Gravitational interactions between fast neutrinos and the formation of bound rotational states

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Abstract

The gravitational forces exerted between fast neutrinos at short distances are examined using Newton's gravitational law, special relativity, and the equivalence principle. It is found that the magnitude of these forces is not negligible and can lead to the formation of bound rotational states with radii in the fm range.

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1 Introduction

During the last two decades the study of neutrino oscillations [1, 2] has shown conclusively that all the three types, or flavors, of neutrinos (ν_e, ν_μ, ν_τ) have small but non-zero rest masses [1, 2, 3]. While neutrinos have very small rest masses (~ 0.04 to $0.4\ eV/c^2$) [1, 2, 3] they have typically quite large (as high as $100\ MeV$) total energies [1, 2]. This implies that their velocities are very near the speed of light and that their Lorentz factors $\gamma(= (1 - \mathrm{v}^2/c^2)^{-1/2})$ are very large. Since the total energy, E, is related to the rest mass, m_o, via the Einstein equation:

$$E = \gamma m_o c^2 \tag{1}$$

it follows that for $E = 100\ MeV$ and $m_o = 0.04\ eV/c^2$ it is $\gamma m_o = 100\ MeV/c^2$ and thus $\gamma = 2.5 \cdot 10^9$.

On the other hand special relativity dictates that when a particle moving on an instantaneous frame S' has a velocity $\mathbf{v}$ and concomitant Lorentz factor γ relative to a laboratory observer in a frame S, then the inertial mass, m_i, equals $\gamma^3 m_o$ [4, 5]. This is obtained after some simple algebra from [4, 5]:

$$F = \frac{d(\gamma m_o \mathrm{v})}{dt} = \gamma^3 m_o \frac{d\mathrm{v}}{dt} \tag{2}$$

Furthermore, according to the equivalence principle the inertial mass, m_i, of a particle equals the gravitational mass, m_g [6, 7], and thus one obtains:

$$m_g = m_i = \gamma^3 m_o \tag{3}$$

Upon substituting the above values, i.e. $\gamma = 2.5 \cdot 10^9$ and $m_o = 0.04\ eV/c^2$, one finds:

$$m_g = m_i = 6.25 \cdot 10^{18}\ GeV/c^2 \tag{4}$$

which, surprisingly, is more than half the value of the Planck mass [6], i.e. $1.221 \cdot 10^{19}\ GeV/c^2$ ($= 2.177 \cdot 10^{-8}\ kg$). Since the magnitude of gravitational and strong forces are expected to merge at energies close to the Planck energy, $1.221 \cdot 10^{19}\ GeV$, it follows that the gravitational forces between such fast neutrinos can be quite significant and thus can lead to the creation of confined states.

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For example the gravitational potential energy, V_g, between two such particles when they are at a distance of 1 fm is:

$$V_g = -\frac{Gm_g^2}{r} = -8.29 \cdot 10^{-12}\ J = -51.74\ MeV \tag{5}$$

whereas for comparison the Coulombic potential energy between a u and a d quark at the same distance is:

$$V_c = -\frac{(2/3)(1/3)e^2}{\epsilon r} = -2.57 \cdot 10^{-14}\ J = -0.32\ MeV \tag{6}$$

, i.e. the Coulombic interaction is a factor of 161 weaker than the gravitational interaction.

Since the strong force interaction between quarks is estimated to be a factor of $\alpha^{-1}(= 137.035)$ stronger than the Coulombic interaction [6, 8, 9] it follows that the magnitude of the gravitational force between fast neutrinos, when accounting for special relativity and the equivalence principle, is comparable to the magnitude of the strong force at the fm range.

This result is at first surprising but stems directly from special relativity, i.e. eqs (2) and (3), from the weak equivalence principle of Eötvös and Einstein [6] (eq. 3), and from Newton's gravitational law without making any assumptions.

It thus becomes interesting to examine what type of bound states such a powerful attractive force can create.

2 Circular states

We thus examine the circular motion of three neutrinos (e.g. three electron neutrinos or antineutrinos) on a circle of radius R (Fig. 1) under the influence of their gravitational attraction.

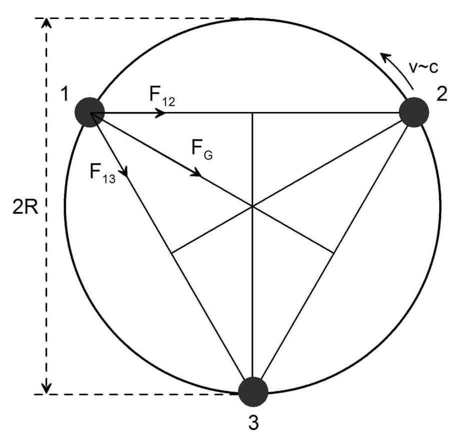

Figure 1: Three particles moving at a constant tangential velocity, $\mathbf{v}$, in a circle of radius R around their center of mass. They are equally spaced. F_{12} and F_{13} are two particle attraction forces and F_G is the resultant, radial, force.

2.1 Equivalence principle and inertial mass

It is important to first examine if equation (3) for the particle inertial and gravitational mass, obtained via equation (2) for linear particle motion, is also applicable when the particle performs a circular motion.

We thus consider a laboratory frame S and an instantaneous inertial frame S' moving with a particle with an instantaneous velocity $\mathbf{v}$ relative to frame S.

It is worth noting that the instantaneous inertial frame S' is defined by the vector $\mathbf{v}$ alone and not by the overall type of motion (e.g. linear or cyclic) performed by the particle [4, 5].

For the laboratory observer in S a particle in the instantaneous frame S' performing a circular motion is indistinguishable from a particle of the same rest mass m_o and velocity, $\mathbf{v}$, performing a linear motion (Fig. 2). Thus one can assign to the frame S' and corresponding velocity $\mathbf{v}$ an inertial particle mass, m_i, by considering a test force, $\mathbf{F}$, parallel to $\mathbf{v}$, acting on the particle. The reason for choosing the test force $\mathbf{F}$ parallel to $\mathbf{v}$ is that according to the theory of special relativity the case where $\mathbf{F}$ and $\mathbf{v}$ are parallel is the only case where the force is invariant, i.e. the force perceived in S and S' is the same [4].

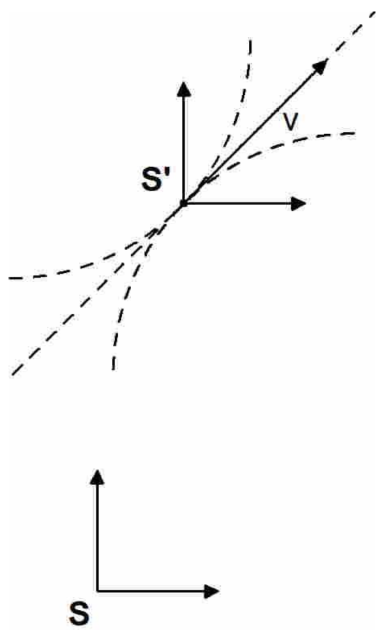

Figure 2: Laboratory frame S and instantaneous inertial frame S', the latter moving with the particle under consideration. The frame S' is uniquely defined by the vector $\mathbf{v}$ alone regardless of the motion (e.g. linear or circular) performed by the particle.

Starting from the general relativistic equation of motion, i.e. from [4, 5]:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \gamma m_o \frac{d\mathbf{v}}{dt} + \gamma^3 m_o \frac{1}{c^2} \left(\mathbf{v} \cdot \frac{d\mathbf{v}}{dt} \right) \mathbf{v} \tag{7}$$

and using the fact that the test force $\mathbf{F}$ is taken to be parallel to $\mathbf{v}$ one obtains:

$$\mathbf{F} = \left[\gamma + \gamma^3 \frac{\mathbf{v}^2}{c^2} \right] m_o \frac{d\mathbf{v}}{dt} = \left[\gamma + \gamma^3 (\gamma^2 - 1)/\gamma^2 \right] m_o \frac{d\mathbf{v}}{dt} = \gamma^3 m_o \frac{d\mathbf{v}}{dt} \tag{8}$$

which is equation (2).

This defines the mass $\gamma^3 m_o$, frequently termed longitudinal mass [5], which is the inertial mass of the particle, m_i, as it equals the ratio of force and acceleration. There seems to be no alternative to this definition of inertial mass [7]. Thus it is $m_i = \gamma^3 m_o$. According to the equivalence principle, m_i also equals the gravitational mass, m_g, of the particle [6], i.e.:

$$m_g = m_i = \gamma^3 m_o \tag{9}$$

which is equation (3). As already noted, for given m_o and v the inertial mass m_i and thus the gravitational mass m_g are both uniquely determined by eq. (9) and their value does not depend on the type of motion (e.g. linear or circular) performed by the particle.

Thus upon considering a second particle of rest mass m_o and instantaneous velocity measure v relative to the observer at S and at a distance r from the first particle, it follows that the inertial and gravitational mass of the second particle is also given by $\gamma^3 m_o$, as in equation (9), and thus one can use these two m_g values in Newton's gravitational law in order to compute the gravitational force, F_G, between the two particles. Thus from:

$$F_G = -\frac{G m_{1,g} m_{2,g}}{r^2} \tag{10}$$

and equation(9) one obtains:

$$F_G = -\frac{G m_o^2 \gamma^6}{r^2} \tag{11}$$

which depends on the 6^{th} power of γ [10] and accounts explicitly for the velocity dependence of the inertial and gravitational mass. It is worth remembering that this equation stems directly from special relativity (eq. 2 or 8), the weak equivalence principle (eq. 3 or 9) and Newton's gravitational law. No other assumptions are involved except that Newton's gravitational law remains valid under relativistic conditions via the use of the relativistic inertial and thus gravitational mass.

Application of equation (11) to the circular motion of Figure 1 gives after some simple trigonometry:

$$F_G = -\frac{G m_o^2 \gamma^6 (R)}{\sqrt{3}\, R^2} \tag{12}$$

2.2 Circular motion

One may now consider the relativistic equation of motion (eq. 7) for the case of a circular orbit of radius R with $F = F_G$. Since in this case $\mathbf{v} \cdot \frac{d\mathbf{v}}{dt} = 0$ one obtains:

$$\mathbf{F}_G = \gamma m_o \frac{d\mathbf{v}}{dt} \tag{13}$$

and since for circular motion it is $|d\mathbf{v}/dt| = \mathrm{v}^2/R$ it follows that:

$$F_G = \gamma m_o \frac{\mathrm{v}^2}{R} \tag{14}$$

Upon combining with equation (11) one obtains:

$$\frac{Gm_o^2\gamma^6(R)}{\sqrt{3}\,R^2} = \frac{\gamma(R)m_o\mathrm{v}^2}{R} \tag{15}$$

i.e., the gravitational force F_G given by equation (12) acts as the centripetal force for the rotational motion.

It must be noted that on the basis of (13) one might be tempted to assign the value γm_o, commonly termed transverse mass [4, 5], to the inertial and thus gravitational mass of each particle. However, as already noted, the mass $m_i(= m_o\gamma^3)$, and thus also m_g, is uniquely determined for given m_o and $\mathbf{v}$, via the colinear to $\mathbf{v}$ test force $\mathbf{F}$, and does not depend on the type of motion (e.g. linear or circular) performed by the particle.

Upon utilizing $\gamma(R) = (1 - \mathrm{v}^2/c^2)^{-1/2}$ in eq. (15) one obtains:

$$R = \frac{Gm_o}{\sqrt{3}\,\mathrm{c}^2}\gamma^5\left(\frac{\gamma^2}{\gamma^2-1}\right) \tag{16}$$

or, equivalently:

$$R = (R_S/(2\sqrt{3}))\gamma^5\left(\frac{\gamma^2}{\gamma^2-1}\right) \tag{17}$$

where $R_S(= 2Gm_o/c^2)$ is the Schwarzschild radius of a particle with rest mass m_o.

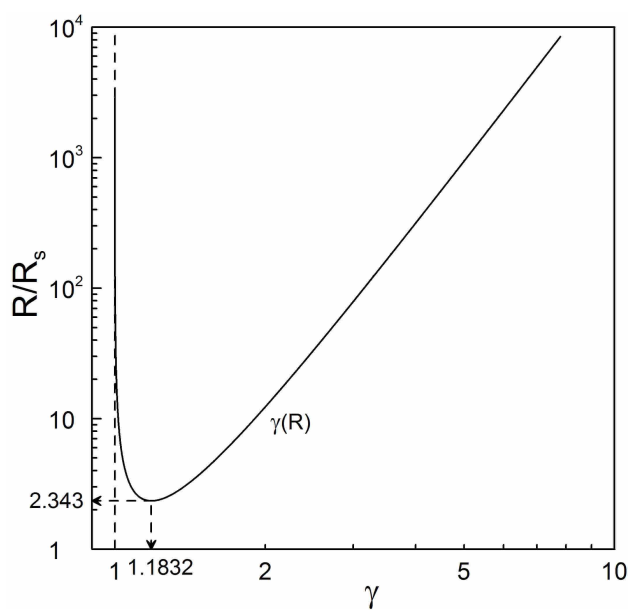

Figure 3: Plot of eq. 17 near the minimum R.

As shown in Figure 3 the function R defined by eq. 17 exhibits a minimum, $R_{\min} = 2.343R_S$, at $\gamma_{\min} = \sqrt{7/5} = 1.1832$ thus $\mathrm{v}_{\min} = \sqrt{2/7}c$. This is the minimum radius for a circular orbit and the corresponding minimum angular momentum is $L_{\min} = \gamma_{\min}m_o v_{\min}R_{\min} = 1.481m_o cR_S = 2.963$ Gm_o^2/c.

The above condition is similar to the criteria $L > Gm^2/c$ or $R > R_S/2$ found for circular orbits in special relativity [11, 12] or $L > 2\sqrt{3}Gm^2/c$ for the Schwarzschild metric in general relativity [12] with orbits around point masses with r^{-1} potentials.

Equation (17) defines two γ branches (Fig. 3), one corresponding to low γ values ($\gamma < 1.1832$) the other corresponding to large γ values ($\gamma > 1.1832$). The first branch corresponds to common Keplerian gravitational orbits. In this case γ and thus the velocity $\mathbf{v}$ decreases with increasing R, e.g. $\mathrm{v} = (Gm_o/(\sqrt{3}R))^{1/2}$ in the non-relativistic case ($\gamma \approx 1$).

The second branch which leads to relativistic velocities defines rotational states where γ and thus v increase with increasing R. These states with $\gamma \gg 1$ are the states of interest to the present model. For $\gamma \gg 1$, e.g. $\gamma > 10^2$, eq. 17 reduces to:

$$R = (R_S/(2\sqrt{3}))\gamma^5 \quad ; \quad \gamma = (2\sqrt{3})^{1/5}(R/R_S)^{1/5} \tag{18}$$

This equation can be used to eliminate R or γ in the force expression of eq. (12). In the former case (i.e. elimination of R) one obtains:

$$F_G = -\frac{\sqrt{3}c^4}{\gamma^4 G} \tag{19}$$

and thus, interestingly, for any fixed value of γ or R, the attractive force is uniquely determined by the familiar G/c^4 parameter of the gravitational field equations of general relativity [13, 14], i.e. $G_{ik} = (8\pi G/c^4)T_{ik}$, which relates the Einstein tensor G_{ik} with the stress-momentum-energy tensor T_{ik} [13, 14, 15].

In the latter case, i.e. elimination of γ, one obtains:

$$F_G = -m_o c^2 \left(\frac{2\sqrt{3}}{R_S}\right)^{1/5} \frac{1}{R^{4/5}} \tag{20}$$

2.3 Potential, translational and total energy

The force equation (20) refers to circular orbits only and thus defines a certain conservative force, since the work done in moving the particles between two points R_1 and R_2, corresponding to two rotational states with radii R_1 and R_2, is independent of the path taken. Therefore, since the force vector is pointing to the center of rotation it follows that a conservative vector field is defined which is the gradient of a scalar potential, denoted $V_G(R)$. The latter is the gravitational potential energy of the three rotating particles when accounting for their rotational motion and corresponds to the energy associated with transfer of the particles from the minimum circular orbit radius $R_{\min}$ to an orbit of radius of interest, R. The function $V_G(R)$ is obtained via integration of eq. (20), i.e., denoting by R' the dummy variable:

$$\begin{aligned} V_G(R) - V_G(R_{min}) &= \int_{R_{min}}^{R} F_G dR' = \\ &= -5m_o c^2 \left(\frac{2\sqrt{3}}{R_S}\right)^{1/5} \left(R^{1/5} - R_{\min}^{1/5}\right) \end{aligned} \tag{21}$$

Noting that $R_{min} = 2.343R_S$ (Fig. 3) and that the value of the Schwarzchild radius, R_S, ($= 2Gm_o/c^2$) for neutrinos is extremely small ($\sim 10^{-63}\,m$) it follows that for any realistic R value (e.g. above the Planck length value of $10^{-35}\ m$) equation (21) reduces to:

$$V_G(R) = -5m_o c^2 (2\sqrt{3})^{1/5} (R/R_S)^{1/5} \tag{22}$$

Thus while the magnitude of the gravitational force acting on the rotating particles increases with decreasing radius, R (eq. 20), the absolute value $|V_G(R)|$ of the gravitational potential energy increases with increasing R Eq. (22). This behavior is reminiscent of asymptotic freedom [16, 17, 18], i.e. the attractive interaction energy is small at short distances and increases significantly with increasing distance R.

In view of Eq. (18) one can rewrite equation (22) as:

$$V_G(R) = -5\gamma m_o c^2 \tag{23}$$

On the other hand the kinetic energy, T, of the three rotating neutrinos is:

$$T(R) = 3(\gamma - 1)m_o c^2 \tag{24}$$

Thus one may now compute the change in total system energy, ΔE, upon formation of the rotational bound state from the three originally free neutrinos. Denoting by f and i the final and initial states and by (RE) the rest energy, one obtains from energy conservation:

$$\begin{aligned} \Delta E &= E_F - E_i = \\ &= [(RE)_f + T_f + V_{G,f}] - [(RE)_i + T_i + V_{G,i}] = \\ &= \left[3m_o c^2 + 3(\gamma - 1)m_o c^2 - 5\gamma m_o c^2\right] - 3m_o c^2 = \\ &= \Delta T + \Delta V_G = -(2\gamma + 3)m_o c^2 \approx -2\gamma m_o c^2 \end{aligned} \tag{25}$$

where the last equality holds for $\gamma \gg 1$ as is the case of interest here.

The same ΔE expression is, of course, obtained regardless of the choice of the reference potential energy state. Thus in view of eqs. (18), (22), (23) and (25) one can summarize the dependence of ΔT, ΔV_G and ΔE on γ and R for $\gamma >> 1$ as:

$$\Delta T = T = 3(\gamma - 1)m_o c^2 \approx 3\gamma m_o c^2 = 3m_o c^2 \left(\frac{2\sqrt{3}R}{R_S}\right)^{1/5} \tag{26}$$

$$\Delta V_G = -5\gamma m_o c^2 = -5m_o c^2 \left(\frac{2\sqrt{3}R}{R_S}\right)^{1/5} \tag{27}$$

$$\Delta E = -(2\gamma + 3)m_o c^2 \approx -2\gamma m_o c^2 = -2m_o c^2 \left(\frac{2\sqrt{3}R}{R_S}\right)^{1/5} \tag{28}$$

The negative sign of ΔE shows that the formation of the bound rotational state starting from the three initially free neutrinos happens spontaneously, is exoergic ($\Delta E < 0$), and the binding energy $BE(= -\Delta E)$ equals $2\gamma m_o c^2$.

2.4 Quantization of angular momentum

We then proceed to identify among the infinity of bound rotational states described by eqs.(26), (27) and (28), each corresponding to a different R, those rotational states where R is an integer multiple of the reduced de Broglie wavelength $\lambdabar(=\hbar/p)$ of the light rotating particles.

Similarly to the Bohr model of the H atom, this can be done by introducing quantization of the angular momentum of the light particles in the form:

$$L = \gamma m_o R c = (2n-1)\hbar \tag{29}$$

Solving for R one obtains:

$$R = \frac{(2n-1)\hbar}{\gamma m_o c} = \frac{3(2n-1)\hbar}{mc} \tag{30}$$

where $m(=3\gamma m_o)$ is the rest mass of the bound rotational state formed.

Using the definition of $R_S(=2Gm_o/c^2)$ one can thus express the ratio R/R_S which appears in equations (26) to (28). It is:

$$\frac{R}{R_S} = \frac{(2n-1)\hbar c}{2\gamma G m_o^2} \tag{31}$$

This ratio is also given by equation (18), i.e.

$$\frac{R}{R_S} = \frac{\gamma^5}{2\sqrt{3}} \tag{32}$$

Consequently from (31) and (32) one obtains:

$$\gamma^6 = \frac{3^{1/2}(2n-1)\hbar c}{G m_o^2} = 3^{1/2}(2n-1)\frac{m_{Pl}^2}{m_o^2} \tag{33}$$

where $m_{Pl} = (\hbar c/G)^{1/2}$ is the Planck mass. Recalling that the mass, m, of the bound rotational state equals $3\gamma m_o$ one thus obtains:

$$m = 3\gamma m_o = 3^{13/12}(2n-1)^{1/6} m_o^{2/3} m_{Pl}^{1/3} \tag{34}$$

i.e. the mass of the bound state has been expressed in terms of m_o and natural constants.

Setting $n=1$, $m_o = 0.04\, eV/c^2$ and using $m_{Pl} = 1.221 \cdot 10^{19}\ GeV/c^2$ one obtains:

$$m = 885.43\ MeV/c^2 \tag{35}$$

which, surprisingly, is in the hadrons mass range and in fact differs less than 6% from the rest mass of the proton (938.272 MeV/c^2) and of the neutron (939.565 MeV/c^2). Exact agreement with the neutron mass, m_n, is obtained for:

$$m_o = 0.043723\, eV/c^2 = 7.7943 \cdot 10^{-38}\ kg \tag{36}$$

This is the value computed from equation (34) for $n=1$ which is assumed to correspond to a neutron, i.e.:

$$m_o = \frac{(m_n/3)^{3/2}}{3^{1/8} m_{Pl}^{1/2}} \tag{37}$$

The thus computed m_o value lies exactly in the estimated electron neutrino mass range [1, 2, 3]. It corresponds to a m_o^2 value of $1.92 \cdot 10^{-3}$ $(eV/c^2)^2$ vs $2.2 \cdot 10^{-3}$ $(eV/c^2)^2$ which is the estimated maximum neutrino mass square value (obtained from the measurable Δm^2 value of the $\nu_\mu \longleftrightarrow \nu_\tau$ oscillations) as extracted from the Super-Kamiokande data [1].

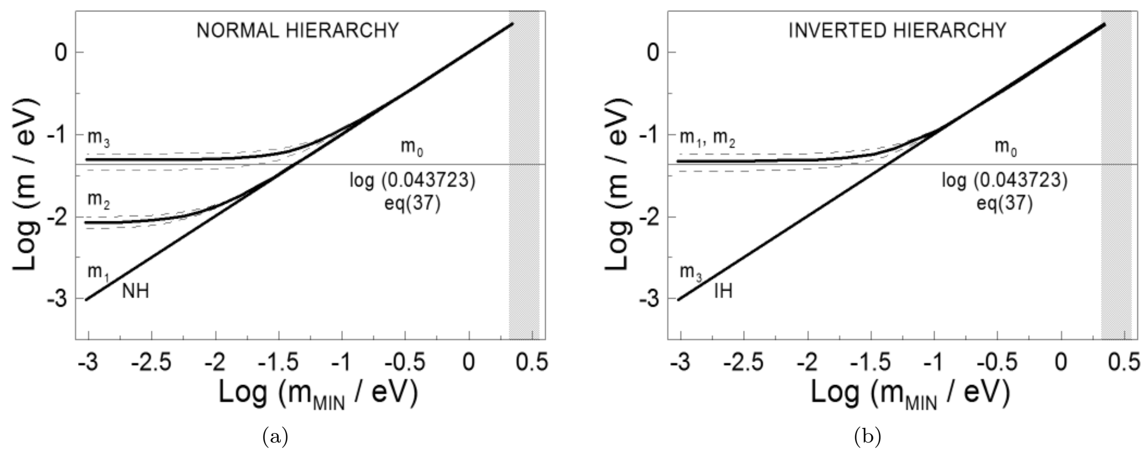

Figure 4: The three light neutrino masses as a function of the lightest mass for the normal (left plot) and inverted (right plot) hierarchy, reprinted from ref. [2] and comparison with equation (37), i.e. $m_o = (m_n/3)^{3/2}/(3^{1/8} m_{Pl}^{1/2}) = 0.043723\, eV/c^2$.

Actually as shown in Figure 4 the m_o value of 0.04373 eV/c^2 (eqs. 36 and 37) practically coincides with the currently computed maximum neutrino mass value both for the normal mass hierarchy ($m_3 >> m_2 > m_1$) and for the inverted hierarchy ($m_1 \approx m_2 >> m_3$) [2].

Also as shown in Figure 5 the computed m_o value is in excellent agreement with the observable effective neutrino mass m_ν measurable in KATRIN [2].

With this m_o value equation (34) can also be written as:

$$m = (2n-1)^{1/6} m_n \tag{38}$$

where m_n is the neutron mass. As shown in Figure 6 this expression is also in very good agreement with experiment regarding the masses of baryons consisting of u, d and s quarks [3] which follow the $(2n-1)^{1/6}$ dependence of equation (38) with an accuracy better than 3% (Fig. 6 and Table 1).

3 Properties of the bound states

3.1 Rest energy and binding energy

As already noted (e.g. equation 34) the total rest plus kinetic energy of the three rotating particles equals $3\gamma m_o c^2$ and constitutes at the same time the rest energy, mc^2, of the composite particle

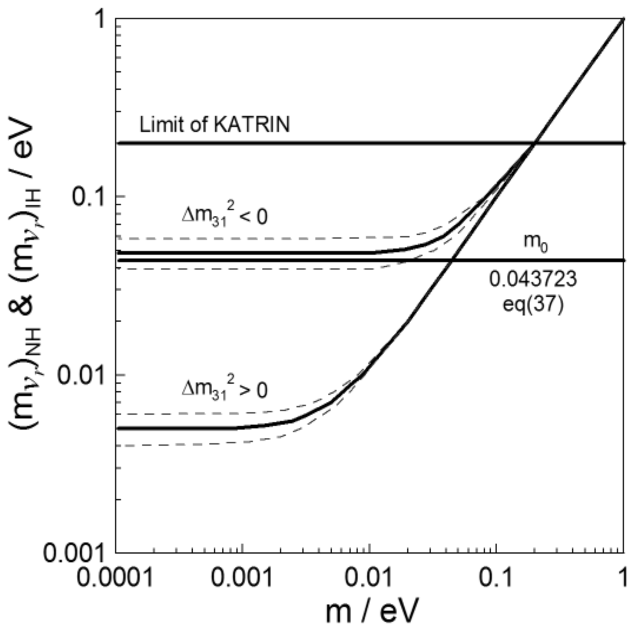

Figure 5: The observable effective mass m_{ν_r} measurable in KATRIN as a function of the lightest mass for the normal (bottom) and inverted (upper) mass ordering [2] and comparison with equation (37). The currently allowed 3σ ranges of the oscillation parameters were used [2].

formed, i.e. of the rotational bound state:

$$mc^2 = 3\gamma m_o c^2 \tag{39}$$

It is useful to note that in the model the rest mass of the three particles, i.e. $3m_oc^2$, does not change when the bound state is formed. The transformation of the kinetic energy of the three rotating particles into rest energy of the bound state, is due to the change in choice of the boundaries of the system. In the former case (three individual rotating particles) the boundaries are geometrically disconnected and encompass each particle individually, in the latter case the system boundary contains all three particles. Thus the formation of the bound rotating state by the three particles provides a simple hadronization mechanism, i.e. generation of rest mass, m, starting from an initial rest mass $3m_o$, according to eq. (39).

It follows from (28) and (39) that:

$$BE = -\Delta E = (2/3)mc^2 \tag{40}$$

Thus the binding energy per light particle is $(2/9)mc^2$, which for $m = m_p = 938.272\ MeV/c^2$, the proton mass, gives an energy of 208 MeV, not far from the estimated particle energy of 150 MeV at the transition temperature of QCD [19].

One may note here that since the potential energy expression (22) does not depend on the number, N, of rotating particles but the kinetic energy, T, does, i.e. $N(\gamma - 1)m_oc^2$, it follows from (25) that, according to the model, stable rotational states cannot be obtained for $N > 5$ since they lead to positive ΔE. The case $N = 2$ is interesting, as it leads to composite masses, m, in the range

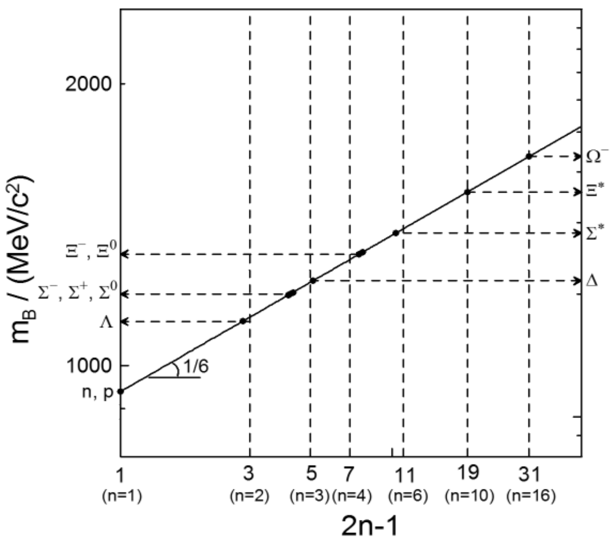

Figure 6: Comparison of the masses, m_B, of the uncharmed baryons, consisting of u, d and s quarks, [3] with equation 38, i.e. $m_B = m_n(2n-1)^{1/6}$ where m_n is the neutron mass.

of mesons, i.e. in the 0.5 GeV/c^2 range.

3.2 Radii and Lorentz factors γ

The hadron radius computed from equation (30) for $n = 1$, i.e.

$$R(n=1) = \frac{3\hbar}{m_n c} = 0.631\ fm \tag{41}$$

equals three times the neutron Compton wavelength and is in very good agreement with the experimental proton and neutron radii values which lie in the 0.6 - 0.7 fm range.

For $n > 1$ the corresponding $R(n)$ values can be computed from equation (30), i.e.

$$R = \frac{(2n-1)\hbar}{\gamma m_o c} \tag{42}$$

by accounting for the γ dependence on $(2n-1)$ given by equation (33), i.e.

$$\gamma(n) = (2n-1)^{1/6}\gamma(n=1) = 7.169 \cdot 10^9 (2n-1)^{1/6} \tag{43}$$

Thus one obtains:

$$R(n) = (2n-1)^{5/6} R(n=1) = 0.631(2n-1)^{5/6} fm \tag{44}$$

The $\gamma(n)$ values are in the range computed in the Introduction for 100 MeV neutrinos. The radii $R(n)$ also lie, interestingly, in the range of hadron, e.g. proton or neutron, radii.

3.3 Lifetimes and rotational periods

The period of rotation $\tau(n)$ of the neutrinos within the composite state, $2\pi R/\mathrm{v} \sim 2\pi R/c$, is, using Eq. 44,

$$\tau(n) = (2n-1)^{5/6}\tau_p = (2n-1)^{5/6} 6.6 \cdot 10^{-24}\ s \tag{45}$$

where $\tau_p = 2\pi R_p/c = 6.6 \times 10^{-24}\ s$ is the rotation period for the proton or the neutron. The time interval $\tau(n)$ provides a rough lower limit for the lifetime of the composite particles, interpreted as baryons, as they can be defined only if the neutrinos complete at least a revolution (Fig. 1). Indeed all the known lifetimes of the baryons are not much shorter than that estimate. The lifetime of the Δ baryons, which is the shortest, is $5.6 \cdot 10^{-24} s$ [3].

3.4 Spins and charges

Up to here the analysis of the three rotating neutrinos model has shown that composite states are formed whose masses reproduce quite well the light hadron mass spectrum. Also since neutrinos are fermions with spin 1/2 [3] one anticipates a spin of 1/2 or 3/2 for the composite state formed which is the case with most hadrons [3].

However many hadrons, e.g. the proton or the Ξ^+, are charged and although the difference in mass, m, from their neutral brethren (i.e. the n or the Ξ^o) is small and of the order of αm, where $\alpha(= e^2/\varepsilon c\hbar = 1/137.0359)$is the fine structure constant, nevertheless the question arises how this charge was introduced in the charged baryons.

One possibility is that in the distant past charged neutrinos existed. Their stronger interaction among themselves and with other particles led to their extinction via formation of hadrons, mesons and neutral neutrinos.

A second possibility is that neutral hadrons were first formed (e.g. neutrons) and then via the β-decay protons were formed. We are not in a position to even speculate about which route is more likely but we can assume as an example that in the final state the charges of the constituent particles are equal to those of u and d quarks, i.e. $(2/3)e$ and $-(1/3)e$. This leads as shown in the next section to very good agreement with experiment regarding magnetic moments.

The Coulombic forces between charged particles with relativistic velocities have been studied in detail [4]. It is well established that Coulomb's law correctly gives the force on the test charge for any velocity of the test charge provided the source charge is at rest [4]. In the simplified geometry of Figure 1 the distance between the two particles remains constant, thus in the reference frame of the source charge the test charge is also at rest, thus Coulomb's law remains valid without any relativistic corrections.

It is thus possible to estimate the Coulomb interaction energy between the rotating particles. In the simplified geometry of Figure 1 the total Coulomb potential energy for the proton (charges 2/3, 2/3, -1/3) vanishes, i.e. denoting $\varepsilon = 4\pi\varepsilon_o$ one obtains:

$$V_{C,p} = \frac{e^2}{\varepsilon\sqrt{3}R}\left[(4/9) - (2/9) - (2/9)\right] = 0 \tag{46}$$

while for the neutron (charges -1/3, -1/3, 2/3) it is negative:

$$V_{C,n} = \frac{e^2}{\varepsilon\sqrt{3}R}\left[(1/9) - (2/9) - (2/9)\right] = -\frac{(e^2/\varepsilon)}{3\sqrt{3}R} \tag{47}$$

, i.e. there is an overall attractive Coulombic interaction.

Upon substituting R from Eq. (20) for the case of the neutron ($n = 1$) one obtains:

$$V_{C,n} = -\frac{e^2}{9\sqrt{3}\varepsilon c\hbar}m_p c^2 = -\frac{\alpha}{9\sqrt{3}}m_n c^2 \tag{48}$$

Thus $V_{C,n}$ is about three orders of magnitude smaller than the rest energy of the neutron. Interestingly as shown in Table 2 the mass differences in baryons which differ only in the charge value (e.g. the N, Σ or Ξ baryons) are generally small (up to 7 MeV/c^2) and the ratio $\Delta m_N/m_N$, $\Delta m_\Sigma/m_\Sigma$ or $\Delta m_\Xi/m_\Xi$ is of the order of 10^{-3}, similarly to the value of the ratio $(V_{C,n}/c^2)/m_p$ obtained from eq. 48. Thus the Coulomb interaction could be the origin of this small difference.

If the Coulomb interaction is taken into consideration the symmetry of the configuration of Fig. 1 is broken as not all three charges are the same. Although the deviation from three-fold symmetry is small, since the Coulombic energy is small, and thus one may still use with good accuracy eq. (47) to estimate the attractive interaction between the three particles forming a neutron, it is conceivable that this broken symmetry may be related to the relative instability of the neutron (lifetime 885.7 s) vs the proton (estimated lifetime $\sim 10^{32}$ s [3]).

3.5 Magnetic moments

It is interesting to compute the magnetic dipole moments, μ, of these bound rotational states. Using the definition of $\mu(= (1/2)qRv)$ and considering the case $n = 1$, corresponding to a proton (which is a uud baryon) with charge $2e/3$ for u and $-e/3$ for d it follows:

$$\mu_p = (1/2)eRc\left[(2/3) + (2/3) - (1/3)\right] = (1/2)eRc \tag{49}$$

Upon substituting $R = R_p = 0.631$ fm one obtains:

$$\mu_p = 15.14 \cdot 10^{-27}\ \mathrm{J/T} \quad (= 3\mu_\mathrm{N}) \tag{50}$$

where μ_N is the nuclear magneton ($5.05 \cdot 10^{-27}$ J/T). This value differs less than 8% from the experimental value of $14.10 \cdot 10^{-27}$ J/T (i.e. 2.79 μ_N) [9, 20].

In the above computation (eq. 49) one assumes that the spin vectors of the three small particles (i.e. uud) are parallel to the vector of rotation of the rotating proton state. If one considers the neutron which is a udd particle and assumes that the spin of one of the two d quarks is parallel with the rotation vector of the rotating neutron state and the spins of the other two particles are antiparallel to the neutron rotation vector then one obtains:

$$\mu_n = (1/2)eRc\left[(-2/3) + (1/3) - (1/3)\right] = -(1/3)eRc \tag{51}$$

and upon substitution of $R = 0.631\ fm$ one obtains:

$$\mu_n = -10.09 \cdot 10^{-27}\ \text{J/T} = -2\mu_N \tag{52}$$

which is in excellent agreement with the experimental value of $-9.66 \cdot 10^{-27}$ J/T $(= -1.913\mu_N)$.

This good agreement seems to imply that the spin contribution of the light particles to the magnetic moment of the rotating state is small and only the spin vector orientation (parallel or antiparallel to the baryon rotation vector) is important.

3.6 Inertial mass and angular momentum

Interestingly it follows from equation 33 that in the case of the neutron or proton ($n = 1$) the inertial and gravitational mass of each rotating particle, $\gamma^3 m_o$, is related to the Planck mass, $m_{Pl} = (\hbar c/G)^{1/2}$, via a very simple equation, i.e.

$$\gamma^3 m_o = 3^{1/4} m_{Pl} = 3^{1/4}\left(\frac{\hbar c}{G}\right)^{1/2} = 1.607 \cdot 10^{19} \quad GeV/c^2 \tag{53}$$

which provides an interesting direct connection between the Planck mass and the gravitational mass of the rotating neutrino model. Gravity is generally expected to reach the level of the strong force at energies approaching the Planck scale ($\sim 10^{19}\ GeV$) [6] which is in good agreement with the model results (eq. 53).

Thus while the relativistic mass, $3\gamma m_o$, of the bound state formed by the three neutrinos corresponds to $\sim 938\ MeV/c^2$, the inertial and gravitational mass $\gamma^3 m_o$ of each of them is in the Planck mass range.

It is worth reminding here Wheeler's concept of geons [13, 14, 15], i.e. of electromagnetic waves or neutrinos held together gravitationally, which had been proposed as a classical relativistic model for hadrons [13]. Similarly to the present case (eq. (53)) the minimum mass of a small geon formed from neutrinos had been estimated [13] to lie in the Planck mass range.

It is interesting to note here that when using the inertial or gravitational mass, $\gamma^3 m_o$, in the definition of the Compton wavelength, λ_c of the particle ($= h/mc$) then one obtains the Planck length ($\sim 10^{-35}\ m$), but when using the mass corresponding to the total energy of the particles, $3\gamma m_o$, then one obtains the proton Compton wavelength ($\sim 10^{-15}\ m$), which is close to the actual distance between the rotating particles.

The model is also qualitatively consistent with a central experimental observation about the strong force [8], i.e. that the normalized angular momentum of practically all hadrons and their excited states is roughly bounded by the square of their mass measured in GeV [8]. Indeed from eq. (29) and (38) one obtains:

$$(L/\hbar)/(m/GeV)^2 = 1.13(2n-1)^{2/3} \tag{54}$$

which is in reasonable qualitative agreement with experiment for small integer n values.

4 Conclusions

A deterministic Bohr type model was developed for the rotational motion of three fast neutrinos using gravity as the attractive force. When accounting for special relativity, for the weak equivalence principle, for Newton's gravitational law and for quantization of angular momentum one finds that the rotational states formed have, surprisingly, most of the observable properties of hadrons, including masses, radii, reduced Compton and de Broglie wavelengths, magnetic moments and angular momenta. It thus appears that the large relativistic inertial and gravitational mass of fast neutrinos makes them suitable for constructing much heavier composite particles, such as hadrons. That hadrons and mesons may consist of neutrinos is not too surprising since, for example, π and K mesons are known to decay to neutrinos [3] and also since neutrinos are known to be emitted in practically all hadron decays and nuclear reactions [3, 9]. Nevertheless much more work will be needed to test the usefulness of such deterministic Bohr type models for the formation of bound states from fast neutrinos.

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Table 1: Experimental [3] and computed (eq. 38) baryon masses

Baryon	Experimental mass value MeV/c^2	$m_n(2n-1)^{1/6}$	n
$N \begin{cases} p \\ n \end{cases}$	938.272 939.565	938.272	1
Λ	1115.68	1126.8	2
Σ^+ Σ^o Σ^- Δ	1189.37 1192.64 1197.45 1232	1226.9	3
Ξ^o Ξ^-	1314.8 1321.3	1297.7	4
$\Sigma*$	1385	1399.2	6
$\Xi*$	1533	1532.6	10
Ω^-	1672	1663.0	16

Table 2: Masses of baryons differing only in the charge q [3]

Baryon	$m_B^{\pm}$ MeV/c^2	$(m_B^{\pm} - m_B^o)/m_B^o$
$N \begin{cases} p \\ n \end{cases}$	938.272 939.565	$-1.376 \cdot 10^{-3}$
Σ^+ Σ^o Σ^-	1189.37 1192.64 1197.45	$-2.74 \cdot 10^{-3}$ $4.03 \cdot 10^{-3}$
Ξ^o Ξ^-	1314.8 1321.3	 $4.94 \cdot 10^{-3}$