Dark matter from primordial metric fields and the term (grad g_{00})^2

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Abstract

It is a well-known truism, inspired on the general theory of relativity, that gravity gravitates. Here we suggest the possibility that dark matter may be caused by the gravitation of the metric. At first sight this seems impossible since the gravitational fields in galaxies and cumuli are so weak that it would seem that second order terms are negligible. Nevertheless, the general theory of relativity tells us that the gravitation due the metric is given by (grad g_{00})^2. Thus, a metric field g_{00} varying fast in the space directions could make a sizeable contribution to the gravitational field despite being a weak field. As a plausible source of such a field consider that during reheating the inflaton field disintegrates into radiation. Those quantum decays that involve higher energies and momenta will produce pockets of metric fields with rapid change in time and space. The expansion of the universe and dissipative processes (including the emission of gravitational waves) eventually result in basically stationary pockets of classical g_{00} field varying rapidly in space that have collapsed along with matter into structures like galaxies and cumuli. These pockets should gravitate precisely by the term mentioned above. They are classical fields and are reminiscent of the cosmic fields that are scattered in our universe.

The dark matter hypothesis. The dark matter hypothesis has been very useful to explain the discrepancy between mass as estimated from its luminosity and as inferred from its gravitational effect. Of course, it is also possible that this discrepancy can be due to the use of incorrect dynamical theories, which in this case would be the general theory of relativity. Many alternative models of dynamics have been presented in the last few years to explain the discrepancy, some covariant, some not.[1, 2] However, several observational results of the last few years, based on the techniques of gravitational lensing, have greatly substantiated the dark matter hypothesis. Dark matter has been, basically, directly observed,[3] and even dark matter cosmological structures.[4] There are several hypothesis as to what is dark matter is, but its actual composition is still a mystery. In this paper we argue for yet another new possible candidate for dark matter.

Gravity gravitates. People often say that the nonlinearity of the equations of general relativity imply that gravity gravitates. Here we raise the question: could it be, perhaps, that the gravitation of gravity in galaxies and clusters constitutes dark matter? We begin with a brief study of the pertinent equations of the general theory of relativity.

Within general relativity the geodesic equation

$$\frac{d^2x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0.$$

serves as an equation of motion for the force of gravity. We first calculate the equation of motion in the Newtonian limit. We consider the particularly simple case of bodies with small velocities interacting solely through a rather weak gravitational field. We can then take the metric to be diagonal and neglect its time dependence. The geodesic equation becomes

$$\frac{d^2x^i}{dt^2} = -\Gamma^i_{00} = \frac{1}{2}g^{ii}g_{00,i} \approx \frac{1}{2}g_{00,i},\ i = 1, 2, 3. \tag{1}$$

Taking the gauge

$$g_{00} = -1 - 2\psi, \tag{2}$$

where ψ is interpreted as the gravitational potential, the equation (1) can be written in the form $\ddot{\mathbf{x}} = -\nabla\psi$, Newton's equation for a particle in a gravitational field. (The signature of the metric is $(- + ++)$.)

In taking the Newtonian limit of general relativity, one approximates matter in the galaxy by a fluid with a density $\rho_M(t, \mathbf{x})$ and negligible pressure. Then the stress tensor has only one nonzero component, $T_{00} = \rho_M$. Einstein's field equation can be written in the alternative form $R_{\mu\nu} = 8\pi G(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$, from which we can immediately infer that $R_{00} = R^\rho{}_{0\rho 0} = 4\pi G\rho_M$, or

$$\Gamma^\rho{}_{00,\rho} - \Gamma^\rho{}_{0\rho,0} + \Gamma^\rho{}_{\rho\eta}\Gamma^\eta{}_{00} - \Gamma^\rho{}_{0\eta}\Gamma^\eta{}_{\rho 0} = 4\pi G\rho_M.$$

Let us take the Newtonian limit of this equation, except that we keep the two terms on the left the side of this equation that involve the g_{00}:

$$-\frac{1}{2}\nabla^2 g_{00} - \frac{1}{4}(\nabla g_{00})^2 = 4\pi G\rho_M. \tag{3}$$

Usually the second term on the left is omitted because it is second order in g_{00} and thus seems negligible for the gravitational field of a galaxy or cumulus. If we substitute in this equation using (2) we can write it in the form

$$\nabla^2\psi = 4\pi G\rho_M + (\nabla\psi)^2, \tag{4}$$

which has the interpretation that both matter density ρ_M and $(\nabla\psi)^2$ are sources of gravity, that is, both matter and gravity gravitate.

Notice the term $(\nabla\psi)^2$ always gravitates with a positive sign, just like matter. Notice, too, that its contribution goes as the square of the *gradient* of the potential. This is a very important point, because even if the gravitational field ψ is small, its gradient does not have to be so. If in the galaxy's halo exist pockets of rapidly varying gravitational field $\psi(x, y, z)$, they could contribute significantly to the average galactic gravitational field $\psi(r)$, where r

is the radial coordinate of the galaxy. Thus the second-order term in (3) does not have to be smaller that the first-order term, and it could even be larger.

These pockets of rapidly varying gravitational field are not gravitational waves, but large volumes of classical, almost stationary but rapidly varying in space, metric fields. In the same way that there exist cosmic magnetic fields scattered throughout the universe, there can exist these metric fields. Both are primordial in origin, a point we shall develop with respect to the gravity pockets.

These pockets differ from other dark matter candidates in that they are not elementary particles and so cannot be observed by experiments involving particle detectors. Like matter, they gravitate and possess inertia. They do not interact with matter at all except gravitationally. In particular, they are impervious to the pressure exerted by cosmic thermal radiation.

Plausibility of the existence of gravity pockets. A plausible primordial origin for gravity pockets is during the reheating of the universe that is supposed to occur after the end of inflation. During reheating, the state of the inflaton φ field falls down a steep potential curve $V(\varphi)$ and eventually oscillates at the concave bottom of the potential. All through this fall and the oscillation at the bottom of the potential the inflaton is decaying into radiation and producing a lot of entropy, and consuming in the process a potential energy density just a few orders of magnitude smaller than E_P^4. The inflaton has a large coherent component φ whose size is model-dependent, but is roughly of the order of Planck's energy: $\varphi \sim E_p = G^{-1/2}$. However, according to quantum field theory, there should exist a perturbative incoherent component, call it φ', which decays into radiation. These decays are stochastic and produce radiation with varying energy and momentum. The ones that have large energy and momentum can produce powerful local modifications in the metric. The usual first-order cosmological perturbation theory[5] does not work in describing these events and the calculation has to be carried out to second-order, since both orders can be of comparable sizes for the reasons stated above.

The stress tensor for the inflaton is

$$T_{\mu\nu} = \varphi_{,\mu}\varphi_{,\nu} - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\varphi_{,\alpha}\varphi_{,\beta} - g_{\mu\nu}V(\varphi),$$

and Einstein's equation during inflation and reheating is

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G\left(T_{\mu\nu} + ...\right) = 8\pi E_P^{-2}\left(T_{\mu\nu} + ...\right), \tag{5}$$

where the ellipsis represents other stress tensors that may be relevant. The value of T_{00} is usually taken to be a few orders of magnitude smaller than E_P^4 to avoid entering a quantum gravity regime (where we cannot calculate),[6] so the right-side of (5) is a few orders of magnitude smaller than E_P^2. We conclude that the differentiated metrics on the left side of (5) are equated to very strong changes on the right during reheating, when the potential $V(\varphi)$ is rapidly diminishing. The volumes surrounding the more energetic disintegrations of the perturbative inflaton become pockets of rapidly changing metric. During this process radiation and gravitational waves are emitted. Eventually the expansion of the universe and dissipative processes conspire to reduce the time dependence of the metric and result in basically stationary gravity pockets of classical g_{00} metric field.

Calculation of virial velocity curves assuming gravity pockets in a weak galactic field. We want to prove that this explanation for dark matter is consistent with the observed virial velocity curves, and also to show how gravity pockets can be treated in a way similar to matter. Regarding the virial velocity curves $v(r)$, it is well-known that for a star outside the visual edge of a galaxy (or a galaxy outside the visual edge of a cluster) they often take a flat horizontal shape.

Consider a galaxy with an average weak, radial field. The g_{00} field within a pocket is also weak, but it varies rapidly in all three directions x, y, and z and gravitates as $(\nabla\psi)^2$ so its contribution does not have to be negligible. Assume the gravity pockets are numbered by i and have volumes V_i. Then the mass-like contribution of a pocket is given by

$$P_i = \int_{V_i} (\nabla\psi)^2 dV_i. \tag{6}$$

(The value of the integral does not depend on where we locate the origin of the coordinates we use to perform it.) It is possible to conclude from (4) and (6), by means of Gauss' Theorem, that if we have a gravity pocket P and a matter lump of mass M, both situated at the origin, their combined gravitational potential is

$$\psi_{M,P}(r) = -\frac{GM}{r} - \frac{P}{4\pi r}. \tag{7}$$

We see that gravity pockets gravitate in a manner very similar to matter lumps.

To proceed with the calculation of the virial velocity curves we need to know the dependence of matter density $\rho_M(r)$ and the gravity pocket density $\rho_P(r)$ on the galaxy's radius r. The present picture of galaxy formation is that, after reheating, radiation and dark matter permeate the universe. Dark matter is attracted by the density fluctuations of primordial quantum origin, and begins collapsing and facilitating the formation of structure. At about the onset of matter domination matter begins to take part in the growth of cosmic structures. If we assume that, in the formation of a galaxy, gravity pockets and matter collapse as a dissipationless gas and undergo violent relaxation,[7] then the resulting matter and gravity pocket densities should decrease as $\rho_M \propto \rho_P \propto r^{-2}$. (This result does not hold for the region near the galactic center). If we assume that $\rho_P \propto r^{-2}$, then the virial velocity curve will be flat, as we shall see.

In the case of elliptical galaxies the same fall for the density ρ_P can be obtained from the use of the Hubble-Reynolds law

$$I = \frac{I_0 r_H^2}{(r + r_H)^2},$$

for surface brightness I, where r_H is usually taken to be quite smaller than the galactic radius r. For radii not near the center of the galaxy we conclude that $I \propto r^{-2}$. If the surface brightness I is proportional to the density brightness ρ_B, and the density brightness is proportional to the matter density ρ_M, and the matter density is proportional to the pocket density ρ_P, then again $\rho_P \propto r^{-2}$.

At any rate let this be our choice for how ρ_P falls with r and with it let us go back to the problem of the virial curves. Assume a galaxy with a matter lump density ρ_M and a gravity pocket density ρ_P, both densities possessing, in the average, spherical symmetry. From the spherical symmetry and (7) it is possible to conclude for this galaxy, that

$$\nabla^2\psi = 4\pi G\rho_M + \rho_P. \tag{8}$$

Now let $\mathbf{F} = -\nabla\psi$ be the gravitational force. Then, from (8) and Gauss' Theorem, we obtain:

$$F = -\frac{G}{r^2}\int_r \rho_M dV - \frac{1}{4\pi r^2}\int_r \rho_P dV,$$

where we have assumed a spherical volume of integration with a radius r. For a radius $r > r_E$, where r_E is the radius of the galaxy's edge, the gravitational force would be given by:

$$F = -\frac{GM}{r^2} - \frac{P}{4\pi r^2} - \frac{1}{r^2}\int_{r_E}^{r} \rho_P(r')r'^2 dr', \tag{9}$$

where the gravity pockets integral has been split in two parts, one integrating up to the edge, and another from the edge on. We are wondering if the third term on the right in (9) does not fall as rapidly as r^{-2}. We take then $\rho_P = Cr^{-2}$ and and use this function in the third term to see how fast it falls with r:

$$-\frac{1}{r^2}\int \rho_P r^2 dr = -\frac{C}{r^2}\int dr' = -\frac{C}{r}. \tag{10}$$

Since this third terms falls more slowly (it can also be argued that the curve has to be continuous), let us simply take $F = -C/r$. The virial equation for a particle in a central force is $2\langle T\rangle = -m\langle Fr\rangle$, or

$$\langle mv^2\rangle = -\langle mFr\rangle = \left\langle m\frac{C}{r}r\right\rangle = mC,$$

from which we conclude that $v^2 = C$, that is, the velocity curve outside the edge is flat.

Final remarks. We posit a new candidate for dark matter, our old friend the g_{00} metric component. We have discussed how it is plausible that, during reheating, quantum decays (the ones involving higher energies and momenta) of the perturbational inflaton φ' result in pockets of rapidly varying metric. The expansion of the universe and dissipative processes eventually modify the pockets of g_{00} field so that they are basically stationary but rapidly varying in the space directions. They gravitate according to the positive-definite term $(\nabla g_{00})^2$ (as required by the general theory of relativity) so that even for weak gravitational fields the value of $(\nabla g_{00})^2$ does not have to be necessarily negligible if g_{00} is varying fast enough. The result, large pockets of rapidly-varying classical g_{00} field, are analogous to the cosmic magnetic fields that exist throughout our universe.

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