Calibrating Holistic Water Resources–Economic Models

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Abstract: This paper presents an approach to calibrating a holistic water resources–economic model, which involves essential water resources and economic components in a consistent model. The model is formulated as an optimization model, with the objective of maximizing economic welfares/profits from water uses. When we apply the model to a baseline scenario against real-world conditions, the economic outputs are often expected to match the observations at the base level, since a wide divergence between model outcomes and actual results is not appropriate for analyzing policy options starting from the baseline. Following the concept of "positive mathematical programming," the holistic water resources–economic model is calibrated to a baseline level using programming constraints and "positive" inferences from baseline observations. The calibration task is complicated by the fact that a large number of interdependent parameters should be calibrated simultaneously. A numerical approach based on hybrid genetic algorithms is used to implement the calibration procedure.

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Introduction

The interdisciplinary nature of water resources problems requires the integration of technical, economic, environmental, social, and legal aspects into a coherent analytical framework. Holistic water resources–economic models (HWEM) (Noel and Howitt 1982; Booker and Young 1994; Cai et al. 2003; Draper et al. 2003; Jenkins et al. 2004) conduct information transfer between hydrologic, agronomic, and economic components endogenously; they presents a simple approach for building truly integrated water resources and economic models. HWEMs are formulated as consistent optimization models, with the objective of maximizing the economic benefits of various water uses. The constraints and relationships involved in these models include: (1) water supply constraints such as hydrologic balance, capacity of water supply facilities, and water quality; (2) production functions such as crop yield functions and profit functions of urban water uses; and (3) policy regulations or economic incentives on water demands (Cai et al. 2001c).

An HWEM can be applied to policy analysis—that is, examining the economic and environmental consequences of a given economic incentive—or searching an optimal policy under certain environmental conditions. However, when the HWEM is solved for the optimal solution driven by the objective with certain given policy or operational options, there is often a significant difference between the model outputs and the observed data. The difference can be found not only with physical and engineering variables such as reservoir storages or flow releases, but also with the economic outputs such as water allocation or irrigated crop yield and area. For example, an HWEM that considers irrigation water supply and profit (Rosegrant et al. 2000; Cai et al. 2003) could result in very different crop acreage from the actual crop acreage under the baseline inputs such as rainfall, crop evapotranspiration, and irrigation system characteristics. The model determines the crop acreage according to the crop profitability, and assumes that other factors such as food demand, soil suitability, and a farmer's experiences on traditional crop patterns can be reflected by variable bounds. A common problem is the so-called "corner solution"—the area of high-valued crops approaches the upper bound while the area of low-valued crops approaches the lower bound. Ideally, if the model has perfect information about the determinants for crop acreage, the problem of "corner solution" may not happen. Unfortunately, it would be very difficult, if not impossible, for this type of model to have an exact simulation of the real-world situation.

In practice, there is usually insufficient data to specify a constraint set that will reproduce the output of a complex model, and the challenge is how to use that incomplete data/information to develop an optimization model that is useful for policy analysis. For an HWEM without a complete set of constraints, we would like to know how we can modify the model so that it will reveal the baseline observations. Theoretically there does not seem to be any reason that one could not completely constrain the model. However, the point is that a model that is fully constrained would not be useful for analysis.

Another challenge for calibrating an HWEM is the need to calibrate water resources, engineering, and economic parameters simultaneously in a consistent framework. Model calibration has been widely studied in water resources economics (Howitt 1995; Rohm and Dabbert 2003) and hydrology fields (Duan et al. 2003), respectively; however, only a few calibration studies have attempted to jointly calibrate hydrologic and economic components

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(e.g., Draper et al. 2003). The difficulty lies not only in the large number of parameters involved in different components, but also in the interdependence among the parameters to be calibrated. In this paper, an integrated framework is developed to calibrate water resources, engineering, and economic outputs simultaneously; the calibration procedure is implemented by using a hybrid genetic algorithm (GA) (Cai et al. 2001a).

In the remainder of this paper, we will introduce the model, describe the calibration of the model including what outputs to match with the observations and what parameters to adjust, and implement the calibration procedure. In addition, we will explain the outputs of the calibration procedure in terms of their physical and economical meaning.

Model

The prototype model was developed for the Maipo River basin, located in a key agricultural region in the metropolitan area of central Chile. The basin is characterized by a very dynamic agricultural sector—serving an irrigated area of about 127,000 ha—and a rapidly growing industrial and urban sector, particularly in the capital city of Santiago with a population of more than 5 million people. More than 90 percent of the irrigated area in the basin depends on water withdrawals from surface flows. Irrigation is of particular importance for perennial crops, like fruit trees and grapes. Agriculture accounts for the majority of total withdrawals.

The model is developed in the context of the river basin, represented as a node-link network, including multiple source nodes (reservoirs, aquifers, river reaches, etc.), multiple demand sites along the river, and consumptive use locations for agricultural, municipal and industrial, and instream water uses (Fig. 1). Agricultural demand sites are delineated according to the irrigation districts. At each agricultural demand site, water is allocated to a series of crops according to their water requirements and economic profitability. Fig. 2 illustrates the scaling-down water allocation processes (from the basin level to the farm level) and the scaling-up economic and environmental consequences (from the farm level to the basin level). Fig. 3 depicts, at various spatial scales, the hydrologic and agronomic processes which are simulated in the model.

Economic benefits from water uses are evaluated for different demand management instruments, including markets in tradable water rights, based on production and benefit functions with respect to water for the agricultural and urban-industrial sectors. The model is formulated as an optimization model in order to maximize water use profit with embedded basin-wide simulation of flow and salinity balance, reservoir operation, irrigation scheduling, and crop growth. The model is a short-term static model in terms of economic processes; however, the hydrologic component simulates storage operations and water balance over 12 time intervals (months) within a one-year period. A detailed formulation of an HWEM is presented in Cai et al. (2006); the application of the model for water trade analysis is provided by Rosegrant et al. (2000). Several of the key relationships involved in the model will be discussed in detail later in this paper.

Normative versus Positive Models

In order to further illustrate why the HWEM should be calibrated to a base level, we introduce the concept of "normative models"

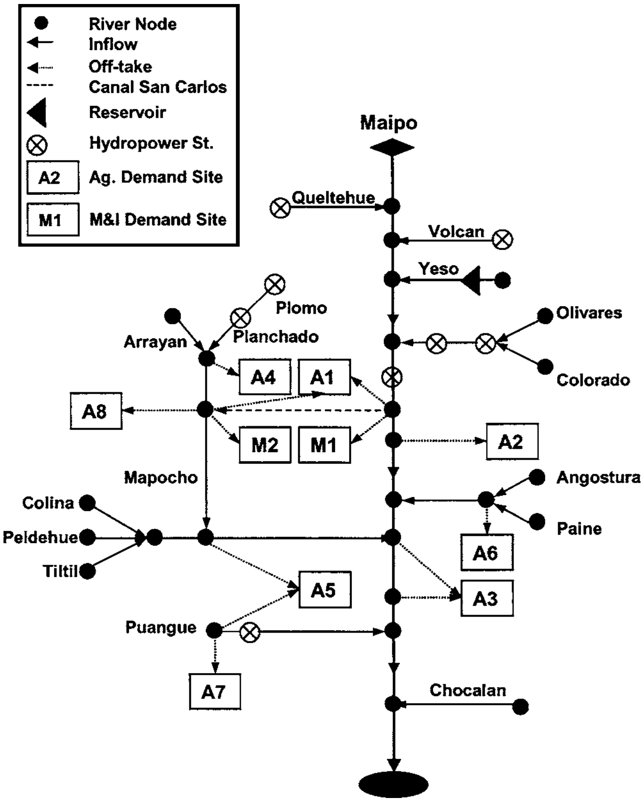

Fig. 1. Node–link network of the Maipo River basin in Chile

and "positive models," and discuss the conditions for the appropriate use of the two types of models in water resources.

The mathematical programming exercise that searches for the optimal solution driven by the objective with certain given policy or operational options, but ignoring the difference between the model outputs and the observed, is referred to as normative programming (Howitt 1995). Since mathematical programming models can rarely capture all real-world constraints explicitly, a normative programming model usually shows a wide divergence between model outcomes and actual results in the base period. Therefore, results from those models are considered indicative. Normative models are often used to analyze alternative scenarios; the difference between the scenario outputs shows the relative impact of policies under different scenarios. Bounds are often placed on variables to avoid outcomes that diverge too much from realistic values; however, tight bounds can be inappropriate for policy analysis.

In the study of agricultural production, Howitt (1995) suggested calibrating a mathematical programming model for agricultural production analysis against real-world conditions (a base year or average over several years). He called the modeling exercise "positive mathematical programming (PMP)." The basic hypothesis of PMP is that "observed behavioral reactions provide a basis for model calibration in a formal manner that is consistent with microeconomic theory" (Howitt 1995). Observations at the base-level are used to construct "positive inferences," which are combined with explicit programming constraints. The core of the approach is to adjust the marginal values of activities (i.e., crop planting) so as to make the marginal values of all activities equal at the base-level (Rohm and Dabbert 2003). The approach was originally developed as a remedy for some of the problems associated with linear programming (LP) by extending a linear objec-

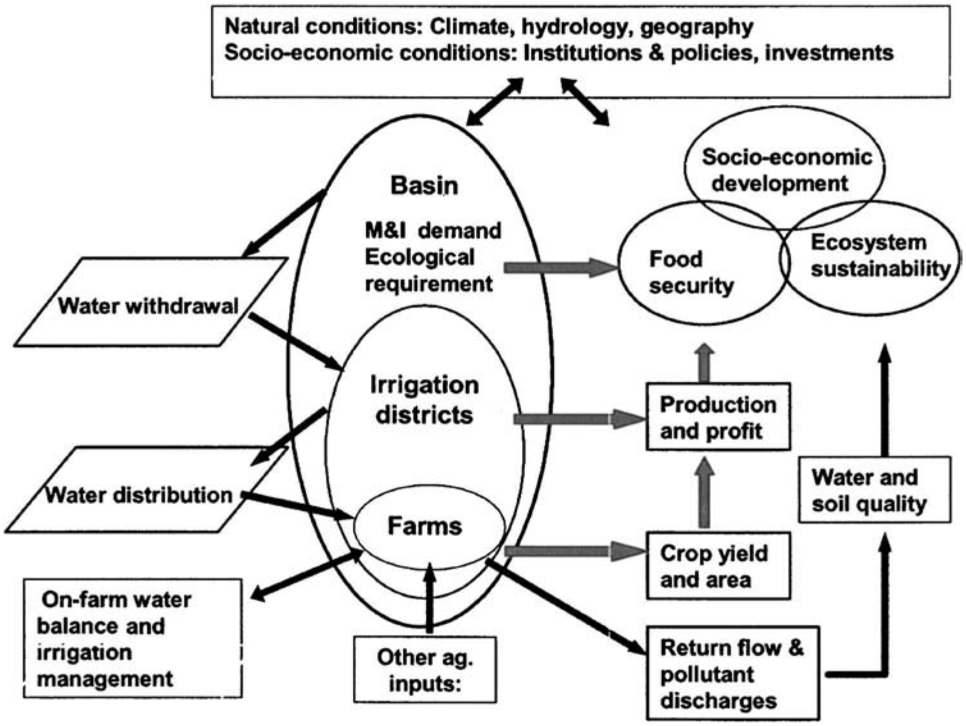

Fig. 2. Scaling-down water allocation processes (from the basin level to the farm level) and scaling-up economic and environmental consequences in the HWEM

tive function to a quadratic function. The coefficients of the nonlinear items in the objective function can be automatically calculated using the dual solution of an LP model.

Do all optimization models need to be calibrated to a baseline? In other words, should all optimization models be positive? This is not true for many optimization models used in science and engineering fields, since the outputs of those models are not necessarily observable. Normative programming has been successfully applied to many types of water resources planning and management models such as groundwater management models (Wagner 1995) and reservoir operation models (Yeh 1985; Wurbs 1993; and Labadie 2004). These models have some common properties, including: (1) the objective function completely represents a certain driving force for a specific purpose of problem solving; (2) the models are usually well-constrained: although there are not enough constraint equations to solve the problem uniquely, all solutions located in the solution space are not only feasible in terms of the constraints, but also reasonable regarding the studying problem; and (3) usually "observed values" do not exist before the model solution. A typical example of this type of model in water resources is the problem of determining minimum required reservoir capacity in order to satisfy a water supply objective under given hydrologic inflows. The problem is formulated as

$$\text{Minimize} \quad K$$

$$\text{s.t.}$$

$$S_{t+1} = S_t + I_t - E_t - RE_t - R \quad \forall\, t = 1,2\,..$$

$$E_t = E(S_t, S_{t+1}) \quad \forall\, t = 1,2\,..$$

$$S_t \leqslant K \quad \forall\, t = 1,2\,.. \tag{1}$$

where K=reservoir storage; S=storage; I=inflow; E=net evaporation from the reservoir surface, which is a function reservoir storage; RE=reservoir release; and R=firm yield (i.e., maximum water supply which can be guaranteed over all time periods), which is given as a parameter. Here the reservoir capacity is a decision variable constrained by water balance in all time periods and its "observed value" is not available.

Another type of reservoir optimization model is programed to search for optimal reservoir operation rules (dynamic reservoir storages and releases) based on reservoir inflows and given water requirements (either for offstream water supply or instream water

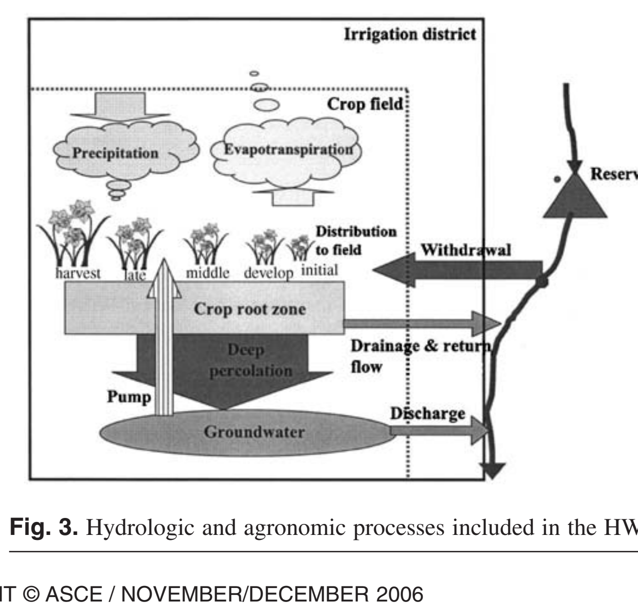

Fig. 3. Hydrologic and agronomic processes included in the HWEM

requirements, or both). The objective of this type of model can be defined to maximize water supply reliability (ReVelle 1999). One example of the formulation of such models is expressed as (Cai and McKinney 1997)

$$\text{Maximize } MRI \quad \text{or} \quad \text{Maximize } (TRI + \omega \cdot MRI)$$

$$.s\,.\,t\,.\, S_{t+1} = S_t + I_t - E_t - RE_t - WS_t \quad \forall\, t = 1,2\,.\,.$$

$$E_t = E(S_t, S_{t+1}) \quad \forall\, t = 1,2\,.\,.$$

$$S_t \leqslant K \quad \forall\, t = 1,2\,.\,.$$

$$RI_t = WS_t / WD_t \quad \forall\, t = 1,2,\,.\,.$$

$$MRI \leqslant RI_t \quad \forall\, t = 1,2,\,.\,.$$

$$TRI = \sum_t WS_t \Big/ \sum_t WD_t \tag{2}$$

where WS = water supply; WD = water demand; RI = ratio of water supply over demand; MRI is defined as the minimum RI over all time periods; and TRI = ratio of total supply over demand in all time periods. The objective is to maximize MRI or a combination of TRI and MRI, with a weight ω for MRI representing the priority on the vulnerability of water supply.

Checking this formulation against the three conditions for a well-defined optimization problem discussed above, there are four concerns. First, the model objective may not exactly represent the real management objectives of *reservoir operation*. The operation rules may be driven by other objectives rather than maximizing the supply-demand ratios. For example, maximizing hydropower generation can also be an objective. This can be usually dealt with by using multiobjective optimization programming. Second, the objective function includes an uncertain item—water demand—which is usually treated as a fixed parameter in the model, but actually changes dynamically with economic incentives and welfares. Even if water demand is treated as a variable, it is very inappropriate to fix the demand by a full set of constraints. Finally, observed values of reservoir storages and releases under a base level of inflow and water demand do exist.

We often find that the modeled "optimal" reservoir operations are very different from the actual. On one hand, this is expected since reservoir management in the real world does not follow the "wise" operations indicated by the model (this is exactly why we need the help of such a model); on the other hand, reservoir managers in the real world may find it difficult to use the "optimal solution" from the model. There can be many reasons for this. From the physical water supply side, a common problem is that the uncertainty of inflows is not fully captured in the model formulation. However, the discussion on the model formulation above may imply that several, more important causes exist.

From a practical perspective, reservoir managers may expect a model that: (1) reveals the actual operations under a baseline scenario that includes the real world physical processes and socioeconomic conditions that exist during the baseline period; and (2) provides an optimal solution if some physical processes and/or socioeconomic conditions are different from the baseline. The analysis from such as model will allow reservoir managers to know what operations or policies could make a difference from the current status and what changes will fit to their priority.

The HWEM that we study here is actually an extension of a traditional water supply system model. It extends the spatial domain to a river basin including multiple reservoirs, river reaches, and water use sites, and maximizes the economic profit of water uses in the whole basin. As argued above, it is difficult for an HWEM to reveal actual decisions and consequences due to the incomplete depiction of the real-world physical status and decision-making processes, which involve many implicit and qualitative factors. Calibration of the HWEM to a base level is then required in order to make it more applicable for policy analysis. In the following, we discuss how to make the HWEM a "positive" model following the concept of PMP.

Calibration of the HWEM

The HWEM is an extended economic model with not only an agricultural production component and nonagricultural profit functions, but also physically based water resources and agroenvironmental components as "constraints." The PMP approach conceptually applies to the HWEM; that is, although there is no complete data to specify a constraint set to reproduce the output at the base level, the economic component of the model can be calibrated to the base level using the observations at the base level as references, without adding new constraints to the model.

However, the quadratic programming and the parameter calculation procedure under the original PMP may not work effectively for the HWEM. Instead of the linear model that originally dealt with by the PMP approach, the HWEM is a complex nonlinear model with nonconvex nonlinearity in the constraint sets (Cai et al. 2001b). Moreover, the HWEM has hydrologic, engineering, agronomic, and economic relationships in a consistent model. The parameters in various components have to be calibrated in an integrated framework due to the interdependence among the parameters. In this study, we develop a new form of formulation to include "positive inferences" based on the baseline observations. We also propose a new approach for the implementation of the integrated calibration procedure. The original PMP, which is applied at the farm level for optimizing crop production, is then extended to optimize water allocation and water supply system operations at the basin (or region) level, as well as crop production at the irrigation district level.

To build a "positive" HWEM, the model calibration needs to match a large number of computed values with observed or recorded values under given climate regime and agricultural inputs. The items include the following: (1) flow through river reaches and reservoirs; (2) water withdrawals for both agricultural demand sites and municipal and industrial sites; (3) crop harvested area and yield; and (4) farmer incomes and profit from industrial and municipal water uses. Therefore, the calibration includes matching a large number of model outputs with the observations and parameters we need to adjust. Because these items are interdependent, they should be handled simultaneously as discussed in the following.

Crop Yield, Area, and Profit

The crop yield function has been developed based on econometric analysis of an agricultural production survey carried out in the Maipo River basin during the months of August to October of 1999 by the Catholic University of Chile. Crop yield has been estimated as a function of several agricultural inputs, including irrigation investment, application of fertilizer, pesticides, machinery, labor, and water. In order to establish a relationship between agricultural inputs and crop yield, a quadratic production function is chosen due to its properties of decreasing marginal returns to additional inputs and substitutability of inputs (Howitt and Msangi 2002). Due to the limited data availability on those inputs, the current model calibration exercise treats all inputs except for water as "invariable." It focuses on the effect of water on crop yield, which is usually subject to major uncertainties. Therefore, crop yield is represented as a quadratic function of water application

$$y_{d,c} = \alpha 0_{d,c} + \alpha 1_{d,c} \cdot w_{d,c} + \alpha 2_{d,c} \cdot w_{d,c}^2 \tag{3}$$

where d=demand site; c=crop; y=crop yield; w=seasonal water application; and α_i, $i=0,1,2$ are coefficients, with $\alpha 0 > 0$, and $\alpha 2 > 0$.

Net crop profit (np) is calculated as

$$np_{d,c} = a_{d,c} \cdot \left(PC_c \cdot y_{d,c} - \sum_{i \in I} IC_{d,c}^i \cdot INPT_{d,c}^i \right) \tag{4}$$

where i=index for inputs including water, irrigation investment, fertilizer, pesticides, labor, machinery, and seed; a=crop area; PC=crop price; IC=input cost; and $INPT$=input per unit of area. If all items on the right-hand side of Eq. (4) are equal to the "observed values" under the base level, net crop profit will be equal to the actual value. In this study, we ignore the uncertainty effects of crop prices and input costs and assume that net crop profit depends on crop acreage and yield. Therefore, once crop area and yield are calibrated, crop profit will be calibrated too.

As described above, crop yield [Eq. (3)] is a quadratic function of water, which is in the form that was used in the PMP approach by Howitt (1995); the coefficients in Eq. (3) are determined by regression based on a field survey (Cai et al. 2005). Our computing experiments showed that the HWEM including the yield function did not automatically result in crop yields and areas that matched the observed values. This, of course, may be due to the error associated with the yield function. However, we would argue that the difference between the modeled and the observed yield might be mainly caused by water allocation among crops and among demand sites, which are characterized by agroenvironmental simulations and institutional constraints involved in the HWEM. In the river basin context, water supply to spatially distributed demand sites is characterized by natural flows, water supply facilities, and institutional rules. If these factors were spatially homogenous, water availability would be homogenous. Then according to the microeconomic theory, all crops at all demand sites would reach an equal marginal value at the base level (Rohm and Dabbert 2003). Obviously spatial homogeneity normally does not exist in the real world; the HWEM attempts to simulate the heterogeneity of water availability by including essential hydrologic, infrastructural, and institutional relationships. Due to the spatial heterogeneity of environmental and institutional factors, the optimal solution of the HWEM may not result in an equal marginal value of water use. For example, at one demand site, a crop has a relatively higher yield than the same crop at other demand sites, but the constraint of water rights for that demand site may prevent the realization of the potential crop yield, which results in a higher marginal value of the crop at that demand site than at others. Cai et al. (2005) provided the marginal gross value of crops and marginal values of water use for crops in the Maipo River basin from a normative HWEM.

Therefore, additional references are needed to make the HWEM positive in terms of crop yield. Following the original PMP approach, we may adjust the coefficient of the nonlinear item α_2 in Eq. (3) for all crops in all demand sites simultaneously. However, we chose an alternative form that was expected to more directly relate to water allocation among crops. By Eq. (3), with other fixed inputs, calibrating crop yield will be realized by adjusting water application. In other words, given $w_{d,c}=w_{d,c}^0$, then $y_{d,c}=y_{d,c}^0$. By substituting $y_{d,c}=y_{d,c}^0$ into Eq. (3), $w_{d,c}^0$ can be solved. By including $w_{d,c}^0$ in the crop profit function, the equation of net crop profit is modified as

$$np_{d,c} = a_{d,c} \cdot \left(PC_c \cdot y_{d,c} - \sum_{i \in I} IC_{d,c}^i \cdot INPT_{d,c}^i - \lambda 1_{d,c} \cdot |w_{d,c} - w_{d,c}^0| \right) \tag{5}$$

where $\lambda 1$ represents a penalty cost for water that differs from the base-level values. If $w_{d,c} > w_{d,c}^0$, $\lambda 1$ is the cost for increasing water use for a crop by one unit; otherwise it is the cost for reducing water use by one unit.

Following the same idea for calibrating crop yield, a nonlinear item of crop area is added to the crop profit function, and the final profit function is formulated as

$$np_{d,c} = a_{d,c} \cdot \left(PC_c \cdot y_{d,c} - \sum_{i \in I} IC_{d,c}^i \cdot INPT_{d,c}^i - \lambda 1_{d,c} \cdot |w_{d,c} - w_{d,c}^0| \right) - \lambda 2_{d,c} \cdot |a_{d,c} - a_{d,c}^0| \tag{6}$$

where if $a_{d,c} > a_{d,c}^0$, $\lambda 2$ represents a penalty cost for increasing the crop area by one unit for a specific crop; otherwise it represents a penalty cost for reducing the area by one unit. Howitt (1995) showed that the ability to adjust some nonlinear parameters in the objective function could improve model calibration. He proved a corollary that stated, "*the number of calibration terms in the objective function must be equal to or greater than the number of independent variables to be calibrated.*" In Eq. (6), the number of calibration terms $\lambda 1$ and $\lambda 2$ are equal to the number of crop yield and area variables ($y_{d,c}$ and $a_{d,c}$).

Water Withdrawals to Irrigation Demand Sites

If actual water withdrawals for irrigation demand sites are available, the computed water withdrawals can be calibrated to actual values. Water withdrawal for an irrigation demand site (ww_d^t for irrigation purpose) is equal to the sum of the water applications over all crops plus conveyance and distribution losses. If we define a conveyance and distribution loss factor as l, then we have

$$ww_d^t \cdot (1 - l_d^t) = \sum_c wcp_{d,c}^t \quad \forall\ d,t \tag{7a}$$

$$ww_d^t = \sum_c wcp_{d,c}^t/(1 - l_d^t) \quad \forall\ d,t \tag{7b}$$

where wcp=crop water application by crop growth stage. The sum of wcp over crop growth stages t equals the seasonal water application [w in Eq. (3)]. The loss coefficient varies over irrigation demand sites d since different demand sites have different irrigation systems and then different water use efficiencies. The loss coefficient also varies with periods t, because in general water loss is higher when the amount of water diversion is higher and water diversions vary over time periods.

Given seasonal water availability, crop water application by stage is determined by crop water requirements in crop growth stages (Cai et al. 2003). By Eq. (7), once crop water application and water loss factor l are determined, ww_d^t can be determined.

Water withdrawals for industrial and domestic demand sites can be calibrated to the observed values by adjusting economic parameters such as price elasticity of demand (Cai et al. 2005). For simplicity, these items were calibrated separately before the integrated program presented in this paper.

Flow through River Reaches and Reservoir Storage–Release Relationship

For a given river reach, the water flow balance at time period t can be represented as

$$of_n^t = if_n^t - \sum_{d\in(n,d)} ww_d^t + \sum_{d\in(d,n)} rf_d^t - ifl_n^t \tag{8}$$

where of=outflow from a river reach; if=inflow to a river reach; ww=water withdrawal, as discussed above; rf=return flow from demand sites; and ifl=instream flow loss which is normally estimated as a fraction of inflow. With n as the index of river reaches, (n,d) and (d,n) represent a set of flow connections between a river reach and one or more demand sites; that is, (n,d) specifies the demand sites that withdraw water from the river reach, and (d,n) specifies the demand sites from which part of the water withdrawal returns back to the river reach. Flow through a river reach can be observed as either the inflow or the outflow; the outflow from one river reach will be the inflow to the next downstream river reach. Given inflow if, water withdrawals ww determined by other relationships [Eq. (7)], and estimated instream flow loss, outflow of can be determined if return flow is specified. In this study, return flow rf from an irrigation demand site to a river reach n is calculated as

$$rf_{(d,n)}^t = \sum_{(d,n)} \left[\sum_c dp_{d,c}^t \cdot \varepsilon_d \cdot (1-\eta_d) + ds_{(d,n)}^t \right] \quad \forall\, d,t \tag{9}$$

where rf=sum of surface drainage, part of the field percolation (dp) from all fields, plus subsurface drainage from a groundwater source associated with an irrigation demand site d; ε=drainage efficiency, defined as the ratio of drainage to field percolation; η specifies the evaporation and seepage loss during the path of surface drainage returning to the river system; and ds=discharge from groundwater to the surface system. These parameters vary with demand site due to different conditions of the drainage systems in different demand sites. A linear relationship is assumed between the discharge ds and the water table head h of an aquifer (Smedema and Rycroft 1990)

$$ds_{(d,n)}^t = (\xi_d \cdot GA_d) \cdot h_d^t \tag{10}$$

where GA=average area of the groundwater tank (approximately the sum of the irrigated area in a demand site). The aquifer is assumed to be a single "tank" associated with a specific irrigation demand site. Tank inflow includes natural recharge, surface water leakage, and deep percolation from irrigation fields; tank outflow includes pumping, groundwater extraction to root zones, and discharge to surface water systems ds. The calibration of return flows is conducted through the determination of appropriate values for the drainage efficiency ε, the evaporation and seepage loss of surface drainage η, and the discharge coefficient ξ. It should be noted that deep percolation [dp in Eq. (9)] and groundwater head [h in Eq. (10)] depend on water application in the crop field and also relate to decisions on water allocation and crop acreage.

Contrary to the water flow balance of a river reach, which ignores the impact of storage, the impact of storage should be accounted for reservoirs. Water balance in a reservoir is written as

$$s_n^t = s_n^{t-1} + if_n^t - \sum_{d\in(n,d)} ww_d^t - e_n^t - r_n^t \tag{11}$$

where s=storage; e=net evaporation from the reservoir surface; and r=release. Reservoir inflow if can be a given parameter or it can be designated as the outflow from the upstream river reach [of in Eq. (12)]. The calibration of water withdrawal from a reservoir has been discussed before. Evaporation can be estimated as a portion of the reservoir storage (ReVelle 1999). With these specifications, only one requirement is left for the calibration of reservoir operations—the relationship between storage and reservoir release to a downstream river reach. If this relationship is characterized by physical facilities (e.g., the design of spillways), then the relationship should be included in the model formulation as an explicit constraint. However, if the relationship is controlled by a designed "operation curve," then, for the base-level case, the storage–release is constrained by the designed reservoir operation curve. For an alternative scenario analysis on reservoir operations, we can search the storage–release relationship for an optimal solution.

To summarize, the calibration of an HWEM involves the adjustment of a large number of parameters with uncertain values in order to make model outputs match the observed values. These items are dependent upon each other in a holistic model, and they must be calibrated simultaneously. Fig. 4 shows a diagram of data that needs to be calibrated and the parameters that are adjusted during the calibration process. This figure also shows the relationships between different items involved in the calibration procedure.

Now the challenge is how to implement the integrated model calibration framework described above. The original PMP automatically calibrates the model in terms of output, input use, objective function values, and dual values. Theoretically the automatic approach will work for the calibration of the HWEM if the basic concavity and convexity conditions apply. However, to search a large number of interdependent economic and physical items simultaneously, a numerical approach might be more realistic. A numerical approach does not need to predetermine an explicit form of the items to calibrate, but searches appropriate values of the items by running the HWEM model that embeds the interdependence among the items in its formulation. In the following, we apply a numerical approach based on a genetic algorithm to the calibration of the HWEM.

Implementing the Calibration Procedure—Genetic Algorithm

The calibration procedure requires that we search for the appropriate values of the parameters shown in Fig. 5, with which the hydrologic and economic outputs from the model at the baseline level match the observations. A trial-and-error search method was found to be very time consuming. The convergence of a trial-and-error method is also in doubt. An advanced search approach based on genetic algorithms was then developed and applied to the implementation of the HWEM calibration.

GAs are a subclass of general artificial-evolution search methods based on natural selection and the mechanisms of population genetics. They belong to a family of optimization techniques in which the solution space is searched by generating candidate solutions with the help of a pseudorandom number generator. These algorithms rely on collective learning processes within a population of individual candidate solutions, each of which represents a

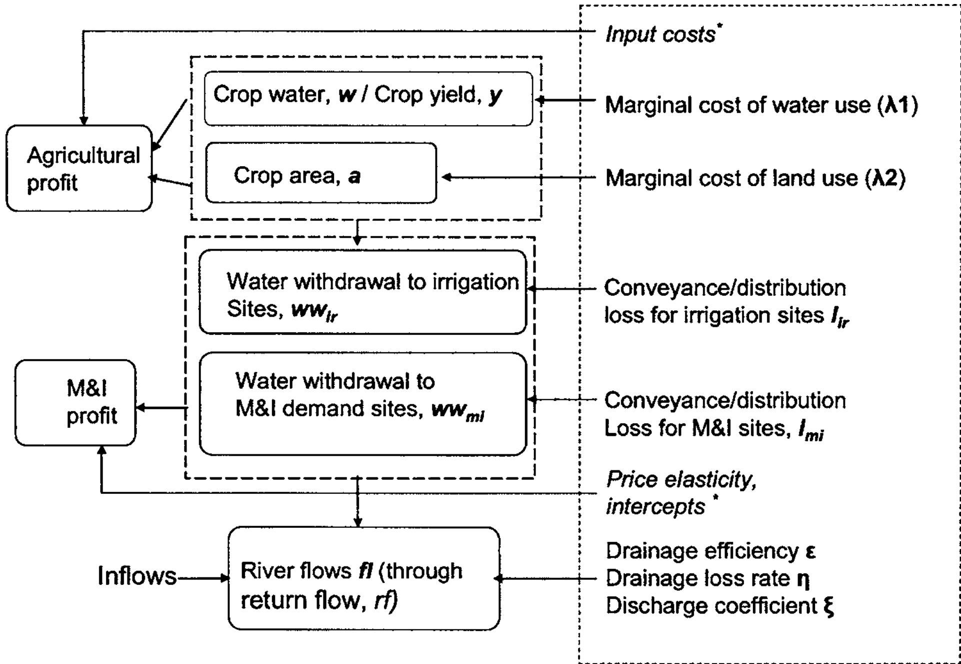

Fig. 4. Integrated framework for the calibration of the HWEM (for simplicity, * items were estimated before the integrated program presented in this study)

point in the space of potential solutions. The basics of GAs were introduced by Goldberg (1989). Since then there have been wide applications of GAs in water resources. Particularly, GAs have been used for calibrating models in water resources, including rainfall-runoff models (Wang 1991; Cheng et al. 2002), water distribution network models (Meier and Barkdoll 2000; Kapelan et al. 2003) and water quality models (Mulligan and Brown 1998).

In this paper, we use a hybrid GA presented by Cai et al. (2001c) to solve the calibration problem of an HWEM. The hybrid GA is an integration of a mathematical programming model and a simple GA (Goldberg 1989). As shown in Fig. 5, in each of the iterations (generations), the GA searches the values of the parameters to calibrate by generating a set of alternatives (individuals) of the parameter values. The GA alternatives are sent to the HWEM, which solves for the model outputs that need to match the observed values, including crop yields and areas, water withdrawals, and flows through river reaches and reservoirs. A fitness program calculates the fitness for each of the alternative sets of parameters generated by the GA based on the difference between the modeled and observed values (items marked by zero), as

$$fit=\left(\sum_{d,c}\left(\frac{a_{d,c}-a0_{d,c}}{a0_{d,c}}\right)^2+\sum_{d,c}\left(\frac{y_{d,c}-y0_{d,c}}{y0_{d,c}}\right)^2+\sum_{d,t}\left(\frac{ww_d^t-ww0_d^t}{ww0_d^t}\right)^2+\sum_{n,t}\left(\frac{of_n^t-of0_n^t}{of0_n^t}\right)^2\right)^{-1} \tag{12}$$

Based on the probabilities calculated from the individual's fitness values, individuals are selected for "mating" to produce offspring for the next generation. This means that strings with a higher value of fitness have a higher probability of participating in reproduction and contributing one or more offspring to the next generation. Therefore, the search is improved generation by generation.

Some details on the GA implementation are given below. Totally there are 364 variables for the GA. Six bits are used to encode one variable, and the total length of one chromosome (gene organization for one individual) is 2,184 bits. Each generation has 100 individuals. By test, the best mutation probability and crossover probability are 0.08 and 0.9, respectively. For each generation, the best 10 solutions are kept for next generation, and this elitism strategy is found to be effective in improving the GA solution.

The ranges of the 364 variables affect the GA performance in terms of the time to convergence. It is a challenge to determine appropriate ranges for so many variables given the interdependence that exists among them. The ranges of the physical items (such as the water distribution/conveyance loss rate l, drainage efficiency ε, and the discharge coefficient ξ) were found in literature. The initial value of the marginal cost of water used the water

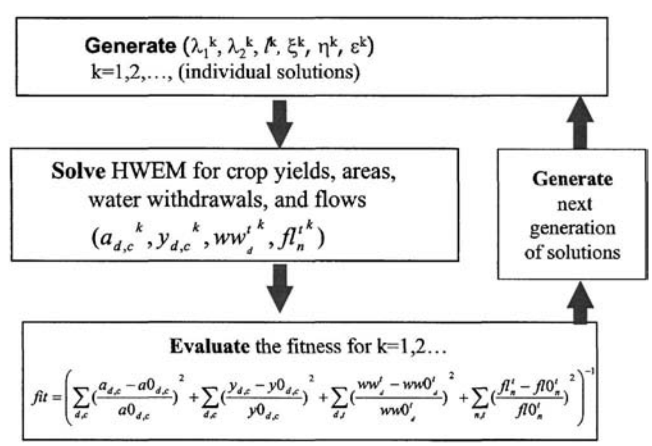

Fig. 5. Hybrid GA—mathematical programming method for the calibration of the HWEM

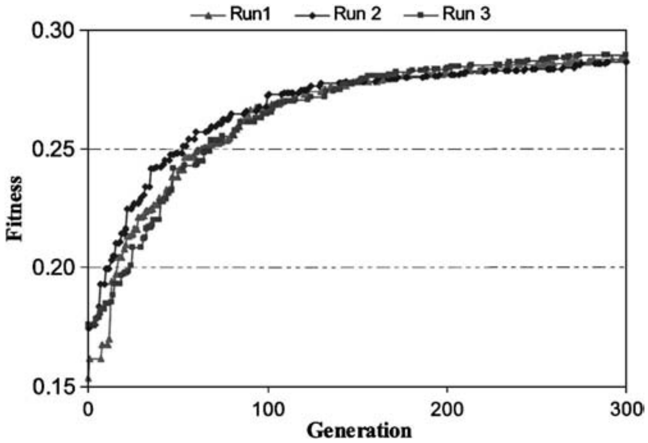

Fig. 6. Three GA runs with different ranges for the GA variables, showing the highest fitness from each generation (ranges were established as described in the text)

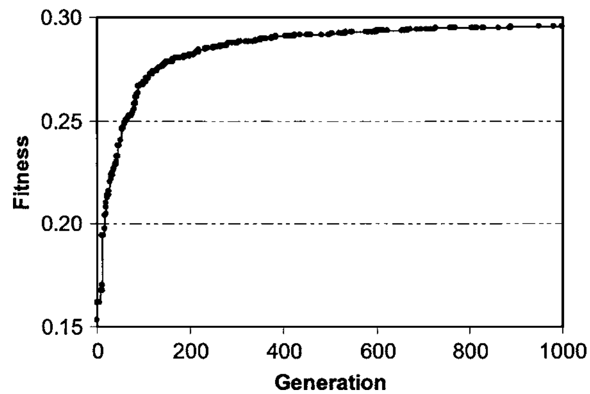

Fig. 7. GA solutions improved with growing generations up to the generation of 1,000

price. The marginal cost of land used the marginal profit of land use were derived from the dual solution of the model. For each variable, the range of the variable was set as the ($iv^1 \cdot lb$, $iv^1 \cdot ub$), i.e., the initial value iv^1 multiplied by a selected lower lb and upper bound ub (e.g., 0.5 and 2.0). To test the robustness of the initial values, the GA ran with the variable ranges ($iv^1 \cdot lb$, $iv^1 \cdot ub$) up to a certain generation, and the best solution from that generation was used to update the initial values (from iv^1 to iv^2); a new set of ranges was established as ($iv^2 \cdot lb$, $iv^2 \cdot ub$). This step was repeated for a series of updates. Fig. 6 plots the results from three runs. Although the results from the three runs are divergent in early generations, they converge to a close fitness value after approximately 300 generations. The GA program is then robust with the chosen initial values. Fig. 7 shows the GA solution up to 1,000 generations. Between 300 and 1,000 generations, the solution is only slightly improved.

Discussion

The fitness value of the best solution found by the hybrid GA is about 0.29, which means the sum of the square errors is $1/0.29 = 3.4$, and the average square error over all items is 0.005 [3.4 divided by 720, the total number of the items to calibrate as shown in Eq. (12)]. Therefore, given this set of parameters for the HWEM, the model outputs match the observed with reasonable errors. Table 1 shows the penalty costs of water and land use, $\lambda 1$ and $\lambda 2$ for selected crops in eight irrigation demand sites, respectively. Generally, lower-valued crops such as annual forage and corn have lower penalty costs. Lower-penalty costs with lower-valued crops will prevent the water and land application at the base level being "taken to" higher-valued crops in order to maintain the area and yield of lower-valued crops at the base level. While higher penalty costs with higher-valued crops prevent those crops using more water and land than the base level.

Penalty costs of land use ($\lambda 2$) vary more significantly among demand sites and crops. The value of $\lambda 2$ is largely dependent on the size of the crop area under the base level; higher values of $\lambda 2$ are associated with larger crop areas. For example, at demand site A7, the penalty costs of land use for grape and peach are even lower than those for lower-valued crops such as wheat, corn, and forage, while at other demand sites, the penalty costs of land use for grape and peach are much higher than other crops. This is because the areas of grape and peach at demand site A7 are very small relative to the other crops, and it seems that there is a larger potential for increasing the area of grape and peach at A7.

The land costs also seem to be affected by the degree of water stress. For crop profitability under larger water stress, there will be a larger driving force to reduce the land of lower-valued crops and to increase the land of high-valued crops. The high land costs then tend to prevent the increase of the higher-valued crop land; while the low land costs allow the maintenance of the crop land for lower-valued crops at the base level. This helps explain the particularly high penalty costs of land use for grape and peach at demand site A6, which suffers a larger water stress than other demand sites.

Table 2 shows the water distribution/conveyance loss coeffi-

Table 1. Calibrated Penalty Costs of Land and Water Use by Crop in Eight Irrigation Demand Sites

Demand sites/crops	Penalty cost of land use, $\lambda 2$ (US\$/ha)					Penalty cost of water use, $\lambda 1$ (US\$/m^3)				
	Wheat	Corn	Annfor	Grape	Peach	Wheat	Corn	Annfor	Grape	Peach
A1	99	153	97	2,727	5,335	1.2E−03	5.7E−04	0.0E+00	1.7E−03	2.6E−03
A2	823	826	125	2,274	3,765	2.1E−04	1.8E−04	2.2E−07	1.6E−03	3.1E−03
A3	74	614	197	1,627	4,220	1.5E−04	1.3E−04	0.0E+00	3.0E−03	3.4E−03
A4	14	178	13	1,303	4,854	9.6E−05	3.4E−04	9.6E−10	1.8E−03	2.8E−03
A5	203	503	210	2,644	1,467	4.0E−05	3.0E−04	1.6E−05	2.4E−03	8.3E−03
A6	61	147	42	6,704	5,075	3.5E−04	1.0E−03	7.9E−06	1.3E−03	2.8E−03
A7	673	124	187	26	33	1.8E−04	4.1E−04	2.3E−05	2.1E−03	1.8E−03
A8	457	54	8	3,413	2,947	2.8E−04	2.4E−04	0.0E+00	4.9E−03	7.5E−04

Table 2. Calibrated Water Distribution/Conveyance Loss Rate by Month in Eight Irrigation Demand Sites

	January	February	March	April	May-July	August	September	October	November	December
A1	0.52	0.45	0.35	0.31	n/a	0.02	0.18	0.22	0.24	0.33
A2	0.36	0.39	0.33	0.28	n/a	0.07	0.24	0.26	0.26	0.37
A3	0.35	0.26	0.33	0.20	n/a	0.06	0.19	0.22	0.35	0.37
A4	0.32	0.34	0.30	0.31	n/a	0.05	0.19	0.25	0.22	0.29
A5	0.48	0.38	0.34	0.25	n/a	0.06	0.11	0.25	0.28	0.31
A6	0.31	0.33	0.30	0.23	n/a	0.03	0.11	0.22	0.25	0.21
A7	0.44	0.40	0.28	0.18	n/a	0.08	0.19	0.18	0.25	0.35
A8	0.40	0.41	0.40	0.33	n/a	0.00	0.00	0.31	0.27	0.37

Note: n/a=not applicable.

Table 3. Calibrated Values of Drainage Efficiency and Discharge Coefficient by Demand Site

Demand sites	A1	A2	A3	A4	A5	A6	A7	A8
Drainage efficiency, ε	0.552	0.571	0.426	0.598	0.455	0.479	0.498	0.488
Discharge coefficient, ξ	0.003	0.003	0.004	0.004	0.003	0.005	0.004	0.003

Table 4. Calibrated Values of Drainage Loss Coefficient by Month

Month	January	February	March	April	May	June	July	August	September	October	November	December
Drainage loss, η	0.440	0.420	0.303	0.354	0.338	0.313	0.367	0.414	0.430	0.459	0.411	0.471

cients by demand site (A1-A8) by month. During May–July (winter in the study area), there is no irrigation. During the summer months (December, January, and February), the coefficients are higher than other months. This is as expected because of larger flow diversions during these months, which usually correspond to larger loss rates in irrigation systems. Also, upstream demand sites (A1, A2, A4, and A8) have relatively higher loss rates than downstream. This is due to the fact that the upper lands in the study basin have larger water availability than the lower lands, and the upstream demand sites have larger water withdrawals relative to their actual requirements.

Table 3 shows the calibrated values of drainage efficiency ε and the drainage loss rate η (evaporation and seepage) by demand site and Table 4 shows the discharge coefficient ξ by month. In general, upstream demand sites (A1, A2, A4, and A8) have higher values of drainage efficiency and lower values of discharge coefficient. This implies that it is more likely that return flow from an upstream irrigation district flows back to the river through surface drainage, and less likely through subsurface flow than a downstream demand irrigation district. In addition, drainage loss rates are higher in summer months due to high evaporation in those moths.

The calibrated HWEM (positive model) and the uncalibrated HWEM (normative model) are compared regarding the impact of changing water prices on water withdrawals. Fig. 8 plots water withdrawals versus water prices from the two models, respectively. As can be seen, under the same water price, the normative model results in a higher water withdrawal than the positive model, although the difference declines when water prices increase. Moreover, it seems that the normative model is more sensitive to the change of water prices; while the positive model is more robust with changing water prices, and it becomes static when the water price is approximately larger than 0.025 $\$/m^3$. A plausible reason is that the uncalibrated model is more "flexible" and has more space for searching a new solution corresponding to the change of water prices.

Regarding the GA-based implementation approach, a couple of points are worthy of further discussion. Although the GA searches a unique set of parameters for the positive model, it cannot be guaranteed that such a unique set of over 300 parameters exists. There might be multiple sets of parameters which result in essentially the same fitness value. This problem, namely "equifinality," was addressed by Beven (2001) and has been studied for hydrologic model calibration and validation. Equifinality may be caused by substitutional relations among parameters. In this specific study, the sets of parameters that result in similar fitness value converge to similar parameter values, which does not show a problem of equifinality. However, equifinality between economic and hydrologic parameters may exist with other cases; and insights of such relations might be valuable to further understand the interactions between human and natural systems.

In addition, to build a positive model, the selection of both hydrologic and economic parameters is important, as demonstrated earlier. First of all, these parameters are known only within a certain range; a sufficient number of parameters need to be adjusted within the plausible range to make the hydrologic and economic outputs match the observed. On the other hand, the number of the parameters should be controlled in order to avoid the computational difficulty.

Conclusions

Positive mathematical programming is more appropriate for an HWEM, which is formulated as a consistent optimization model with embedded water balance and crop production simulation. This paper presents a procedure to convert an existing normative

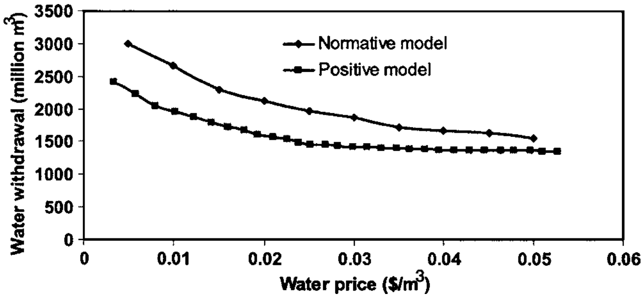

Fig. 8. Compare the "positive" and "normative" models in terms of the impact of water prices on irrigation water withdrawals

model to a positive one through calibrating the major model outputs of the HWEM to the base level. We identify some economic and hydrologic parameters that need to be adjusted to match crop areas and yields, water withdrawals, and flows through river reaches and reservoirs with the observed values. Due to the interdependence among a large number of the parameters to calibrate, an integrated framework based on a hybrid GA is applied to implement the calibration. The algorithm is successful in finding the values for the set of parameters, which make the model outputs match the observed with acceptable errors. The values are explainable and understandable from the point of view of the physical and economical implications. The difference between the calibrated and the uncalibrated models (the positive model versus the normative model) in responding to the change of water prices has demonstrated the significance of the model calibration work.

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References

Beven, K. J. (2001). "Calibration, validation and equifinality in hydrological modeling." *Model validation: Perspectives in hydrological science*, M. G. Anderson and P. D. Bates, eds., Wiley, New York, 43–55.

Booker, J. E., and Young, R. A. (1994). "Modeling intrastate and interstate markets for Colorado River water resources." *J. Envir. Econom. Manage.*, 26(1), 66–87.

Cai, X., and McKinney, D. C. (1997). "A multiobjective analysis model for negotiations in regional water resources allocation." *Proc., Aesthetics in the Construct Environment ASCE*, New York.

Cai, X., McKinney, D. C., and Lasdon, L. (2001a). "Solving nonlinear water management models using a combined genetic algorithm and linear programming approach." *Adv. Water Resour.*, 24(6), 667–676.

Cai, X., McKinney, D. C., Lasdon, L. S., and Watkins, D. (2001b). "Solving large nonconvex water resources management models using generalized benders decomposition." *Oper. Res.*, 49(2), 235–245.

Cai, X., Rosegrant, M. W., McKinney, D. C., and Ringler, C. (2001c). "Holistic water resources-economic modeling." *Proc., Bridging the Gap: Meeting the World's Water and Environmental Resources Challenges*, ASCE, Reston, Va.

Cai, X., McKinney, D. C., and Lasdon, L. S. (2003). "Integrated hydrologic-agronomic-economic model for river basin management." *J. Water Resour. Plann. Manage.*, 129(1), 4–17.

Cai, X., Ringler, C., and Rosegrant, M. W. (2006). "Modeling water resources management at the basin level: Methodology and application to the Maipo River basin." *Rep.*, The International Food Policy Research Institute, Washington D.C.

Cheng, C. T., Ou, C. P., and Chau, K. W. (2002). "Combining a fuzzy optimal model with a genetic algorithm to solve multi-objective rainfall-runoff model calibration." *J. Hydrol.*, 268(5), 202–211.

Draper, A. J., Jenkins, M. W., Kirby, K. W., Lund, J. R., and Howitt, R. E. (2003). "Economic-engineering optimization for California water management." *J. Water Resour. Plann. Manage.*, 129(3), 155–164.

Duan, Q., Gupta, H., Sorooshian, S., Rousseau, A., and Turcotte, R. (2003). "Calibration of watershed models." *Proc., Water Science and Application 6*, American Geophysical Union, Washington, D.C.

Goldberg, D. E. (1989). *Genetic algorithms*, Addison-Wesley, Reading, Mass.

Howitt, R. E. (1995). "Positive mathematical programming." *Am. J. Agric. Econom.*, 77(5), 329–342.

Howitt, R. E., and Msangi, S. (2002). "Reconstruction disaggregate production functions." *Selected paper presented at the 2002 Am. Ag. Econ Asso., (AAEA) annual meeting*, Long Beach, CA.

Jenkins, M. W. et al. (2004). "Optimization of California's water supply system: Results and insights." *J. Water Resour. Plann. Manage.*, 130(4), 271–280.

Kapelan, Z. S., Savic, D. A., and Walters, G. A. (2003). "Multiobjective sampling design for water distribution model calibration." *J. Water Resour. Plann. Manage.*, 129(6), 466–479.

Labadie J. (2004). "Optimal operation of multireservoir systems: State-of-the-art review." *J. Water Resour. Plann. Manage.*, 130(2), 93–113.

Meier, R. W., and Barkdoll, B. D. (2000). "Sampling design for network model calibration using genetic algorithms." *J. Water Resour. Plann. Manage.*, 126(4), 245–250.

Mulligan, A. E., and Brown, L. C. (1998). "Genetic algorithms for calibrating water quality models." *J. Environ. Eng.*, 124(3), 202–211.

Noel, J. E., and Howitt, R. E. (1982). "Conjunctive multibasin management: An optimal control approach." *Water Resour. Res.*, 18(4), 753–763.

ReVelle C. (1999). *Optimizing reservoir resources—Including a new model for reservoir reliability*, Wiley, New York.

Rohm, O., and Dabbert, S. (2003). "Integrating agri-environmental programs into regional production models: An extension of positive mathematical programming." *Am. J. Agric. Econom.*, 85, 254–265.

Rosegrant, M., Ringler, C., McKinney, D. C., Cai, X., Keller, A., and Donoso, G. (2000). "Integrated economic-hydrologic water modeling at the basin scale: The Maipo River basin." *J. Agricultural Economics*, 24(1), 33–46.

Smedema, L. K., and Rycroft, O. W. (1990). *Land drainage*, Batsford, London, U.K.

Wagner, B. J. (1995). "Recent advances in simulation-optimization groundwater management modeling." *Rev. Geophys.*, 33, 1021–1028.

Wang, Q. J. (1991). "The genetic algorithm and its application to calibrating conceptual rainfall-runoff models." *Water Resour. Res.*, 27(9), 2467–2471.

Wurbs R. (1993). "Reservoir-system simulation and optimization models." *J. Water Resour. Plann. Manage.*, 119(4), 455–472.

Yeh W. (1985). "Reservoir management and operations models: A state-of-the-art review." *Water Resour. Res.*, 21(12), 1797–1818.