Boson origin of superconducting phase

Tian De Cao

Department of Physics, Nanjing University of Information Science & Technology, Nanjing 210044, China

London presented a relation between suppercurrent density and magnetic vector potential[1], Pippard suggested a non-local relation[2], and the similar relation can be derived with the BCS theory and some strongly correlated models. However, pairs may have wavefunctions, and this leads the supercurrent to being related to a phase function[3,4]. That is to say, the whole microscopic relation between suppercurrent and magnetic vector potential has to found. This paper suggests a microscopic equation on the basis of boson features of superconducting pairs and finds some new results. Some reproduced results are qualitatively in agreement with experiments, while some new results have to be tested. Particularly, it is found that the phase function is determined by the Bose-Einstein distribution of the pairs, and the phase function establishes the relation between supercurrent and magnetic vector potential, this is beneficial to explaining why superconductivity requires pairs.

We assume that the superconducting pairs (for spin singlet pairs) could be described with the bosons, and the bosons have the excitation energies $\Omega_{\vec{q}} \equiv \Omega(\vec{q}, \vec{A}_{\vec{q}})$ in magnetic field. Because the Bose-Einstein distribution highlights the effects of low energy particles, for approximate isotropic systems [5], $\Omega(\vec{q}, e\vec{A}) = \Omega(q, eA) = \Omega(0,0) + \alpha q^2/2 + e^2 \beta A^2/2$, this leads to this supercurrent density equation

$$\vec{j}_s(\vec{q}) = -2eC[\alpha\vec{q} + e\beta\vec{A}_{\vec{q}}]n_B[\Omega_{\vec{q}}] \tag{1}$$

where the constant number α and β can be determined below, and the Bose-Einstein distribution is

$$n_B(\Omega_{\vec{q}}) = \frac{1}{e^{(\Omega_{\vec{q}} - 2\mu)/k_B T} - 1} \tag{2}$$

where 2μ is the chemical potential of the boson systems while μ is the chemical potential of other electrons. The determination of the chemical potential will not be discussed in this paper.

Whether Eq.(1) is reasonable should be examined with experiments. The equation is different from the London equation and Pippard equation, it seems more reasonable, and we will find this. We express $\sum_{\vec{q}} n_B(\Omega_{\vec{q}}) = n_s$, and define the phase function

$$\theta(\vec{x}) = \frac{1}{n_s}\int d^3q n_B(\Omega_q) e^{i\vec{q}\cdot\vec{x}} \tag{3}$$

this phase function is only determined by the Bose distribution. Eq.(3) shows that the phase function reflects the collective behavior of pairs. Using Eq.(1) and (3), we have $\vec{j}(\vec{x}) = -2eC\alpha n_s \vec{\nabla}\theta(\vec{x}) - 2e^2C\beta n_s \int \theta(\vec{x}' - \vec{x})\vec{A}(\vec{x}')d^3x'$. When the temperature is below the Bose condensation temperature, $T < T_B$, we can express $\theta(\vec{x}) = \frac{1}{n_s}\int d^3q[n_B^{(0)} + n_B^{(1)}(\Omega_{\vec{q}})]e^{i\vec{q}\cdot\vec{x}} = \frac{1}{n_s} n_B^{(0)}\delta(\vec{x}) + \frac{1}{n_s}\int d^3q n_B^{(1)}(\Omega_{\vec{q}})e^{i\vec{q}\cdot\vec{x}}$, thus $\vec{j}(\vec{x}) = -2eC\alpha n_s \vec{\nabla}\theta(\vec{x}) - 2e^2C\beta n_B^{(0)} n_s \vec{A}(\vec{x}) - 2e^2C\beta n_s \int \theta^{(1)}(\vec{x}' - \vec{x})\vec{A}(\vec{x}')d^3x'$ in the real space, where $n_B^{(0)}/n_s$ expresses the ratio of particle number under the Bose-Einstein condensation to the total particle number. The term including $\theta^{(1)}$ could be neglected when $T << T_B$. Having compared with the quantum mechanics form $\vec{j} = \frac{-en_s}{m_e}(\hbar\vec{\nabla}\theta + 2e\vec{A})$, we find $C\alpha = \frac{\hbar}{2m_e}$ and $C\beta = \frac{1}{m_e n_B^{(0)}}$, thus Eq.(1) can be rewritten in the form

$$\vec{j}_s(\vec{q}) = -\frac{e}{m_e}[\hbar\vec{q} + \frac{2e}{n_B^{(0)}}\vec{A}(\vec{q})]n_B(\Omega_{\vec{q}}) \tag{4}$$

$$\vec{j}_s(\vec{x}) = -\frac{e\hbar}{m_e}n_s\vec{\nabla}\theta(\vec{x}) - \frac{2e^2}{m_e}n_s\frac{1}{n_B^{(0)}}\int\theta(\vec{x}'-\vec{x})\vec{A}(\vec{x}')d^3x' \tag{5}$$

$$= \frac{-en_s}{m_e}(\hbar\vec{\nabla}\theta + 2e\vec{A}) - \frac{2e^2}{m_e}n_s\frac{1}{n_B^{(0)}}\int\theta^{(1)}(\vec{x}'-\vec{x})\vec{A}(\vec{x}')d^3x' \tag{6}$$

It is necessary to note $n_B^{(0)} \neq 0$ unless the electron systems are in the normal state. Eq.(5) shows that the two terms in supercurrent density are strictly correlated through the phase function $\theta(\vec{x})$, and this should be very appropriate. However, this result could not be found in the quantum mechanics form $\vec{j} = \frac{-en_s}{m_e}(\hbar\vec{\nabla}\theta + 2e\vec{A})$ or others.

The Meissner effect has been explained with the London equation or the Pippard equation, from which we know that the penetration depth decrease with increasing n_s or others. In the past, the temperature dependence of the penetration depth has been attributed to the change of the number of pairs or the change of the energy gap. The Meissner effect can be explained with Eq.(5) or (6), too. However, Eq.(4) shows that the temperature dependence of the penetration depth is strictly correlated to the Bose-Einstein distribution, this is interesting but has to be confirmed by comparing possible detail calculations with experiments. Eq.(4) obviously shows that the penetration depth will decrease with the decreasing of temperature, because of the increase of both n_s and $n_B^{(0)}$, this qualitative change is consistent with the well-known results, while possible quantitative examination have to be investigated.

Eq.(6) shows that the local relation in the London equation is due to the Bose-Einstein condensation. We will find that the magnetic flux quantum in a superconducting ring is also correlated to the condensation. Using Eq.(6), because of $\vec{j}_s=0$ inside the superconductor, we have $\frac{-en_s}{m_e}(\hbar\vec{\nabla}\theta+2e\vec{A})-\frac{2e^2}{m_e}n_s\frac{1}{n_B^{(0)}}\int\theta^{(1)}(\vec{x}'-\vec{x})\vec{A}(\vec{x}')d^3x'=0$. When $T<<T_B$, the Bose-Einstein condensation is dominant, the term including $\theta^{(1)}$ can be neglected, this leads to the result $\oint\vec{A}\cdot d\vec{l}=-\frac{1}{2e}\hbar\oint\vec{\nabla}\theta\cdot d\vec{l}=\pm\frac{1}{e}\hbar n\pi=\pm\frac{h}{2e}n=\pm n\phi_0$. Generally, we find

$$\oint\vec{A}\cdot d\vec{l}=\pm n\phi_0+\oint\vec{g}\cdot d\vec{l} \qquad (7)$$

where $\vec{g}(\vec{x})=\frac{1}{n_B^{(0)}}\int\theta^{(1)}(\vec{x}'-\vec{x})\vec{A}(\vec{x}')d^3x'$, and $\vec{g}(\vec{x})\neq c\vec{A}(\vec{x})$. The superconducting transition temperature is expressed as T_c, and the Bose-Einstein condensation temperature is expressed as T_B. Two cases may appear in various superconductors. The first case is $T_c<T_B$, and this leads Eq.(6) to being the quantum mechanics result $\vec{j}=\frac{-en_s}{m_e}(\hbar\vec{\nabla}\theta+2e\vec{A})$, the magnetic flux quantum in a superconducting ring could be kept. The second case is $T_c>T_B$. Eq.(7) shows that the term including $\vec{g}$ may break the magnetic flux quantum. In this second case, two ways could keep the magnetic flux quantum, one is to decrease temperature, and another is to use a ring thick enough. However, when the ring is large enough, the fluctuation of the earth magnetic field could not be neglected, and the measurement of magnetic field may produce a large discrepancy. Therefore, the possible way to keep the magnetic flux quantum is decreasing temperature. Because this case corresponds to $T_c > T_B$, this ring favors to be made from high temperature superconductors. This clearly shows that the increasing temperature may break the magnetic flux quantum, and this appearance easily occurs in high-T_c superconducting ring. The magnetic flux quantum has been examined with some low-temperature superconductors [6,7] under some discrepancy, while the origin of this discrepancy has to be investigated, and our suggestion above may shed light on the origin.

A superconductor has its critical current density $\vec{j}_c$, when the current density is larger than the critical current, $\vec{j} \geq \vec{j}_c$, the DC resistance of the superconductor is not zero. This could be understood with Eq.(5). Using $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}_s$, we get

$$\frac{1}{\mu_0}\vec{\nabla} \times \vec{B} = -\frac{e\hbar}{m_e} n_s \vec{\nabla}\theta(\vec{x}) - \frac{2e^2}{m_e} n_s \frac{1}{n_B^{(0)}} \int \theta(\vec{x}' - \vec{x}) \vec{A}(\vec{x}') d^3 x' \qquad (8)$$

It shows that $\vec{A} \neq 0$ but $\vec{B}$=0 is possible in a superconductor. Moreover, $\vec{A}$=0 is impossible in the superconducting region of a superconductor. If we take the Coulomb gauge $\vec{\nabla} \cdot \vec{A}$=0, we establish this equation

$$\frac{1}{\mu_0}\nabla^2 \vec{A} = \frac{e\hbar}{m_e} n_s \vec{\nabla}\theta(\vec{x}) + \frac{2e^2}{m_e} n_s \frac{1}{n_B^{(0)}} \int \theta(\vec{x}' - \vec{x}) \vec{A}(\vec{x}') d^3 x' \qquad (9)$$

thus the magnetic vector potential $\vec{A}$ is knew if it has been given in the surface of a superconductor. The magnetic vector potential in Eq. (9) can be such a distribution that the current determined by Eq(5) arrives at its maximum, the critical value. If we exert some external field on the superconductor, the magnetic vector potential should consist of two parts, $\vec{A}_{total} = \vec{A} + \vec{A}_{ext.}$, $\vec{A}$ is still determined by Eq.(9) (in superconducting state), while $\vec{A}_{ext.}$ is not.

In summary, Eq.(1) shows some new physics, some are in agreement with experiments, while some have to be tested. Eq(1) leaves many problems to be investigated. For example, what is the form of $\Omega(0,0)$? $\Omega(0,0)$ may correlate with the medium of electron pairing. What is the form of $\theta(\vec{x})$? $\theta(\vec{x})$ must be determined by computer. However, this paper should be beneficial for one to understand some general problems of superconductivity.

References

1. London, E. Superfluids, Vol.1(Wiley, New York,1954).
2. Pippard, A. B. An Experimental and Theoretical Study of the Relation between Magnetic Field and Current in a Superconductor. Proc.Roy.Soc., **A216**,547 (1953).
3. Byers, N. & Yang, C. N. Theoretical considerations concerning quantized magnetic flux in superconducting cylinders. Phys. Rev. Lett. **7**, 46-49 (1961).
4. The Feynman Lectures on Physics, Vol.3, Chao.21, 21-5.
5. Cao, T. D. arXiv:1010.3896.
6. Doll, R. & Näbauer, M. Experimental Proof of Magnetic Flux Quantization in a Superconducting Ring. Phys. Rev. Lett. 7, 51-52(1961).
7. Deaver, B. S. & Fairbanks, W. M. Experimental Evidence for Quantized Flux in Superconducting Cylinders. Phys. Rev. Lett. 7, 43-46(1961).