires extensive calculations, which limits its practical application. Thus, it remains a major challenge as how to obtain cam outline expansion both efficiently and accurately. This problem catches the attention of researchers in the world

Many researchers have already carried out studies on this cam outline expansion problem. HSIEH[1] presented a simple yet comprehensive method for the design and machining of a cylindrical cam with a meshing indexing disc. CHEN, et al[2], deduced the equations for planar profile expansion and methods of calculating pressure angle by applying 3D expansion formula of the follower's motion orbit, and the minimum base radius can be determined by using MATLAB software. HSIEH and LIN[3] applied the homogenous transformation matrix for the measurement of cam profiles on coordinate measuring machines. LIN, et al[4], established a systematic design approach for cam mechanism with moderate speeds. LEE, et al[5], developed a new method to generate toolpath, which combines the advantages of the creating method and sculpturing method for machining a spatial cam. Based on the geometric relationships at the contact point between a planar cam profile and its follower, TSAY, et al[6], proposed that analytical descriptions could be utilized to determine the follower displacement curve and its derivatives.

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GRANT and SONI[7] worked out the analytic expression for cam contour surface of conical cam mechanism with oscillating tapered roller follower in 3D space by applying the theory of envelopes to one parameter family of surfaces. TASY and LIN[8] defined the planar or spatial cam profile by the envelope of its follower surfaces represented in a parametric form in different relative positions of the cam and the follower. YIN, et al[9], used the web-based remote design system to formulate spatial cam mechanisms based on mathematical models. NISHIOKA[10] evaluated the offset error of cylindrical cam. However the equations obtained using the above methods are too complicated, the geometrical relationship is not very strong, and thus limit their practical use in cam design.

With the application of scientific computation and CAD/CAM technology, we have made unprecedented progress in improving the design and manufacturing processing of spatial cam. CHEN, et al[11], proposed a “3D expansion method” to design cam mechanism by dividing the cam-follower’s motion. They also presented a series of manufacturing plans for cams with different motion requirements. This “3D expansion method” solved the cam’s NC manufacturing problem and was used in practice with the help of CAD software. On the basis of CHEN, et al[12, 13], this paper presents a further development of method for designing conical cam contour outline expansion. The existing conical cam outline expansion methods were analyzed, and a new method is proposed to eliminate common error in the existing methods.

2 Analysis of Error in the Existing Cam Outline Expansion Methods

Conic cam mechanism has two types, i.e., conic cam mechanism with translating follower and that with oscillating follower. Fig. 1 shows the conic cam with translating roller follower. The follower’s translating direction vector interacts with the conic camshaft’s axis. The angle is γ, which is also the half of the cone’s angle. When the conic cam rotates, the translating roller follower moves passively following the grooved obit of the cam, the follower then makes the roll to translate back and forth. Because the outline expansion problem is relatively simple for conic cam with translating follower, this paper will be devoted to analyze the outline expansion problem for conic cam mechanism with oscillating follower.

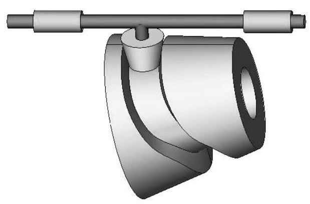

Fig. 1. Conic cam mechanism with translating roller follower

Fig. 2 is the 3D model of conic cam mechanism with oscillating follower (including spatial roller, follower and oscillating bar frame). The oscillating roll’s axis is perpendicular to the camshaft in space. *MN* is the common perpendicular line of the two axes. If point *N* is set to be the origin which is the intersection point between *MN* and the camshaft axis, the vector is set to be *Z*-axis which is parallel to the oscillating roll’s axis, the vector of common perpendicular line is set to be *Y*-axis, then a right-hand coordinate system can be built.

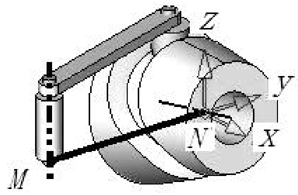

Fig. 2. Mechanism of spatial cam with oscillating follower

Here is the assumption. Let us suppose that ψ is the oscillating angle of the oscillating bar, l is the length of the oscillating bar, φ is the rotation angle of the spatial cam, R is the radius of the conic cam, γ is half of the cone angle, and a is the distance from the rotation axis of oscillating bar to the rotary axis of spatial conic cam. SHI and WU[14] introduced a method that can be used to design cam outline expansion curve for conic cam mechanism with translating follower and oscillating follower. The design process is relatively simple for conical cam mechanism with translating follower. Thus we emphasize on the cam contour expansion for conic cam mechanism with oscillating follower.

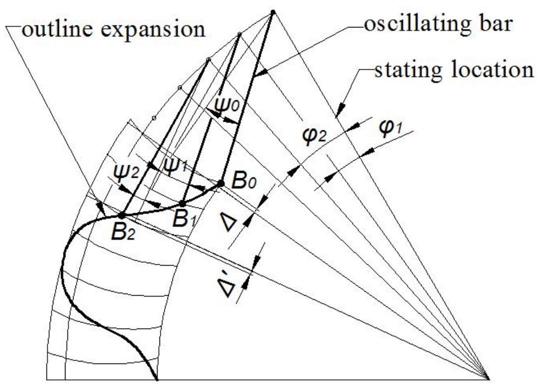

Fig. 3. Design of outline expansion curve

As shown in Fig. 3, on the oscillating bar’s moving plane, there will be two points at most, which are the cross points between the arc locus of the oscillating follower and conic camshaft’s axis. The other points on the locus are deviated from the conical camshaft’s axis. In Fig. 3, according to the value of the conical cam rotary angle φ and the motion law for the cam-follower, the value of oscillating angle ψ can be obtained. Then the corresponding point can be drawn on the displacement curve. A series of such points can be obtained and connected smoothly to form the displacement curve of the cam mechanism in theory. Consequently, the condition expression is available for the allowable pressure angle, the radius of base circle and the radius of curvature.

In Fig. 3, when the rotation angle of the conic cam $\varphi = 0$ (stating location), the starting point is B_0 for the axis of the oscillating bar, the angle of the oscillating bar is ψ_0, then the deviated distance is Δ from axis of follower to axis of conic cam. B_1, B_2 are the locations when the rotary angle of conic cam is φ_1, φ_2 and the oscillating angle of oscillating bar is ψ_1, ψ_2 respectively. Δ' is the deviated distance from the axis of the follower to the axis of the conic cam at B_2 location. In Fig. 3, the arc line on the conic expansion plane is drawn where the radius is the length of the oscillating bar.

As shown in Fig. 4, if a normal section through point B_2 on the conic cam, when the deviate distance is Δ' from axis of the follower to the axis of the conic cam, the corresponding distance on the basic circle expansion plane is the distance of the linear expansion of arc B_2A_2. Obviously the distance of the linear expansion of arc B_2A_2 is more than Δ'. In other words, the projection curve is an arc when locus traced by axis of the follower is projected onto the surface of the conic cam. When the surface of the conic cam surface is expanded, the projected curve could not be an arc. Therefore such design method would induce inherent errors, as demonstrated in Fig. 3

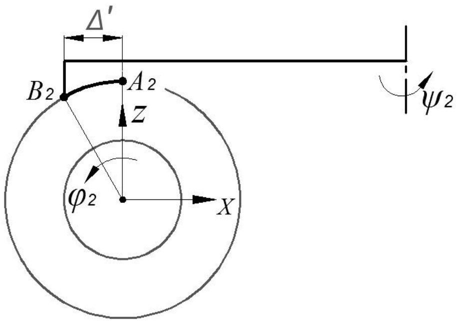

Fig. 4. Corresponding arc

3 3D Expansion Method for the Oscillating Follower

As evaluated in part 2, there are errors when the existing cam outline expansion method is used to design conic cam with oscillating follower. According to the locus of the follower, a new method is proposed to solve the 3D curve outline expansion problem with the use of rotary angle of conic cam.

During the movement of the conic cam mechanism, the conic cam rotates and drives the cam-follower oscillating in the grooved orbit of the cam. The cam-follower has two movements, one is the rotation relative to the cam, the other is the arc oscillation in the oscillating roll's moving plane, as shown in Fig. 5. The planar arc oscillating movement can be further divided into two linear movements, the linear movement along *Y* direction and perpendicular linear movement along *X* direction. Hence the movement of the related conical cam-follower is composed of one rotation and two linear movements, which is a 3D movement.

In Fig. 5, *MN* is the common perpendicular line of the two axes, and *M*, *N* is the respective perpendicular point. When the conic cam rotates, the distance from the point on the cam contour surface to *N* is kept constant, the location of the cam changes with the rotary angle φ. Now if we adopt the principle of inversion in the analysis and suppose that the conic cam is stationary, the cam-follower, the bar frame and the roller that rotate around *N* point and the *YZ* plane will turn around in the opposite direction.

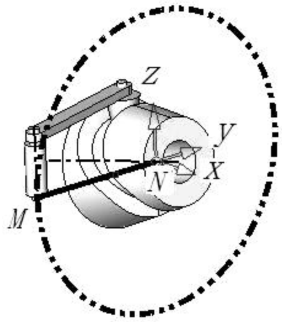

Fig. 5. Relative locus traced by oscillating bar

As shown in Fig. 6, following the double dot line, the rotation plane of machine frame is expanded to a line and is perpendicular to the surface, on which the oscillating bar is oscillating. The direction of the expanded line (i.e. φ, direction of the spatial cam rotary angle) is parallel to *Z*-axis in the coordinate frame. Consequently the arched motion of oscillating follower rotating with spatial cam will be expanded to a 3D cylinder surface motion.

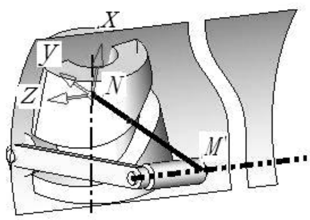

Fig. 6. Expansion of cylindrical surface

From the view of cam-follower's motion law, the general relationship can be deduced from the displacement of the roller, and the conic cam rotary angle can be expressed as

$$S = f(\varphi), \tag{1}$$

where S —Displacement of the roller, mm;
φ —Conic cam rotary angle, (°).

The cam-follower's motion law curve can be projected onto the expanded cylinder surface. As shown in Fig.7, for each point (φ, S) on the follower's displacement curve, a corresponding point can be found on the projected 3D expansion curve, and the value (x, y, z) can be established as previously described (see Ref. [11]):

$$\begin{cases} x = S \\ y = \sqrt{l^2 - S^2} - a \\ z = \varphi \end{cases}, \tag{2}$$

where S —Displacement of the roller, mm;
l —Length of the oscillating bar, mm;
a —Distance from the axis of oscillating follower to the axis of conic cam, mm;
φ —Conic cam rotary angle, (°).

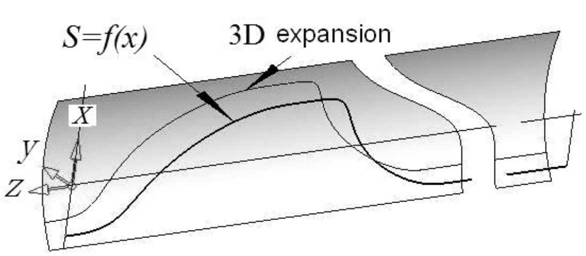

Fig. 7. 3D expansion curve

4 Deduction of Curve Equation for Cam Contour Expansion

The 3D expansion curve shown in Fig. 7 is the oscillating follower's motion expansion curve. It is not the cam contour expansion curve. As shown in Fig. 8, the cross point is from the follower's axis and the surface of conic cam. When the conic cam rotates, the locus of the cross point B forms an intersection curve, which is the cam contour expansion curve.

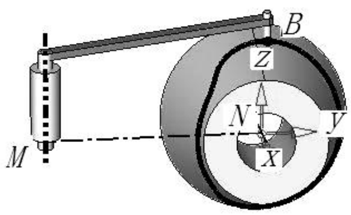

Fig. 8. Contour expansion curve of conic cam

Because the axis of the oscillating follower deviates from the axis of the conical cam, as shown in Fig. 9, when the displacement of the cam-follower is S, the corresponding cross point is B, the rotary angle of conic cam is φ.

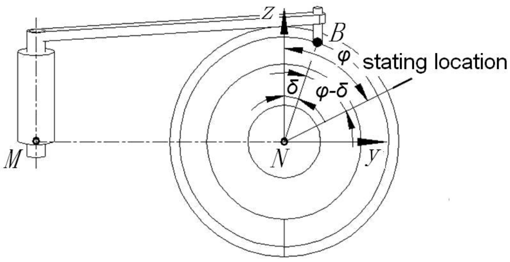

Fig. 9. Deviation angle

But in fact the cross point of the oscillating follower and the surface of the cam is delayed by an angle δ relative to the rotary angle φ. If the rotate angle between the cross point and the starting point is circumferential angle, then the cam's circumferential angle is $\varphi - \delta$, which corresponds to the displacement S, and the expression of deviate angle δ is

$$\delta = \arcsin \frac{y}{R} = \arcsin \frac{\sqrt{l^2 - S^2} - a}{R}. \tag{3}$$

Suppose that the circumferential angle of cam is expressed in form of polar coordinate system, as shown in Fig. 10, the value (ρ, θ) is

$$\begin{cases} \rho = \rho_0 - S_1 \\ \theta = (\varphi - \delta)\sin\gamma \end{cases}. \tag{4}$$

Where ρ is polar radius, which is the distance from the cross point B to the cone top, mm; θ is the polar angle, meaning the circumferential angle of conic cam $\varphi - \delta$, (°); γ is the half angle of conic cam, (°); S_1 is the length of the generatrix from point B to the datum ρ_0, mm; and $R = (\rho_0 - S_1)\sin\gamma$.

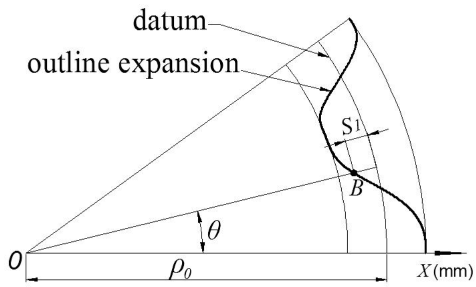

Fig. 10. Contour expansion in the form of polar coordinate system

Substituting Eq. (3) into Eq. (4), the equation in the form of polar coordinate system can be expressed as

$$\begin{cases} \rho = \rho_0 - S_1 \\ \theta = \left[\varphi - \arcsin \dfrac{\sqrt{l^2 - S^2} - a}{(\rho_0 - S_1)\sin\gamma} \right] \sin\gamma \end{cases}. \tag{5}$$

As shown in Fig. 11, P is the datum plane of the oscillating angle of oscillating bar, $BD \perp P$, $BD = S$, BC is the length of the generatrix from point B to the datum, $BC = S_1$, and the derived equation is

$$S_1 = S + (\rho_0 - S_1)[1 - \cos\arcsin \frac{\sqrt{l^2 - S^2} - a}{(\rho_0 - S_1)\sin\gamma}] \times \sin^2\gamma, \tag{6}$$

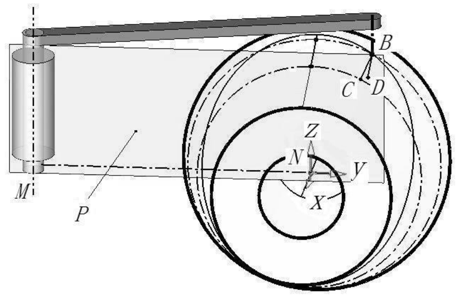

Fig. 11. Space geometry relation

5 Conic Cam Contour Expansion

Let us assume that the movement $f(\varphi)$ of the oscillating follower changes as sine acceleration motion curve, as shown in Fig. 12.

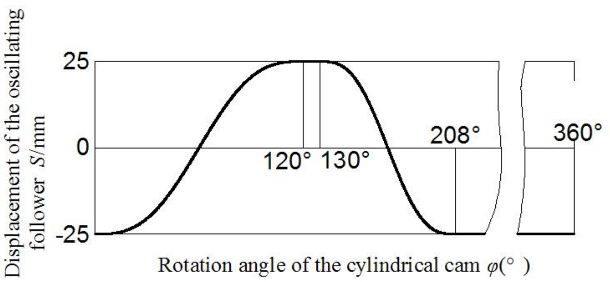

Fig. 12. Sine acceleration motion curve

Suppose the rise phase curve function is

$$S = f_1(\varphi) = 50(\frac{\varphi - 60}{120} - \frac{1}{2\pi}\sin\frac{2\pi}{120}\varphi), \tag{7}$$

where $0 \le \varphi \le 120$.

Suppose the return curve function is

$$S = f_2(\varphi) = 50[\frac{1}{2} - \frac{\varphi - 130}{78} + \frac{1}{2\pi}\sin\frac{2\pi}{78}(\varphi - 130)], \tag{8}$$

where $130 \le \varphi \le 208$.

Suppose $l = 251$, $a = 251.3$, $\gamma = 10^0$, $\rho_0 = 230.35$, according to Eqs. (5)–(8), a program is compiled and run. The curve figure is obtained by using Matlab software. A series of value X, Y can be extracted and copied onto the paste board. After data being processed, the cam contour expansion curve can then be drawn in Cartesian coordinate system by using "PLINE" command in AutoCAD software, as shown in Fig. 13.

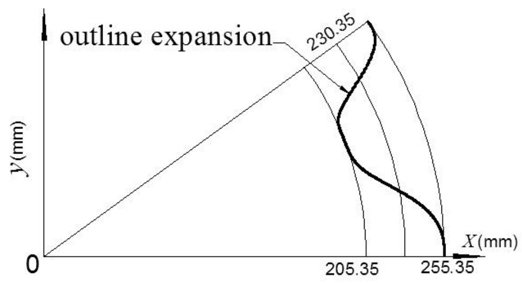

Fig. 13. Cam contour expansion curve

When we compare the cam contour expansion curve obtained using the available method[14] to the one using the new method we introduced here, the difference, as displayed in Fig. 14, can easily observed.

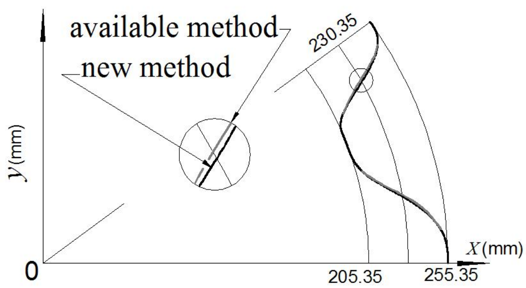

Fig. 14. Comparison of cam contour expansion curves

Because the deviate angle δ is zero for conic cam with translating follower, the curve's equation for cam contour in polar coordinate system can be simplified as

$$\begin{cases} \rho = \rho_0 - S \\ \theta = \varphi \sin\gamma \end{cases}. \tag{9}$$

Therefore the cam contour expansion curve figure for conic cam with translating follower can be obtained conveniently by adopting the same method.

6 Conclusions

(1) This paper pointed out the errors in the commonly used conical cam contour planar expansion method and proposed to include diverge angle in the calculation of conical cam contour expansion. This correction eliminates the common error in the current methods.

(2) Applying a new method of expansion of the follower's trajectory, this paper constructed three-dimensional expression of conical cam follower

trajectory. Based on geometric relations of conical cam, this paper deduced planar polar curve equation of conical cam contour expansion.

(3) Using MATLAB and AutoCAD software, this paper provided an example and verified the new conical cam contour design. The new method is simple in design but with high precision. It can be used not only for the design of contour expansion of conical cam with swing follower, but also for the design of contour expansion of other space cam.

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