

Is Reality Digital or Analog?

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Abstract

A report of a discussion with Isaac Newton.

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Some years ago I happened to be listening to a lecture given by Gregory Chaitin on what he called as "digital philosophy." [1] In the following night, just before falling asleep, I was lying on my bed wondering what Isaac Newton, the discoverer of calculus and arguably the greatest physicist of all times, would have said about Chaitin's ideas.¹⁾ Gradually, my thoughts took a form of a firm decision: I would go and see Isaac Newton in person and ask from himself.

It was not so easy to arrange an interview with Isaac Newton. He had a reputation of a difficult person, and it was very uncertain, whether he would receive me in the first place. Then I got it: Newton would certainly be interested in the physics of our times. I decided to send him a collection of the best possible textbooks of physics I could find, together with copies of the original papers on black hole entropy and radiation written by Bekenstein and Hawking. [2, 3] I also sent to Newton a letter, where I introduced myself and inquired for a possibility to meet him in person.

A month later there was a letter in my mailbox. It read:

"Will receive you with pleasure. Meet me in my London residence on the 18th of November, at 3 p. m. in 1700.

Sincerely Yours,

Isaac Newton."

I am not going to bore the reader with a detailed description of how I managed to arrive to London of the year 1700. To make a long story short I just

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tell that exactly at 3 p. m. on the 18th of November in 1700 I was standing on the doorsteps of Isaac Newton. With some hesitation I knocked the door...

To my astonishment the door was opened by a beautiful lady in her early twenties. I introduced myself, and a warm, welcoming smile came to her face. "Please, come in," she said. "My uncle will receive you in his study. This way, please." At that point I remembered that some time after Newton had left from Cambridge to London to become a Warden and, later, a Master of Mint, his niece Catherine Barton moved into his house as a housekeeper. [4] Together with Ms. Barton I ascended the stairs to Newton's study. "I am so happy that you sent all those books to my uncle", she said exuberantly. "My uncle has been somewhat depressed lately, but ever since he received your books he has been in high spirits."

We arrived to Newton's study. Ms. Barton knocked the door and we heard a gentle voice "come in." She opened the door, I stepped in, and there was the great man himself.

From everything I had read I expected to see a short and somewhat aloof and inward bound person with an untidy appearance. On the contrary, he seemed extremely alert, he was well-dressed, and his long, white hair was carefully combed. He was very short, it is true, hardly five feet, but one immediately forgot his short stature after looking at his large, bright and intelligent eyes.

There was no need to formal introductions, because I had already introduced myself in a letter, and Isaac Newton certainly did not need any introductions. So I asked right away: "Is reality digital or analog?" Without any moments of hesitation he said: "Why, digital of course." I was rather taken aback by his quick answer and asked: "How do you know?" "Because I have calculated it," was his response.

This was quite a news. With some enthusiasm I asked, whether he would show me his calculation. "Of course," he said. In his study he had a large, white, wooden box. When he opened the box, I observed that it was full of papers. He perused the contents of the box for a while and said: "No, it is not here. Now I remember: I performed the calculations on yesterday evening by the fireplace, and before going to sleep I left my calculations on the mantelpiece." He went to see the fireplace. "No, it is not here, either. Where can it be?" He opened the door and shouted: "Catherine!" "Yes, uncle," said Ms. Barton. "Have you seen the papers, which I left on the mantelpiece last night?" asked Newton. "Let me think," said Ms. Barton. "There was indeed some papers on the mantelpiece this morning. To me they seemed to be of no importance, and so I burned them in the fireplace."

Isaac Newton went back to his study. He looked out of the window, and for a moment he seemed utterly crushed. Then, suddenly, he burst into a laugh. I was so surprised that I could not avoid of beginning to laugh, too. We laughed together, and finally he sat on an armchair, gesturing me to do the same.

"I became convinced of the digital nature of reality already in my youth," he said at last. "I made some experiments with prisms and sunlight, you know, and I observed that although white light, when passing through a prism, dissolves into light with different colours, the colours thus produced cannot be dissolved

further by another prism. For instance, red light stays red, while passing through another prism, blue light stays blue, and so on. So it was natural to conclude that light consists of particles. Different particles produce different colours, and white light is produced as a mixture of those particles. As I am aware now, my chain of reasoning was far from flawless, and Robert Hooke was actually quite correct in many points in his critique of my particle theory of light.²⁾ Nevertheless, it took me into a whirl of speculations. If light consists of particles, then why should not matter as well? Maybe all matter is made of some ultimate constituents with a very small number of intrinsic properties, I thought, and all properties of matter may finally be reduced to the properties of its constituents. In this sense, the behavior of matter is digital, rather than analog. At that time I had no experimental support for my speculations, but reading the books you kindly sent me I learned that light consists of particles, which are known as photons, and all matter is made of atoms. So I was right after all, wasn't I?" concluded Newton with a grin.

"Yes, you were, indeed," said I. "And what are your current thoughts about space and time?"

"When I read about Einstein's special and general theories of relativity," he replied, "I was very impressed. From the very beginning I liked those theories enormously. I had no difficulties whatsoever to accept them. Indeed, so simple and so beautiful those theories are that I wonder why I did not discover them by myself, when I was in my prime of invention. The Equivalence Principle, for instance, was entirely within my grasp. Unfortunately, the idea of the constancy of the speed of light with respect to all inertial observers never dawned to me. but even that I could have deduced, if I had had the courage to draw the ultimate conclusions from my first law of motion implying, in effect, an equivalence of all inertial observers."³⁾

As a specialist in general relativity with an avid interest in the history of science I was forced to admit to myself that Newton was not bragging, but simply telling the truth. He thought for a moment, and then he continued:

"The ultimate problem, of course, is how to make general relativity compatible with quantum mechanics.⁴⁾ To me it seems likely that spacetime, in the same way as matter, has a sort of atomic structure. In other words, there must be some ultimate constituents of space and time."

"That is very interesting," said I. "Why do you think so?"

"When speculating on the possible atomic structure of spacetime," he said, "one is very much in the same position as I was in my youth when speculating on the possible atomic structure of matter. There is no direct observational evidence. However, there is a very important piece of indirect theoretical evidence."

"And what is that?" asked I.

"The result, obtained by Stephen Hawking, my worthy successor in the Lucasian chair, that black holes emit spontaneous thermal radiation. I must confess that understanding Hawking's important result was very hard to me. However, I think that I have now mastered every detail of its standard derivation, [3] and I am convinced that it is true. Black hole indeed has certain temperature,

which is inversely proportional to its mass. And it has entropy, too, which - in the natural units, where all natural constants are set to unity - is one-quarter of its event horizon area."⁵⁾

"And what does that have to do with the atomic structure of space and time?" asked I.

"Do you still remember the thermodynamical reason for the particle nature of light?" asked Newton.

"Yes," said I. "When considering the thermodynamical properties of electromagnetic radiation - light, for example - one finds that if one adds up the energies carried by the different frequencies of the radiation in a given temperature, the total energy density of the radiation becomes infinite, which is absurd, of course. However, if one assumes that the radiation consists of particles, each of them carrying an energy, which is proportional to the frequency of the radiation, the energy density miraculously becomes finite, and the resulting expression for the intensity of the radiation agrees with observations."

"Precisely," said Newton. "When considering the thermodynamical properties of black holes, one meets with similar unphysical infinities." At this point he went to his white wooden box and pulled out an empty sheet of paper. From his desk he picked up a quill pen, beginning to write equations on the paper. "The thermodynamical properties of any system," he continued, "may be deduced from its partition function

$$Z(\beta) = \sum_E g(E) e^{-\beta E}, \quad (1)$$

where β is the inverse temperature of the system, and we have summed over all possible energies E of the system.⁶⁾ Of course, if the energy spectrum is continuous, the sum must be replaced by an integral. $g(E)$ tells the degeneracy of a state with energy E . In other words, it tells the number of the microscopic states associated with the same total energy E of the system. Can you recall what is entropy?"

"Well," said I, "if all microstates associated with the same macroscopic state of the system have equal probabilities, then the entropy of the system is, in natural units, the natural logarithm of the number of those microstates..."

"...which means that the number of the associated microstates is the exponential of the entropy," said Newton. "As you may remember, the radius of the Schwarzschild black hole, which is the simplest possible black hole is, in natural units, $R = 2M$, where M is the mass of the hole.⁷⁾ Hence it follows that the event horizon area of the hole is $A = 4\pi R^2 = 16\pi M^2$. Because the entropy of a black hole is one-quarter of its event horizon area we find, identifying the mass M of the hole with its total energy E , that the partition function of the Schwarzschild black hole is

$$Z(\beta) = \sum_E e^{4\pi E^2} e^{-\beta E}. \quad (2)$$

As far as we know, there is no upper limit for the mass of a black hole, and so we observe that the sum in our partition function diverges.⁸⁾ Actually, it

diverges very badly, because the function $e^{4\pi E^2} e^{-\beta E}$ goes very rapidly towards the positive infinity, when the energy E is increased, no matter what is β ."

Newton laid his quill pen on his writing desk and concluded: "Since the sum in Eq. (2) diverges, a black hole has no well-defined partition function. Without partition function we cannot deduce its thermodynamical properties. Hence we meet with an unphysical infinity, which is somewhat similar to the one we encounter when considering the energy density of electromagnetic radiation."

"Do you have any ideas of how to get rid of that infinity?" asked I.

"The reason for a divergent partition function," replied Newton, "was that we identified $g(E)$ with the exponential of the black hole entropy, which was assumed to be one-quarter of its event horizon area. One is therefore inclined to doubt the general validity of the law of the simple proportionality between the area and the entropy of a black hole. Of course, it is valid when the temperature of the hole is very low. There is no question about that. An interesting issue is, whether the area law of black hole entropy holds even when the temperature of the hole is very high. In my opinion we should modify the function $g(E)$ such that the resulting entropy of the black hole is proportional to the event horizon area at low temperatures, but not necessarily at high temperatures. The main goal is to make the partition function convergent."

"How are you going to find an appropriate modification of $g(E)$?" asked I.

"I construct the event horizon out of discrete constituents," answered Newton, picking up his quill pen again, and moisturing it with ink. "Each constituent is assumed to carry an area which, in the natural units, is an integer times an appropriate constant, and the total area of the horizon is assumed to be the sum of the areas of those constituents. In other words, we write the total area of the horizon as:

$$A = 16\pi\alpha^2(n_1 + n_2 + n_3 + \dots + n_N), \quad (3)$$

where N is the number of the constituents, and α is a pure number to be determined later. The quantum numbers n_1, n_2, \dots, n_N are non-negative integers, and they determine the areas of the individual constituents of the horizon. More precisely, if we pick up a constituent j , ($j = 1, 2, \dots, N$) the area contributed by that constituent to the total area of the horizon is $A_j = 16\pi\alpha^2 n_j$, where n_j is a non-negative integer."⁹⁾

"To me your idea seems very simple and natural," said I. "What is now the degeneracy of those states of the black hole, which have the same energy?"

"Because the event horizon area of a black hole is $A = 16\pi M^2$," replied Newton, "the mass M of the hole is determined by its horizon area. Identifying the mass of the hole with its energy we find, using Eq. (3), that the possible energies of the hole are

$$E_n = \alpha\sqrt{n}, \quad (4)$$

where n is a non-negative integer determined by the quantum numbers n_j such that

$$n = n_1 + n_2 + n_3 + \dots + n_N. \quad (5)$$

In other words, the energy of the hole is uniquely determined by the sum of the quantum numbers n_j . There are several ways to write the non-negative integer n as a sum of N non-negative integers n_j , and this gives a rise to the degenerate states.”

Newton moistured his quill pen again with ink and then continued: ”When a constituent is in vacuum, *i. e.* $n_j = 0$, it does not contribute to the physical properties of the horizon, and hence it is natural to identify the microscopic states with the different combinations of the non-vacuum states of its constituents. As a consequence, the degeneracy $g(E_n)$ of a state with energy E_n is the number of ways of writing the positive integer n as a sum of at most N positive integers n_j . More precisely, it is the number of ordered strings (n_1, n_2, \dots, n_m) , where n_1, n_2, \dots, n_m are positive integers, $1 \leq m \leq N$, and $n_1 + n_2 + \dots + n_m = n$. The number of ways of writing a positive integer n as a sum of m positive integers is the same as is the number of ways of arranging n balls in a row in m groups by putting $(m - 1)$ divisions in the $(n - 1)$ empty spaces between the balls. The position for the first division may be chosen in $(n - 1)$ ways, for the second in $(n - 2)$ ways, and so on. The total number of the combinations for the positions of the divisions is therefore $(n - 1)(n - 2) \cdots (n - m + 1)$. However, because the divisions are identical, we must divide this number by the number of the possible orderings of the divisions, which is $(m - 1)(m - 2) \cdots 2 \cdot 1 = (m - 1)!$. Hence the total number of ways of writing a positive integer n as a sum of exactly m positive integers is¹⁰⁾

$$\binom{n-1}{m-1} := \frac{(n-1)(n-2) \cdots (n-m+1)}{(m-1)!}, \quad (6)$$

and the degeneracy of the state with energy E_n is

$$g_1(E_n) := \sum_{m=1}^N \binom{n-1}{m-1}, \quad (7)$$

provided that $N \leq n$. The number m cannot be greater than n , and therefore the degeneracy is

$$g_2(E_n) := \sum_{m=1}^n \binom{n-1}{m-1}, \quad (8)$$

whenever $N \geq n$. So we obtain an expression

$$Z(\beta) = \sum_{n=1}^N \left[\sum_{m=1}^n \binom{n-1}{m-1} e^{-\beta\alpha\sqrt{n}} \right] + \sum_{n=N+1}^{\infty} \left[\sum_{m=1}^N \binom{n-1}{m-1} e^{-\beta\alpha\sqrt{n}} \right] \quad (9)$$

for the partition function of a black hole.”

”And your partition function is convergent?” asked I.

”Absolutely,” said Newton. ”Using Eq. (6) we find that when $n \gg N$, we have¹¹⁾

$$\sum_{m=1}^N \binom{n-1}{m-1} \sim \frac{1}{(N-1)!} (n-1)^{N-1}. \quad (10)$$

For fixed N the sum

$$\sum_{n=N+1}^{\infty} (n-1)^{N-1} e^{-\beta\alpha\sqrt{n}}$$

will certainly converge, and therefore our partition function will converge as well.”

”So you have managed to show that if the event horizon of a black hole consists of a finite number of discrete constituents, then the partition function of the hole is well-defined and finite,” said I. ”Very fine. What about the entropy of the hole at low temperatures?”

”You are familiar with the Binomial Theorem, I suppose?” asked Newton.

”Of course,” said I. ”If n is a positive integer, the Binomial Theorem implies that

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k \quad (11)$$

for any reals a and b .”

”When the temperature of the black hole is very low, we have $\frac{T}{N} \ll 1$, which means that a great majority of the constituents of the event horizon are in vacuum,” said Newton. ”In this limit the degeneracy of a state with energy E_n is given by the function $g_2(E_n)$ of Eq. (8). Putting $a = b = 1$ in the Binomial Theorem we find:

$$g_2(E_n) = \sum_{m=1}^n \binom{n-1}{m-1} = (1+1)^{n-1} = 2^{n-1}, \quad (12)$$

and hence the entropy of the hole is, in the low temperature limit:

$$S = \ln(2^{n-1}) = (n-1) \ln 2. \quad (13)$$

Since the event horizon area $A = 16\pi\alpha^2 n$, we get for large n , taking $\alpha = \frac{1}{2}\sqrt{\frac{\ln 2}{\pi}}$:

$$S = \frac{1}{4}A, \quad (14)$$

which is the well-known law for black hole entropy.”

”Excellent!” I exclaimed. ”That is really impressive!”

”Elementary,” said Newton. ”Anyway, our calculation may be used as an argument for an atomic structure of spacetime. Assuming that the event horizon of a black hole consists of a finite number of discrete constituents, we may both remove the unphysical infinities from its partition function, and obtain a correct expression for the black hole entropy at low temperatures. If the event horizon of a black hole consists of some fundamental constituents, then why should not spacetime as a whole as well? After all, effects very similar to the spontaneous thermal radiation of black holes may be observed, at least in principle, even in flat spacetime, where no black holes are present. When reading Wald [11] I learned that an observer in a uniformly accelerating motion will detect thermal

radiation of particles even when all inertial observers detect a vacuum.¹³⁾ To me it seems likely that to explain effects like this one must assume a sort of atomic structure of spacetime. In this sense spacetime, in the same way as matter and indeed reality itself, is digital, rather than analog.”

”Although I am very impressed by your derivation of the area law of black hole entropy at low temperatures, I must confess that I am still sceptical,” said I. ”In Einstein’s general relativity the concept of distance plays a fundamental role in the sense that if we know the distances between the points of spacetime, we know the so-called metric tensor $g_{\mu\nu}$ of spacetime. The metric tensor, in turn, determines the geometric and the causal properties of spacetime.¹⁴⁾ If spacetime really has an atomic structure, then what happens to the concepts of distance, time and causality?”

”When the appropriate natural constants are put in the calculations we just made,” said Newton, ”one finds that the possible areas of the constituents of the black hole event horizon may be expressed in terms of the quantum numbers n_j as $4n_j(\ln 2)\ell_{Pl}^2$, where the quantity ℓ_{Pl} is written in terms of the gravitational constant G , Planck’s constant \hbar and the speed of light c in the form:

$$\ell_{Pl} := \sqrt{\frac{\hbar G}{c^3}} \approx 1.6 \times 10^{-35} m. \quad (15)$$

As I have read from your books, this quantity is known as the Planck length. The result suggests that at the Planck length scale the concepts of distance, time and causality will probably lose their meaning. However, at macroscopic scales, where the number of the constituents of the spacetime region under consideration is very large, these concepts may be used to describe the statistical properties of spacetime.”

”What will replace distance as the fundamental concept in your model?” asked I.

”The concept of area,” was Newton’s firm answer. ”At macroscopic scales one may reduce the concept of distance to the concept of area. Actually, that is a trivial consequence of the fact that the dimension of spacetime at macroscopic scales is four.”

”To me that seems highly non-trivial,” said I.

”Well,” said Newton, showing admirable patience to my slow intellect. ”Consider first a tetrahedron. A tetrahedron is a three-dimensional object with 4 vertices not lying on the same plane. Each vertex is connected to every other vertex by an edge. As a result there are 6 edges and 4 triangles in a tetrahedron. A natural four-dimensional generalization of a tetrahedron has 5 vertices not lying in the same three-dimensional space, each vertex being connected, again, to every other vertex by an edge. As I read from *Gravitation*, [15] this object is known as a four-simplex. An interesting property of a four-simplex is that the number of its edges is 10, which is the same as is the number of its triangles.¹⁵⁾ So there is a one-to-one relationship between the edges and the triangles of a four-simplex, and we may not only express the triangle areas in terms of the edge lengths, but the edge lengths may also be expressed in terms of the triangle

areas. Hence we observe that whenever we pick up 5 points in a four-dimensional spacetime such that those points do not lie in the same three-dimensional space, the distances between the points may be expressed in terms of the triangle areas of a four-simplex having those points as its vertices. In this sense the concept of distance may really be reduced, in four-dimensional spacetime, to the concept of area."

"I see," said I. "It is trivial. I suppose it is natural to assume that the triangles in your four-simplex, in the same way as the event horizons of black holes, consist of discrete constituents, each of them contributing an area of the form $4n_j(\ln 2)\ell_{P_l}^2$ to the triangle area?"

"That is right," said Newton. "As a consequence, we may reduce the distances between the points of spacetime, and thereby the metric tensor $g_{\mu\nu}$, to the quantum states of those constituents. Since the metric tensor $g_{\mu\nu}$ determines the causal properties of spacetime, it seems to me likely that there exists a still unknown law of nature, closely related to the Second Law of Thermodynamics, which implies that when we consider spacetime at macroscopic scales, the quantum states of its constituents are distributed in such a way that the everyday notions of time and causality are recovered. However, that is just speculation."

Obviously, our discussion was coming to an end. I got up from the armchair to thank Newton for an interesting discussion, when suddenly Ms. Barton rushed into the room. "Uncle," she said. "I managed to find your papers. They were not the ones, which I burned in the fireplace this morning, but you had left them on the breakfast table. Here they are!"

To my surprise Newton handed the papers to me and said: "Please, keep these. I think that you will need them more than I. After all, it would be a strong violation of causality, if I published them by myself."

"I think that you will find your way out by yourself," said Ms. Barton, observing that I was just about to leave. "I will," said I. Saying good-bye to Isaac Newton and his charming niece I left Newton's study and closed the door behind me.

When descending the stairs I could not resist the temptation of having a look at those papers. Obviously, they constituted a finished draft of an extensive research report, written with a miniature, but still clear and precise hand. Its front page carried the title:

THE COMPLETE QUANTUM THEORY OF GRAVITATION

Near the end of the draft my eyes picked up a sentence: "So we observe that the fundamental equation of quantum gravity is..."

I stumbled. I fell on my face and came rolling down the stairs. I felt pain in my shoulder and all papers flew out of my hands...

I was lying on the floor beside of my bed. It was eight o'clock in the morning.

NOTES

1) Isaac Newton lived during the years 1642-1727.

2) Robert Hooke (1633-1707) was a rival and an antagonist of Isaac Newton. His original achievements included, among other things, significant improvements on a microscope, and a discovery of cells.

3) The Equivalence Principle states, in very broad terms, that it is not possible to decide, on grounds of local observations, whether one is in a gravitational field, or in an accelerating frame of reference. Historically, it was the starting point of Einstein's general theory of relativity, which explains gravity by means of the properties of space and time. The constancy of the speed of light with respect to all inertial observers, in turn, is the starting point of Einstein's special theory of relativity. Among other things, it implies that space and time are not absolute, but they depend on the observer's state of motion. This is in marked contrast with Newton's classical mechanics, where space and time are considered absolute. Hence Newton's favorable opinion on Einstein's special and general theories of relativity is most remarkable.

4) A theory, which makes general relativity and quantum mechanics compatible with each other is known as a quantum theory of gravitation, or quantum gravity, in short. Construction of the quantum theory of gravity is generally considered as the greatest challenge of theoretical physics.

5) By definition, a black hole is a region of space, where the gravitational field is so strong that not even light can escape from that region. The boundary of a black hole is known as event horizon. A black hole is created, among other things, if a very large star collapses under its own weight after burning off its nuclear fuel. It was a great discovery of Stephen Hawking that black holes are not completely black after all, but they emit radiation by means of certain quantum-mechanical processes taking place in the vicinity of the event horizon. An importance of this result lies in the fact that it brings together general relativity, quantum mechanics and thermodynamics, the fundamental theories of physics.

6) The parameter β may be written in terms of the absolute temperature T of a system, in natural units, as $\beta = \frac{1}{T}$. If we know the partition function of a system, we may obtain expressions for various thermodynamical quantities as functions of T . For instance, the average energy of a system is

$$E_{ave} = -\frac{d}{d\beta} \ln Z(\beta).$$

7) In terms of the gravitational constant G and the speed of light c the radius of the Schwarzschild black hole may be written as

$$R = \frac{2GM}{c^2}.$$

For instance, if the mass M of the hole is one solar mass, then R is about three kilometers.

8) Solutions to the problem of diverging partition function have been suggested in Refs. [5] and [6].

9) One of the consequences of Newton's model is that the event horizon area of a black hole has an equally spaced spectrum. The idea of an equally spaced horizon area spectrum was raised first by Bekenstein in Ref. [7], and later by Bekenstein and Mukhanov in Ref. [8].

10) For instance, the number 5 may be written as a sum of 3 positive integers in

$$\binom{5-1}{3-1} = \frac{(5-1)(5-2)}{(3-1)!} = 6$$

ways. Indeed, we have:

$$5 = 2 + 2 + 1 = 2 + 1 + 2 = 1 + 2 + 2 = 1 + 1 + 3 = 1 + 3 + 1 = 3 + 1 + 1.$$

Finding the microscopic states associated with the given macroscopic state of a black hole is one of the greatest problems of black hole physics. The problem has been considered in the context of string theory in Ref. [10], and in that of loop quantum gravity in Ref. [11]. It should be noted that in Newton's model different orderings of the same quantum numbers n_j represent different microscopic states.

11) When $n \gg N$, the leading term in the sum on the left hand side of Eq. (10) is the one, where $m = N$. Using Eq. (6) one finds that the leading term of $\binom{n-1}{N-1}$ is given by the right hand side of Eq.(10).

12) This is known as the *Unruh effect*. [12] An accelerating observer experiences the so-called Rindler horizon, which has properties very similar to those of the event horizon of a black hole. It is possible that the Unruh effect could be explained by means of an appropriate discrete model of spacetime. [13, 14]

13) In general relativity one defines between spacetime points (x^0, x^1, x^2, x^3) and $(x^0 + dx^0, x^1 + dx^1, x^2 + dx^2, x^3 + dx^3)$ the so-called *line element*

$$ds^2 = \sum_{\mu, \nu=0}^3 g_{\mu\nu} dx^\mu dx^\nu,$$

where $g_{\mu\nu}$ is the metric tensor at the point (x^0, x^1, x^2, x^3) . The distance between these points is the square root of the modulus of ds^2 . A curve connecting two points of spacetime is *spacelike*, if the line element at its every point is positive, and *timelike*, if it is negative. Hence the metric tensor determines both the metric-, and the causal properties of spacetime.

14) The numbers of edges and triangles of a four-simplex, respectively, are the same as are the numbers of ways of picking up two and three vertices out of five. Two vertices may be picked up in $\frac{5 \cdot 4}{1 \cdot 2} = 10$ ways, and three vertices may also be picked up in 10 ways, because $\frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} = 10$.

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