

Iterative Learning Control Algorithm with a Fixed Step

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Abstract: Iterative Learning Control (ILC) captures interests of many scholars because of its capability of high precision control implement with no need of mathematical models, and it is widely applied in control engineering. Presently, most ILC algorithms still follow the original ideas of ARIMOTO, in which the iterative-learning-rate is composed by the control error with its derivative and integral values. This kind of algorithms will result in inevitable problems such as huge computation, big storage capacity for algorithm data, and also weak robust. In order to resolve these problems, an improved iterative learning control algorithm with fixed step is proposed here which breaks the primary thought of ARIMOTO. In this algorithm, the control step is set only according to the value of the control error, which could enormously reduce the computation and demanded storage size, also improve the robust of the algorithm by not using the differential coefficient of the iterative learning error. In this paper, the convergence conditions of this proposed fixed step iterative learning algorithm is theoretically analyzed and testified. Then the algorithm is tested through simulation researches on a time-variant object with randomly set disturbance through calculation of step threshold value, algorithm robustness testing, and evaluation of the relation between convergence speed and step size. Finally the algorithm is validated on a valve-serving-cylinder system of a joint robot with time-variant parameters. Experiment results demonstrate the stability of the algorithm and also the relationship between step value and convergence rate. Both simulation and experiment testify the feasibility and validity of the new algorithm proposed here. And it is worth to notice that this algorithm is still simple and keeps strong robust after improvements, which provides new ideas to the research of iterative learning control algorithms.

Key words: iterative learning control, fixed step, time variant system, simulating study, robot control

1 Introduction

It was UCHIYAMA, who firstly created the concept of ILC in 1978 on robot trajectory track^[1], but because Ref.[1] was written in Japanese, little attention had been paid to the context till ARIMOTO, et al^[2], at Osaka University published their pioneering works based on essence of ILC. They put forward a D type learning regulation for ILC:

$$u_{k+1}(t) = u_k(t) + L\dot{e}_k(t), \quad k = 1, 2, \dots, N, \quad (1)$$

where, k is the iteration number; $u_{k+1}(t)$ is the $k+1$ th iteration learning control input at time t ; $u_k(t)$ is the k th iteration learning control input at time t ; L is the iterative learning rate; $\dot{e}_k(t)$ is the derivative of the control error $e_k(t)$; N is a natural number.

Generally, the methods of ILC have many advantages and are widely applied in control systems^[3-8]. For example they are suitable to many linear or nonlinear dynamic systems with high uncertainty, but in simple algorithm forms they demand less knowledge about the plant. The computation for the methods is not complicated. Moreover, it is easy to obtain high precision control with ILC. Consequently, how to apply them to solve problems has been the interest of many scholars. In particular, there is extensive research on combining ILC with other methods to enhance performance. For instance, ILC integrated with fuzzy sliding-mode-control is used on hydraulic servo system and achieves a satisfying control effect^[9]. TAYEBI, et al^[10], proposed robust ILC and applied on robot control. BRISTOW^[11] reported about a method combining a time-varying filter with ILC for systems with uncertainty. WANG, et al^[12], put forward fuzzy adaptive ILC. MI, et al^[13], combined ILC with PID for ABS system for vehicles and QIAN, et al^[14], designed parallel ILC and PID control to alleviate the speed fluctuating for speed control on synchronous motor.

The above ILC methods are just the varied or improved ones from the original ILC idea proposed by ARIMOTO in

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that the iterative learning rate is deduced from linear or nonlinear combination form control errors and their derivative and integral, the crux for these methods is to find a suitable iterative learning rate to ensure sound convergence and stability^[15]. And in the process of computing iterative learning rate, the data needed to be stored includes reference trajectory data, control errors and integrals. Therefore conventional ILC methods demand controllers with a big storage capacity and quick computing speed. Consequently, the ILC approaches would challenge the commonly used control units such as Single-Chip-Micyoco (SCM) and small sized Programmable Logic Controller (PLC) with two problems. One is that SCM is not good at dealing with floating data, let alone mass data computation. Another is that SCM has a small storage capacity, for example, for 89C52 SCM, the inner storage size is 8 KB, while the maximum outside extended storage size is 32 kB.

Therefore, it is essential to find new methods that demand less memory and computation resources but have satisfying control precision and convergence speed based on ILC. Different from ARIMOTO's idea, a new approach for ILC with a fixed step is proposed. The essence of the new method is to select a suitable fixed step for ILC, while the positive or the negative of the control error will decide the plus or minus of the control step, and the value of the control error itself is irrelevant to control law design. The following sections include how to design ILC with fixed steps, the stability analyses for the new approach, and the validity verification of the method through simulation researches and real plant control.

2 Scheme of ILC with Fixed Step

Inspired by the principle of ILC, meanwhile taking into factors such as nonlinear property of a plant, incomplete model of a system and outside disturbance, we propose a new ILC method with a fixed step as illustrated in Fig. 1.

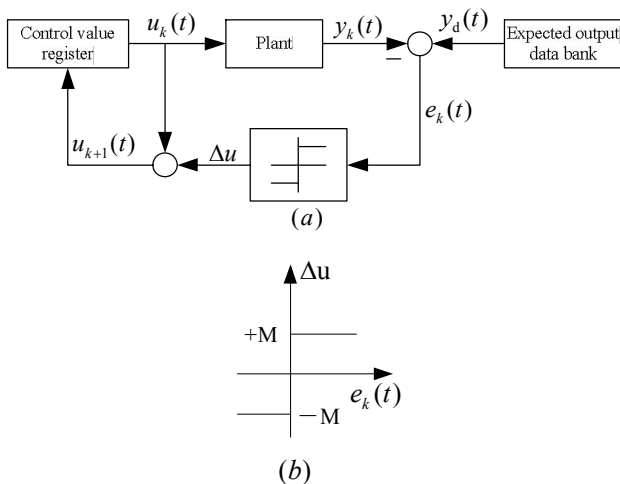


Fig. 1. Scheme of ILC with the fixed step

In Fig. 1, $y_k(t)$ is the k th iterative output at time t ; Δu is the iterative learning increment value; $y_d(t)$ is expected output at time t ; M is the step for the iterative learning increment value.

Fig. 1 and Eq. (1) show that the iterative learning step for the new ILC has special definition. Replacing $L\dot{e}_k(t)$ in Eq. (1) by Δu results in

$$u_{k+1}(t) = u_k(t) + \Delta u, \quad k = 1, 2, \dots, N. \quad (2)$$

We further defined

$$\Delta u = \text{sgn}(e_k(t)) \cdot M, \quad M > 0. \quad (3)$$

When compared with conventional ILC, the iterative learning increment value expressed in Eq. (3) has three apparent advantages: less computation with only addition or subtraction but no time-consuming, laborious differential and integral operating; smaller storing requirement with only $y_d(t)$, $u_{k+1}(t)$, and M to be stored; smoother operating process for the system because no error amplification result forms differential term.

3 Convergence Analyses for the Method

Take a linear system with repetitive output task as example, and let equation in status space be

$$\begin{cases} \dot{\mathbf{x}}_k(t) = \mathbf{A}(t)\mathbf{x}_k(t) + \mathbf{B}(t)u_k(t) \\ y_k(t) = \mathbf{C}(t)\mathbf{x}_k(t) \end{cases}, \quad (4)$$

where, $k = 1, 2, \dots, N$; $\mathbf{x}_k(t) \in \mathbf{R}^n$ is the k th iterative state vector at time t ; $u_k(t) \in \mathbf{R}$ is the k th iterative input at time t ; $y_k(t) \in \mathbf{R}$ is the k th iterative output at time t ; $\mathbf{A}(t)$, $\mathbf{B}(t)$, and $\mathbf{C}(t)$ are matrices to the equation of state for the system.

Define

$$(1) \quad f(t) = \mathbf{C}(t) \int_0^t \Phi(t, \tau) \mathbf{B}(\tau) d\tau, \quad t \in [0, T],$$

where, T is the period of one cycle; $\Phi(t, \tau)$ is the state transfer matrix for the matrix $\mathbf{A}(t)$.

$$(2) \quad \Delta = \max(f(t)).$$

Theorem

To a system described by Eq. (4) under control by algorithm of ILC with the fixed step M , only if $f(t) > 0$ ($t \in [0, T]$), then when $k \rightarrow \infty$, $y_k(t)$ will convergent to $y_d(t)$ within $t \in [0, T]$ and the iterative control error will meet the expression $|e| < M \cdot \Delta$.

Proof

It is presumed that the system initial state is set the same for each of iterations. Thus it is ensured that each of

iterations has the same initial condition, that is

$$\mathbf{x}_{k+1}(0) = \mathbf{x}_k(0). \quad (5)$$

At the very beginning of the k th iteration, the solution to the system described by Eq. (4) to the control input $u_k(t)$ is

$$\mathbf{x}_k(t) = \Phi(t, 0)\mathbf{x}_k(0) + \int_0^t \Phi(t, \tau)\mathbf{B}(\tau)u_k(\tau)d\tau, \quad (6)$$

$$\mathbf{x}_{k+1}(t) = \Phi(t, 0)\mathbf{x}_{k+1}(0) + \int_0^t \Phi(t, \tau)\mathbf{B}(\tau)u_{k+1}(\tau)d\tau. \quad (7)$$

In the $k+1$ th iteration, the iterative control error is

$$e_{k+1}(t) = y_d(t) - y_{k+1}(t). \quad (8)$$

In the k th iteration, the iterative control error is

$$e_k(t) = y_d(t) - y_k(t). \quad (9)$$

Combining Eq. (8) and Eq. (9) results in

$$\begin{aligned} e_{k+1}(t) - e_k(t) &= y_k(t) - y_{k+1}(t) \\ &= \mathbf{C}(t)[\mathbf{x}_k(t) - \mathbf{x}_{k+1}(t)]. \end{aligned} \quad (10)$$

Applying Eq. (6) and Eq. (7) to Eq. (10) results in

$$\begin{aligned} e_{k+1}(t) - e_k(t) \\ = \mathbf{C}(t) \int_0^t \Phi(t, \tau)\mathbf{B}(\tau)[u_k(\tau) - u_{k+1}(\tau)]d\tau. \end{aligned} \quad (11)$$

Applying Eq. (2) to Eq. (11) results in

$$e_{k+1}(t) - e_k(t) = -\Delta u \cdot \mathbf{C}(t) \int_0^t \Phi(t, \tau)\mathbf{B}(\tau)d\tau. \quad (12)$$

Applying Eq. (3) to Eq. (12) results in

$$e_{k+1}(t) = e_k(t) - \text{sgn}(e_k(t))M \cdot \mathbf{C}(t) \int_0^t \Phi(t, \tau)\mathbf{B}(\tau)d\tau. \quad (13)$$

And according to definition (1), Eq. (13) can be rewritten as

$$e_{k+1}(t) = e_k(t) - \text{sgn}(e_k(t))M \cdot f(t). \quad (14)$$

From Eq. (3), $M > 0$ is known. Therefore, for the iterative control error in Eq. (14), only if $f(t) > 0$ ($t \in [0, T]$), the system expressed in Eq. (4) is convergent under the ILC with the fixed step.

Applying definition (2) to Eq. (14) results in

$$e_{k+1}(t) = e_k(t) - \text{sgn}(e_k(t))M \cdot \Delta. \quad (15)$$

From Eq. (15) it can be concluded that under the novel

control law in Eq. (2) the system by Eq. (4) is convergent, and absolute value of iterative control error is confined by formulation:

$$|e| < M \cdot \Delta. \quad (16)$$

Permit completed.

Because Δ is decided by the system nature in Eq. (16), the value of $|e|$ is proportional to the value of M : the bigger M is, the bigger the iterative control error is. But in most cases, a compromise must be made between the convergent speed and the control error constraint in a control process. Therefore the value of M cannot be selected randomly as a very small value.

In the theorem, the convergent condition for a system in Eq. (4) is

$$f(t) = \mathbf{C}(t) \int_0^t \Phi(t, \tau)\mathbf{B}(\tau)d\tau, \quad t \in [0, T].$$

But to a time-variant system, $\Phi(t, \tau)$ has a complicated form, and it is necessary to rely on computer or other means to obtain $f(t)$. The following is the example of derivation for $f(t)$ in two typical linear systems.

3.1 A is a upper triangular matrix and its dimension is 2×2

For a system by Eq. (4), let

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{pmatrix}, \quad \mathbf{B} = (b_1 \quad b_2)^T, \quad \mathbf{C} = (c_1 \quad c_2),$$

then the transfer matrix for A is

$$e^{\mathbf{A}(t-\tau)} = \begin{pmatrix} e^{a_{11}(t-\tau)} & \frac{a_{12}(e^{a_{11}(t-\tau)} - e^{a_{22}(t-\tau)})}{a_{11} - a_{22}} \\ 0 & e^{a_{22}(t-\tau)} \end{pmatrix}. \quad (17)$$

Applying Eq. (17) to

$$f(t) = \mathbf{C} \int_0^t e^{\mathbf{A}(t-\tau)} \mathbf{B} d\tau$$

results in

$$f(t) = h_1(e^{a_{11}t} - 1) + h_2(e^{a_{22}t} - 1), \quad (18)$$

where $h_1 = \frac{1}{a_{11}}(b_1c_1 + \frac{a_{12}b_2c_1}{a_{11} - a_{12}})$,

$$h_2 = \frac{1}{a_{22}}(b_2c_2 - \frac{a_{12}b_2c_1}{a_{11} - a_{22}}).$$

3.2 A is a diagonal matrix

For a system by Eq. (4), let

$$A = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \lambda_n \end{pmatrix}, \quad \lambda_1 \neq \lambda_2 \neq \dots \neq \lambda_n,$$

$$B = (b_1 \quad b_2 \quad \dots \quad b_n)^T,$$

$$C = (c_1 \quad c_2 \quad \dots \quad c_n),$$

then the transfer matrix for A is

$$e^{A(t-\tau)} = \begin{pmatrix} e^{\lambda_1(t-\tau)} & 0 & 0 & 0 \\ 0 & e^{\lambda_2(t-\tau)} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & e^{\lambda_n(t-\tau)} \end{pmatrix}. \quad (19)$$

Applying Eq. (19) to

$$f(t) = C \int_0^t e^{A(t-\tau)} B d\tau,$$

results in

$$\begin{aligned} f(t) &= \int_0^t (b_1 c_1 e^{\lambda_1(t-\tau)} + b_2 c_2 e^{\lambda_2(t-\tau)} + \dots + b_n c_n e^{\lambda_n(t-\tau)}) d\tau \\ &= \frac{b_1 c_1}{\lambda_1} (e^{\lambda_1 t} - 1) + \frac{b_2 c_2}{\lambda_2} (e^{\lambda_2 t} - 1) + \dots + \frac{b_n c_n}{\lambda_n} (e^{\lambda_n t} - 1). \end{aligned} \quad (20)$$

Meeting condition $f(t) > 0$ requires

$$\begin{aligned} f(t) &= \frac{b_1 c_1}{\lambda_1} (e^{\lambda_1 t} - 1) + \frac{b_2 c_2}{\lambda_2} (e^{\lambda_2 t} - 1) + \dots + \frac{b_n c_n}{\lambda_n} (e^{\lambda_n t} - 1) > 0, \\ t &\in [0, T]. \end{aligned} \quad (21)$$

4 Simulation Verification

In order to verify the feasibility as well as robustness and validity of our approach for ILC with fixed steps, simulations are carried out on a second-order plant with time-variant property. And the matrices for the plant in the state space are

$$A = \begin{pmatrix} -1 & 1 \\ 0 & -2 - 0.01t \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad C = (0 \quad 1).$$

The expected output trail is $y_d(t) = 12t^2(1-t)$ and the initial state is considered as $x_k(0) = 0$ ($k = 1, 2, \dots, N$). In the whole control process iterative control error must meet $|e(t)| < 0.01$, $t \in [0, 0.8]$.

4.1 Computation the iterative step M

From the parameter of the plant, we can get

$$\Delta = \max_{t \in [0, 0.8]} f(t) = 0.3991. \quad (22)$$

To meet $|e| < 0.01$ in Eq. (16), we can set M under condition of $M < 0.0251$.

4.2 Digital simulations

Based on the principle illustrated in Fig. 1 for a fixed step ILC, considering the control input limitation for the plant, the scheme for the plant based on our assumptions is shown in Fig. 2.

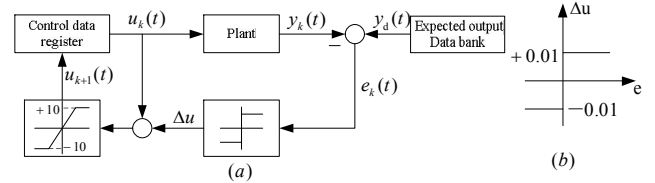


Fig. 2. Principle of simulation plant

To the plant, the simulation time span belongs to $t \in [0, 0.8]$ s, sampling time period is $\Delta t = 0.001$ s, and M is 0.01 V. After 844 iterations with the approach the iterative control output and corresponding expected results are shown in Fig. 3 at iteration 844 and the control error curve is shown in Fig. 4. In order to attest the convergence at the same time sequence for all iterations, control errors at time 0.6 s are shown in Fig. 5.

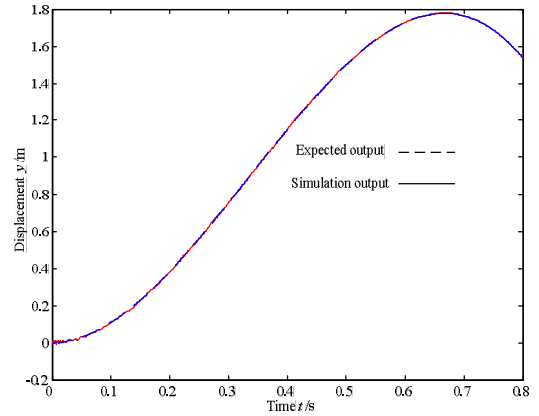


Fig. 3. Expected and simulation output

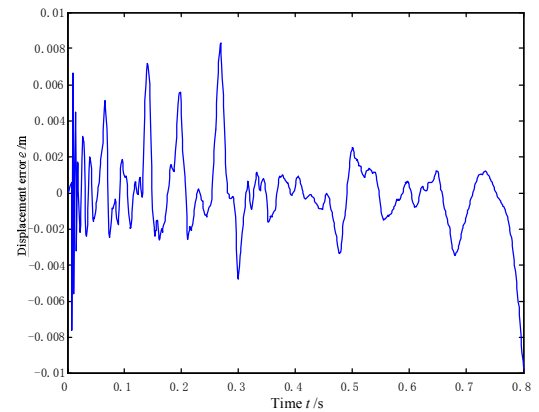


Fig. 4. Iterations control error curve

Fig. 3 shows that for the simulating plant the output meets expected value well under ILC with a fixed step after 844 iterations. And from data in Fig. 4, it can be seen that

in the iterative process the control error satisfies demand. Results from Fig. 5 show that the control error decreases and approaches zero with the increasing iterative times.

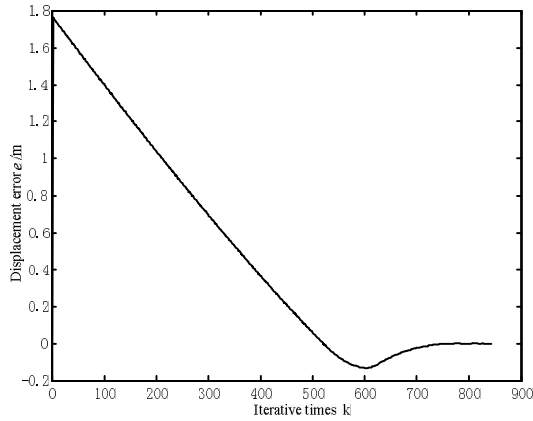


Fig. 5. Control errors at time 0.6 s

4.3 Robustness test

Supposing a random unrepeatable disturbance with amplitude 0.02 is mixed in the expected output signal, then the new expected output has the form as follows:

$$y_k(t) = y_k(t) + 0.02d(), \tag{23}$$

where $d()$ is the random function.

The control parameters in simulation are set as described in section 3.2. Fig. 6 shows the resultant control errors of the 852th iteration and Fig. 7 shows control errors at time 0.6s for all iterations.

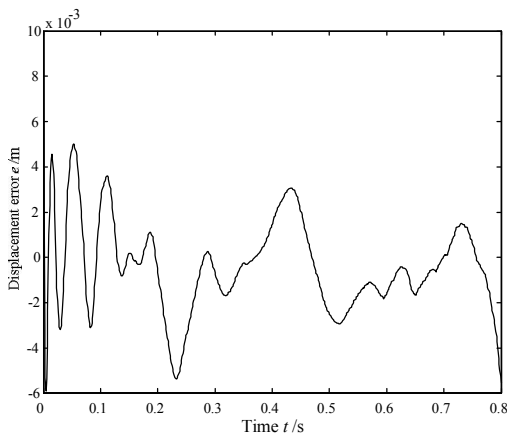


Fig. 6. Control errors of the 852th iteration

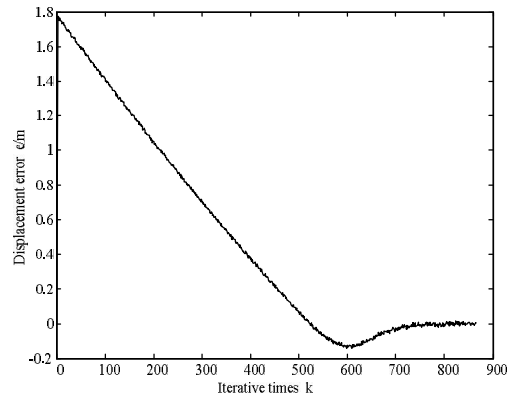


Fig. 7. Control errors at time 0.6s for all iterations

From Fig. 6 and Fig. 7, the control aim $|e| < 0.01$ is achieved after 852 iterations and the iteration number almost keeps the same with the proposed method, although disturbance amounts to double size of the control aim. Although the method with fixed step has a simpler structure, the comparable simulation results proof robustness.

4.4 Relation between M and convergence speed

Eq. (16) shows that both convergence speed and control precision are closely related to M and comparative simulation is carried out to further demonstrate the relation among them. Statistic data in table shows the relation between step M and iteration numbers at convergent point, where the control aim is $|e| < 0.01$ and k is iteration number needed for convergence.

Table Step value and iteration number

Parameter	Value					
Step M/V	0.005	0.010	0.015	0.020	0.025	0.040
Iteration number k	1 683	844	562	424	341	UA

Note: UA is unachievable.

Data in table show that the bigger value M is, the faster convergence speed is. But the value of M is restricted by the control precision as described in Eq. (16).

5 Experimental Verification

After convergence analysis and simulation study of the ILC with fixed steps, the method is adapted further in order to verify its ability to work for real control projects.

The plant is the second joint of a printing robot ПБ-211 shown in Fig. 8. The second joint of ПБ-211 robot is a swaying cylinder controlled by a servo valve. The transfer function for the plant is shown in Eq. (24):

$$\frac{\theta}{u} = \frac{\frac{k_Q}{B_Q} k_x}{\frac{J_t V_t}{4E_y B_Q^2} s^3 + (\frac{J_t k_{ce}}{B_Q^2} + \frac{B_t V_t}{4E_y B_Q^2}) s^2 + (\frac{B_t k_{ce}}{B_Q^2} + 1) s}, \quad (24)$$

and parameters can be referred to Ref. [9].



Fig. 8. Robot P B -211 system

During operation of the robot, the temperature of the hydraulic oil will rise, this will cause the parameters including E_y , B_t and k_{ce} to change. Therefore the plant is a time-variant system.

The expected output for the second joint of the P B -211 robot is:

$$y_d(t) = 10 \sin(\pi t / 2), \quad t \in [0, 2]s. \quad (25)$$

The control error limit is set as $|e| < 0.3$ with $M = 0.001V, 0.005V, 0.01V$ and $0.05V$ in order to test the method.

5.1 M=0.001V

The control results of different iterations are shown in Fig. 9. After 315 iterations the system reaches $|e| < 0.3$.

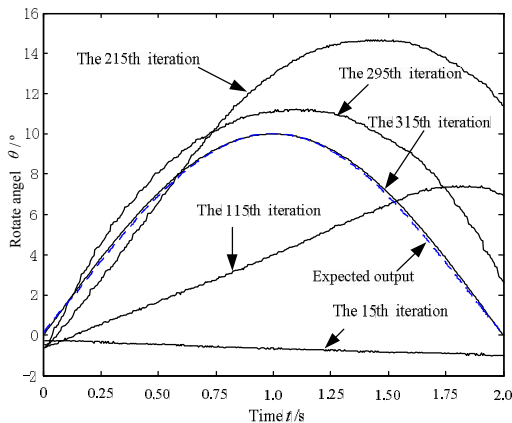


Fig. 9. Control results for $M=0.001V$

5.2 M=0.005V

The control results of different iterations are shown in Fig. 10. After 126 iterations the system reaches $|e| < 0.3$.

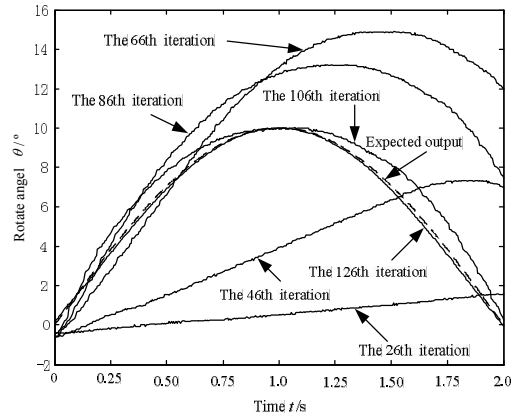


Fig. 10. Control results for $M=0.005V$

5.3 M=0.01V

The control results of different iterations are shown in Fig. 11. After 96 iterations the system reaches $|e| < 0.3$.

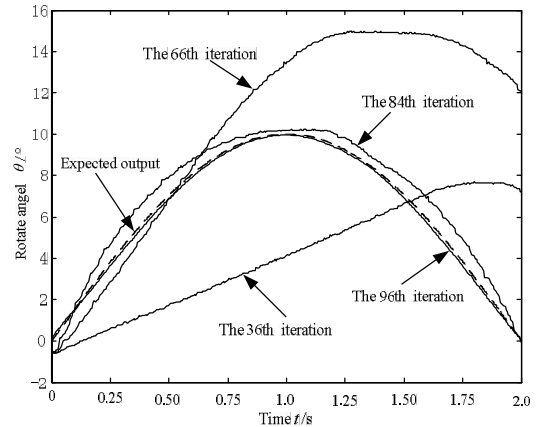


Fig. 11. Control results for $M=0.01V$

5.4 M=0.05V

The control results of different iterations are shown in Fig. 12. Fig. 13 shows the results of iteration 500 and under the value of M , the plant cannot reach the condition $|e| < 0.3$ within the considered time span.

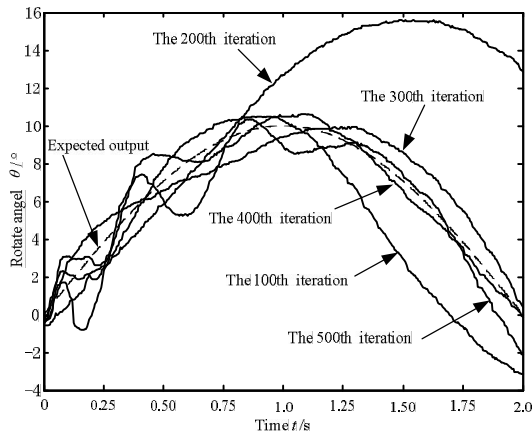


Fig. 12. Control results for $M=0.05V$

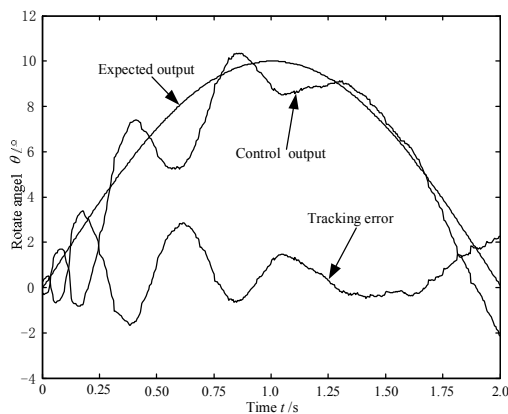


Fig. 13. Results of iteration 500 with $M=0.005V$

From the above results of the robot joint comparative experiments, the following conclusions can be drawn:

(1) The algorithm of ILC with fixed steps has validity in real application.

(2) In a relative wide scope of the step value, good tracking can be achieved under the demanded control precision.

(3) The convergence can speed up by increasing the step value, but the control precision will be deteriorated more or less.

(4) Within the permissible range of the respond speed, control precision can be enhanced by decrease of the step value.

6 Conclusions

(1) On the basis of the structure of the algorithm, the stability of the new ILC approach with a fixed step is proofed theoretically with the corresponding convergence condition.

(2) The convergence speed of iterative learning control algorithm with a fixed step is closely related to step M . The bigger the value of M is, the faster the convergence rate is. But the value of M is restricted by the control precision.

(3) The feasibility and merits of the approach are verified through both simulation system and real tracking control of a robot joint with time-variant parameters.

(4) Compared with the conventional ILC, our approach has the following advantages: simpler algorithm structure, less computation and storing capacity demand, no derivative amplifying of error, and thus enhances anti-disturbance ability of the method. Considering permissible ranges of control precision and convergence speed there is a wide scope to select a suitable step, which means good robustness of the mean.

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Biographical notes

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