Violation of the Rotational Invariance in the CMB Bispectrum

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We investigate a statistical anisotropy on the Cosmic Microwave Background (CMB) bispectrum which can be generated from the primordial non-Gaussianity induced by quantum fluctuations of a vector field. We find a new configurations in the multipole space of the CMB bispectrum given by $\ell_1 = \ell_2 + \ell_3 + 2, |\ell_2 - \ell_3| - 2$ and their permutations, which violate the rotational invariance, such as an off-diagonal configuration in the CMB power spectrum. We also find that in a model presented by Yokoyama and Soda (2008), the amplitude of the statistically anisotropic bispectrum in the above configurations becomes as large as that in other configuration such as $\ell_1 = \ell_2 + \ell_3$. As a result, it might be possible to detect these contributions in the future experiments and then it would give us novel information about the physics of the early Universe.

§1. Introduction

The current cosmological observations, in particular Cosmic Microwave Background (CMB), tell us that the Universe is almost isotropic and primordial density fluctuations are almost Gaussian random fields. However, in keeping with the progress of the experiments, there have been a lot of works that verify the possibility of the small deviation of the statistical isotropy, e.g., so-called "Axis of Evil". The analyses of the power spectrum with employing the current CMB data suggest that the deviation of the statistical isotropy is about 10% at most (e.g.[1)−6)]). Toward more precise measurements in the future experiments, there are a lot of theoretical discussions about the effects on the CMB power spectrum[7)−,11)] e.g., the presence of the off-diagonal configuration of the multipoles in the CMB power spectrum, which vanishes in the isotropic spectrum.

As is well known, it might be difficult to explain such kind of statistical anisotropy in the standard inflationary scenario. However, recently, there have been several works about the possibility of generating the statistically anisotropic primordial density fluctuations in order to introduce non-trivial dynamics of the vector field[12)−.24)] In Ref. 14), the authors considered a modified hybrid inflation model where a waterfall field couples not only with an inflaton field but also with a massless vector field. They have shown that due to the effect of fluctuations of the vector field the primordial density fluctuations may have a small deviation from the statistical isotropy and also the deviation from the Gaussian statistics. If the primordial density fluctuations have the deviation from the Gaussian statistics, it produces the non-zero higher order spectra (corresponding to higher order correlation functions), e.g., the bispectrum (3-point function), the trispectrum (4-point function) and so on. Hence,

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in the model presented by Ref. 14), we can expect that there are characteristic signals not only in the CMB power spectrum but also in the CMB bispectrum.

With this motivations, in this paper, we calculate the CMB statistically-anisotropic bispectrum sourced from the curvature perturbations generated in the modified hybrid inflation scenario proposed by Ref. 14), based on the useful formula presented in Ref. 25). Then, we find the peculiar configurations of the multipoles never appearing in the isotropic bispectrum, like off-diagonal components in the CMB power spectrum.

This paper is organized as follows. In the next section, we briefly review the inflation model where the scalar waterfall field couples with the vector field and calculate the bispectrum of curvature perturbations based on Ref. 14). In §3, we give an exact form of the CMB statistically-anisotropic bispectrum and analyze the behavior of it by using the numerical computations. Finally, we devote the final section to the summary and discussion.

Throughout this paper, we obey the definition of the Fourier transformation as

$$f(\boldsymbol{x}) \equiv \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \tilde{f}(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \ . \tag{1·1}$$

§2. Statistically-anisotropic non-Gaussianity in curvature perturbations

In this section, we briefly review the mechanism for generating the statistical-anisotropic bispectrum induced by primordial curvature perturbations proposed in Ref. 14), where the authors set the system like the hybrid inflation that there are two scalar fields: inflaton ϕ and waterfall field χ, and a vector field A_μ coupled with a waterfall field. The action is given by

$$S = \int dx^4 \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi \partial_\nu \phi + \partial_\mu \chi \partial_\nu \chi) - V(\phi, \chi, A_\nu) \right. \\ \left. - \frac{1}{4} g^{\mu\nu} g^{\rho\sigma} f^2(\phi) F_{\mu\rho} F_{\nu\sigma} \right] \ . \tag{2·1}$$

Here $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the vector field A_μ, $V(\phi, \chi, A_\mu)$ is the potential of fields and $f(\phi)$ denotes a gauge coupling. In order to guarantee the isotropy of the background Universe, we need the condition that the energy density of the vector field is negligible in the total energy of the Universe and we assume the small expectation value of the vector field. Therefore, we neglect the effect of the vector field on the background dynamics and also the evolution of the fluctuations of the inflaton. In the standard hybrid inflation (only with the inflaton and the waterfall field), the inflation suddenly ends due to the tachyonic instability of the waterfall field which is triggered when the inflaton reaches a critical value $\phi_{\rm e}$. In the system described by Eq. (2·1), however, $\phi_{\rm e}$ may fluctuate due to the fluctuation of the vector field and it generates additional curvature perturbations.

Using the δN formalism,[26),27),28),29)] the total curvature perturbation on uniform energy density hypersurface at the time of the end of inflation $t = t_{\rm e}$ can be estimated in terms of the perturbation of the e-folding number as

$$\begin{aligned}\zeta(t_{\rm e}) &= \delta N(t_{\rm e}, t_*) \\ &= \frac{\partial N}{\partial \phi_*}\delta\phi_* + \frac{1}{2}\frac{\partial^2 N}{\partial \phi_*^2}\delta\phi_*^2 + \frac{\partial N}{\partial \phi_{\rm e}}\frac{d\phi_{\rm e}(A)}{dA^\mu}\delta A^\mu_{\rm e} \\ &+\frac{1}{2}\left[\frac{\partial N}{\partial \phi_{\rm e}}\frac{d^2\phi_{\rm e}(A)}{dA^\mu dA^\nu} + \frac{\partial^2 N}{\partial \phi_{\rm e}^2}\frac{d\phi_{\rm e}(A)}{dA^\mu}\frac{d\phi_{\rm e}(A)}{dA^\nu}\right]\delta A^\mu_{\rm e}\delta A^\nu_{\rm e} \ . \end{aligned} \tag{2·2}$$

Here t_* is the time when the scale of interest crosses the horizon during the slow-roll inflation. Assuming the sudden decay of all fields into radiations just after the inflation, the curvature perturbations on uniform energy density hypersurface become constant after the inflation ends. Hence, at the leading order, the power spectrum and the bispectrum of curvature perturbations are respectively derived as

$$\begin{aligned}\left\langle \prod_{n=1}^{2}\zeta(\boldsymbol{k_n})\right\rangle &= (2\pi)^3 N_*^2 P_\phi(k_1)\delta\left(\sum_{n=1}^{2}\boldsymbol{k_n}\right) \\ &+N_{\rm e}^2\frac{d\phi_{\rm e}(A)}{dA^\mu}\frac{d\phi_{\rm e}(A)}{dA^\nu}\langle\delta A^\mu_{\rm e}(\boldsymbol{k_1})\delta A^\nu_{\rm e}(\boldsymbol{k_2})\rangle \ , \end{aligned}\tag{2·3}$$

$$\begin{aligned}\left\langle \prod_{n=1}^{3}\zeta(\boldsymbol{k_n})\right\rangle &= (2\pi)^3 N_*^2 N_{**}[P_\phi(k_1)P_\phi(k_2) + 2 \text{ perms.}]\delta\left(\sum_{n=1}^{3}\boldsymbol{k_n}\right) \\ &+N_{\rm e}^3\frac{d\phi_{\rm e}(A)}{dA^\mu}\frac{d\phi_{\rm e}(A)}{dA^\nu}\frac{d\phi_{\rm e}(A)}{dA^\rho}\langle\delta A^\mu_{\rm e}(\boldsymbol{k_1})\delta A^\nu_{\rm e}(\boldsymbol{k_2})\delta A^\rho_{\rm e}(\boldsymbol{k_3})\rangle \\ &+N_{\rm e}^4\frac{d\phi_{\rm e}(A)}{dA^\mu}\frac{d\phi_{\rm e}(A)}{dA^\nu}\left(\frac{1}{N_{\rm e}}\frac{d^2\phi_{\rm e}(A)}{dA^\rho dA^\sigma} + \frac{N_{\rm ee}}{N_{\rm e}^2}\frac{d\phi_{\rm e}(A)}{dA^\rho}\frac{d\phi_{\rm e}(A)}{dA^\sigma}\right) \\ &\times[\langle\delta A^\mu_{\rm e}(\boldsymbol{k_1})\delta A^\nu_{\rm e}(\boldsymbol{k_2})(\delta A^\rho\star\delta A^\sigma)_{\rm e}(\boldsymbol{k_3})\rangle + 2 \text{ perms.}] \ , \end{aligned}\tag{2·4}$$

where $P_\phi(k) = H_*^2/(2k^3)$ is the power spectrum of the fluctuations of the inflaton, $N_* \equiv \partial N/\partial\phi_*$, $N_{**} \equiv \partial^2 N/\partial\phi_*^2$, $N_{\rm e} \equiv \partial N/\partial\phi_{\rm e}$, $N_{\rm ee} \equiv \partial^2 N/\partial\phi_{\rm e}^2$, and $\star$ denotes the convolution. Here, we assume that $\delta\phi_*$ is a Gaussian random field and $\langle\delta\phi A^\mu\rangle = 0$.

In the Coulomb gauge where $\delta A_0 = 0$ and $k_i A^i = 0$, the evolution equation of the fluctuations of the vector field is given by

$$\mathcal{A}_i'' - \frac{f''}{f}\mathcal{A}_i - a^2\partial_j\partial^j\mathcal{A}_i = 0 \ , \tag{2·5}$$

where $\mathcal{A}_i \equiv f\delta A_i$, $'$ denotes the derivative with respect to the conformal time and we neglect the contribution from the potential term. When $f \propto a, a^{-2}$ with appropriate quantization of the fluctuations of the vector field, we have the scale-invariant power spectrum of δA^i on superhorizon scale as[14),18),30)]

$$\left\langle\delta A^i_{\rm e}(\boldsymbol{k_1})\delta A^j_{\rm e}(\boldsymbol{k_2})\right\rangle = (2\pi)^3 P_\phi(k) f_{\rm e}^{-2} P^{ij}(\hat{\boldsymbol{k_1}})\delta\left(\sum_{n=1}^{2}\boldsymbol{k_n}\right) \ , \tag{2·6}$$

where a is the scale factor, $P^{ij}(\hat{\boldsymbol{k}}) = \delta^{ij} - \hat{k}^i\hat{k}^j$, $\hat{}$ denotes the unit vector, and $f_{\rm e} \equiv f(t_{\rm e})$. Therefore, substituting this expression into Eq. (2·3), we can rewrite the power spectrum of the primordial curvature perturbations, ζ, as

$$\left\langle \prod_{n=1}^{2} \zeta(\boldsymbol{k_n}) \right\rangle \equiv (2\pi)^3 P_\zeta(\boldsymbol{k_1}) \delta \left(\sum_{n=1}^{2} \boldsymbol{k_n} \right) , \tag{2·7}$$

$$P_\zeta(\boldsymbol{k}) = P_\phi(k) \left[N_*^2 + \left(\frac{N_{\rm e}}{f_{\rm e}} \right)^2 q^i q^j P_{ij}(\hat{\boldsymbol{k}}) \right] , \tag{2·8}$$

where $q_i \equiv d\phi_{\rm e}/dA^i, q_{ij} \equiv d^2\phi_{\rm e}/(dA^i dA^j)$. We can divide this expression into the isotropic part and the anisotropic part as[7)]

$$P_\zeta(\boldsymbol{k}) \equiv P_\zeta^{\rm iso}(k) \left[1 + g_\beta \left(\hat{\boldsymbol{q}} \cdot \hat{\boldsymbol{k}} \right)^2 \right] , \tag{2·9}$$

with

$$P_\zeta^{\rm iso}(k) = N_*^2 P_\phi(k)(1+\beta) \ , \quad g_\beta = -\frac{\beta}{1+\beta} \ , \tag{2·10}$$

where $\beta = (N_{\rm e}/N_*/f_{\rm e})^2 |\boldsymbol{q}|^2$. The bispectrum of the primordial curvature perturbation given by Eq. (2·4) can be written as

$$\left\langle \prod_{n=1}^{3} \zeta(\boldsymbol{k_n}) \right\rangle \equiv (2\pi)^3 F_\zeta(\boldsymbol{k_1}, \boldsymbol{k_2}, \boldsymbol{k_3}) \delta \left(\sum_{n=1}^{3} \boldsymbol{k_n} \right) , \tag{2·11}$$

$$\begin{aligned} F_\zeta(\boldsymbol{k_1}, \boldsymbol{k_2}, \boldsymbol{k_3}) = & \left(\frac{g_\beta}{\beta} \right)^2 P_\zeta^{\rm iso}(k_1) P_\zeta^{\rm iso}(k_2) \\ & \times \left[\frac{N_{**}}{N_*^2} + \beta^2 \hat{q}^a \hat{q}^b \left(\frac{1}{N_{\rm e}} \hat{q}^{cd} + \frac{N_{\rm ee}}{N_{\rm e}^2} \hat{q}^c \hat{q}^d \right) P_{ac}(\hat{\boldsymbol{k_1}}) P_{bd}(\hat{\boldsymbol{k_2}}) \right] \\ & +2 \text{ perms.} . \end{aligned} \tag{2·12}$$

Here $\hat{q}^{cd} \equiv q^{cd}/|\boldsymbol{q}|^2$ and we have assumed that the fluctuation of the vector field δA^i almost obeys Gaussian statistics, hence $\langle \delta A_{\rm e}^\mu(\boldsymbol{k_1}) \delta A_{\rm e}^\nu(\boldsymbol{k_2}) \delta A_{\rm e}^\rho(\boldsymbol{k_3}) \rangle = 0$.

Hereinafter, for calculating the CMB bispectrum explicitly, we adopt a simple model whose potential looks like an Abelian Higgs model in unitary gauge as[14)]

$$V(\phi, \chi, A^i) = \frac{\lambda}{4}(\chi^2 - v^2)^2 + \frac{1}{2} g^2 \phi^2 \chi^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} h^2 A^\mu A_\mu \chi^2 \ , \tag{2·13}$$

where λ, g, h are the coupling constants, m is the inflaton mass, and v is the vacuum expectation value of χ. Since the effective mass squared of the waterfall field is given by

$$m_\chi^2 \equiv \frac{\partial^2 V}{\partial \chi^2} = -\lambda v^2 + g^2 \phi_{\rm e}^2 + h^2 A^i A_i = 0 \ , \tag{2·14}$$

and the critical value of the inflaton $\phi_{\rm e}$ can be obtained as

$$g^2 \phi_{\rm e}^2 = \lambda v^2 - h^2 A^i A_i \ , \tag{2·15}$$

we can express β, q^i and q^{ij} in Eq. (2·12) in terms of the model parameters as

$$\hat{q}^i = -\hat{A}^i \ , \quad \hat{q}^{ij} = -\frac{1}{\phi_{\rm e}}\left[\left(\frac{g\phi_{\rm e}}{hA}\right)^2 \delta^{ij} + \hat{A}^i\hat{A}^j\right] \ , \quad \beta \simeq \frac{1}{f_{\rm e}^2}\left(\frac{h^2 A}{g^2\phi_{\rm e}}\right)^2 \ , \tag{2·16}$$

where we have used $N_* \simeq N_e \simeq 1/\sqrt{2\epsilon}$ with ϵ being the slow-roll parameter and $|\boldsymbol{A}| \equiv A$. Substituting these quantities into Eq. (2·12), the bispectrum of primordial curvature perturbations is obtained as

$$F_\zeta(\boldsymbol{k_1}, \boldsymbol{k_2}, \boldsymbol{k_3}) = C P_\zeta^{\rm iso}(k_1) P_\zeta^{\rm iso}(k_2) \hat{A}^a \hat{A}^b \delta^{cd} P_{ac}(\hat{\boldsymbol{k_1}}) P_{bd}(\hat{\boldsymbol{k_2}}) + 2 \text{ perms.} \ , \tag{2·17}$$

$$C \equiv -\frac{g_\beta^2}{N_{\rm e}\phi_{\rm e}}\left(\frac{g\phi_{\rm e}}{hA}\right)^2 \ . \tag{2·18}$$

Note that we have neglected the terms which are suppressed by the slow-roll parameters in the above expression. The current CMB observations suggest $g_\beta < \mathcal{O}(0.1)$ (e.g.[1),2)]) and $1/(N_{\rm e}\phi_{\rm e})$ is equivalent to the slow-roll parameter $\eta \equiv V_{\phi\phi}/V$ with $V_{\phi\phi} \equiv \partial^2 V/\partial\phi^2$. Hence, the overall amplitude of the bispectrum in this model, C, seems not to be enough large to be detected. However, even if $g_\beta \ll 1$ and $\eta \ll 1$, C can become large depending on the value of the coupling constants, g and h, the expectation value of A and also the critical value of the inflaton field, $\phi_{\rm e}$. Then, in the next section, we closely investigate the CMB bispectrum generated from the primordial bispectrum given by Eq. (2·17) and discuss a new characteristic feature of the CMB bispectrum induced by the statistical anisotropy of the primordial bispectrum.

§3. CMB statistically-anisotropic bispectrum

In this section, we give a formula of the CMB bispectrum generated from the primordial bispectrum, which has statistical anisotropy due to the fluctuations of the vector field, given by Eq. (2·17). We also discuss the special signals of this CMB bispectrum, which vanish in the statistically-isotropic bispectrum.

3.1. *Formulation*

The CMB fluctuation can be expanded in terms of the spherical harmonic function as

$$\frac{\Delta X}{X} = \sum_{\ell m} a_{X,\ell m} Y_{\ell m}(\hat{\boldsymbol{n}}) \ , \tag{3·1}$$

where $\hat{\boldsymbol{n}}$ is an unit vector pointing toward a line-of-sight direction, and X denotes the intensity ($\equiv I$) and the polarizations ($\equiv E, B$). The coefficient, $a_{\ell m}$, generated from primordial curvature perturbations, ζ, is expressed as[25),31)]

$$a_{X,\ell m} = 4\pi(-i)^\ell \int_0^\infty \frac{k^2 dk}{(2\pi)^3} \zeta_{\ell m}(k) \mathcal{T}_{X,\ell}(k) \quad (\text{for } X = I, E) \ , \tag{3·2}$$

$$\zeta_{\ell m}(k) \equiv \int d^2\hat{\boldsymbol{k}} \zeta(\boldsymbol{k}) Y_{\ell m}^*(\hat{\boldsymbol{k}}) \ , \tag{3·3}$$

where $\mathcal{T}_{X,\ell}$ is the time-integrated transfer function of scalar modes as calculated in Refs. 32), 33). Using these equations, the CMB bispectrum generated from the bispectrum of the primordial curvature perturbations is given by

$$\left\langle \prod_{n=1}^{3} a_{X_n,\ell_n m_n} \right\rangle = \left[\prod_{n=1}^{3} 4\pi (-i)^{\ell_n} \int_0^\infty \frac{k_n^2 dk_n}{(2\pi)^3} \mathcal{T}_{X_n,\ell_n}(k_n) \right] \left\langle \prod_{n=1}^{3} \zeta_{\ell_n m_n}(k_n) \right\rangle , \quad (3\cdot4)$$

with

$$\left\langle \prod_{n=1}^{3} \zeta_{\ell_n m_n}(k_n) \right\rangle = \left[\prod_{n=1}^{3} \int d^2 \hat{\boldsymbol{k_n}} Y^*_{\ell_n m_n}(\hat{\boldsymbol{k_n}}) \right] \times (2\pi)^3 \delta\left(\sum_{n=1}^{3} \boldsymbol{k_n} \right) F_\zeta(\boldsymbol{k_1}, \boldsymbol{k_2}, \boldsymbol{k_3}) \ . \quad (3\cdot5)$$

We expand the angular dependencies which appear in the Dirac delta function, $\delta(\boldsymbol{k_1} + \boldsymbol{k_2} + \boldsymbol{k_3})$, and the function, $F_\zeta(\boldsymbol{k_1}, \boldsymbol{k_2}, \boldsymbol{k_3})$, given by Eq. (2·17) with respect to the spin spherical harmonics as

$$\delta\left(\sum_{n=1}^{3} \boldsymbol{k_n} \right) = 8 \int_0^\infty y^2 dy \left[\prod_{n=1}^{3} \sum_{L_n M_n} (-1)^{L_n/2} j_{L_n}(k_n y) Y^*_{L_n M_n}(\hat{\boldsymbol{k_n}}) \right] \times I^{0\,0\,0}_{L_1 L_2 L_3} \begin{pmatrix} L_1 & L_2 & L_3 \\ M_1 & M_2 & M_3 \end{pmatrix} , \quad (3\cdot6)$$

$$\hat{A}^a \hat{A}^b \delta^{cd} P_{ac}(\hat{\boldsymbol{k_1}}) P_{bd}(\hat{\boldsymbol{k_2}}) = -4 \left(\frac{4\pi}{3} \right)^3 \sum_{L,L',L_A=0,2} I^{01-1}_{L11} I^{01-1}_{L'11} I^{000}_{11L_A} \begin{Bmatrix} L & L' & L_A \\ 1 & 1 & 1 \end{Bmatrix} \times \sum_{MM'M_A} Y^*_{LM}(\hat{\boldsymbol{k_1}}) Y^*_{L'M'}(\hat{\boldsymbol{k_2}}) Y^*_{L_A M_A}(\hat{\boldsymbol{A}}) \times \begin{pmatrix} L & L' & L_A \\ M & M' & M_A \end{pmatrix} , \quad (3\cdot7)$$

where the 2×3 matrices of a bracket and a curly bracket denote the Wigner-$3j$ and $6j$ symbols, respectively, and

$$I^{s_1 s_2 s_3}_{l_1 l_2 l_3} \equiv \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ s_1 & s_2 & s_3 \end{pmatrix} \ . \quad (3\cdot8)$$

Here, we have used the expressions of an arbitrary unit vector and a projection tensor as

$$\hat{r}_a = \begin{pmatrix} \sin\theta_r \cos\phi_r \\ \sin\theta_r \sin\phi_r \\ \cos\theta_r \end{pmatrix} = \sum_m \alpha_a^m Y_{1m}(\hat{\boldsymbol{r}}) \ , \quad (3\cdot9)$$

$$P_{ab}(\hat{\boldsymbol{r}}) = \delta_{ab} - \hat{r}_a \hat{r}_b = -2 \sum_{L=0,2} I^{01-1}_{L11} \sum_{M m_a m_b} Y^*_{LM}(\hat{\boldsymbol{r}}) \alpha_a^{m_a} \alpha_b^{m_b} \begin{pmatrix} L & 1 & 1 \\ M & m_a & m_b \end{pmatrix} , \quad (3\cdot10)$$

with

$$\alpha_a^m \equiv \sqrt{\frac{2\pi}{3}} \left(\begin{array}{c} -m(\delta_{m,1} + \delta_{m,-1}) \\ i\,(\delta_{m,1} + \delta_{m,-1}) \\ \sqrt{2}\delta_{m,0} \end{array} \right) , \qquad (3\cdot11)$$

and summation rules of the Wigner symbols as discussed in the Appendix of Ref. 25) *). With integrating these spherical harmonics over each unit vector, the angular dependencies on $\boldsymbol{k_1}, \boldsymbol{k_2}, \boldsymbol{k_3}$ can be reduced to the Wigner-$3j$ symbols as

$$\int d^2\hat{\boldsymbol{k_1}} Y^*_{\ell_1 m_1} Y^*_{L_1 M_1} Y^*_{LM} = I^{0\,0\,0}_{\ell_1 L_1 L} \left(\begin{array}{ccc} \ell_1 & L_1 & L \\ m_1 & M_1 & M \end{array} \right) , \qquad (3\cdot13)$$

$$\int d^2\hat{\boldsymbol{k_2}} Y^*_{\ell_2 m_2} Y^*_{L_2 M_2} Y^*_{L' M'} = I^{0\,0\,0}_{\ell_2 L_2 L'} \left(\begin{array}{ccc} \ell_2 & L_2 & L' \\ m_2 & M_2 & M' \end{array} \right) , \qquad (3\cdot14)$$

$$\int d^2\hat{\boldsymbol{k_3}} Y^*_{\ell_3 m_3} Y^*_{L_3 M_3} = (-1)^{m_3} \delta_{L_3,\ell_3} \delta_{M_3,-m_3} . \qquad (3\cdot15)$$

From these equations, we obtain an alternative explicit form of the bispectrum of $\zeta_{\ell m}$ as

$$\begin{aligned}
\left\langle \prod_{n=1}^{3} \zeta_{\ell_n m_n}(k_n) \right\rangle = &-(2\pi)^3 8 \int_0^\infty y^2 dy \sum_{L_1 L_2} (-1)^{\frac{L_1+L_2+\ell_3}{2}} I^{0\,0\,0}_{L_1 L_2 \ell_3} \\
&\times P_\zeta^{\rm iso}(k_1) j_{L_1}(k_1 y) P_\zeta^{\rm iso}(k_2) j_{L_2}(k_2 y) C j_{\ell_3}(k_3 y) \\
&\times 4 \left(\frac{4\pi}{3}\right)^3 (-1)^{m_3} \sum_{L,L',L_A=0,2} I^{01-1}_{L11} I^{01-1}_{L'11} \\
&\times I^{0\,0\,0}_{\ell_1 L_1 L} I^{0\,0\,0}_{\ell_2 L_2 L'} I^{000}_{11L_A} \left\{ \begin{array}{ccc} L & L' & L_A \\ 1 & 1 & 1 \end{array} \right\} \\
&\times \sum_{M_1 M_2 M M' M_A} Y^*_{L_A M_A}(\hat{\boldsymbol{A}}) \left(\begin{array}{ccc} L_1 & L_2 & \ell_3 \\ M_1 & M_2 & -m_3 \end{array} \right) \\
&\times \left(\begin{array}{ccc} \ell_1 & L_1 & L \\ m_1 & M_1 & M \end{array} \right) \left(\begin{array}{ccc} \ell_2 & L_2 & L' \\ m_2 & M_2 & M' \end{array} \right) \left(\begin{array}{ccc} L & L' & L_A \\ M & M' & M_A \end{array} \right) \\
&+2 \text{ perms. } . \qquad (3\cdot16)
\end{aligned}$$

This equation implies that due to the vector field $\boldsymbol{A}$ the CMB bispectrum has a direction-dependence and hence the dependency on m_1, m_2, m_3 can not be confined only to a Wigner-$3j$ symbol, namely

$$\left\langle \prod_{n=1}^{3} \zeta_{\ell_n m_n}(k_n) \right\rangle \neq (2\pi)^3 \mathcal{F}_{\ell_1 \ell_2 \ell_3}(k_1, k_2, k_3) \left(\begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{array} \right) . \qquad (3\cdot17)$$

*) Equation (3·10) is easily derived by using the expression with a divegenceless vector described in Ref. 34). Equation (3·11) leads to the orthogonality relation as

$$\alpha_m^a \alpha_a^{m'} = \frac{4\pi}{3} (-1)^m \delta_{m,-m'} . \qquad (3\cdot12)$$

This fact truly means the violation of the rotational invariance in the bispectrum of the primordial curvature perturbations and leads to the statistical anisotropy on the CMB bispectrum.

Let us consider the explicit form of the CMB bispectrum. Here we set the coordinate as $\hat{\boldsymbol{A}} = \hat{\boldsymbol{z}}$. Then, with substituting Eq. (3·16) into Eq. (3·4) and using the relation $Y^*_{L_A M_A}(\hat{\boldsymbol{z}}) = \sqrt{(2L_A+1)/(4\pi)}\delta_{M_A,0}$, the CMB bispectrum is expressed as

$$\begin{aligned}
\left\langle \prod_{n=1}^{3} a_{X_n,\ell_n m_n} \right\rangle =& -\int_0^\infty y^2 dy \left[\prod_{n=1}^{3} \frac{2}{\pi} \int_0^\infty k_n^2 dk_n \mathcal{T}_{X_n,\ell_n}(k_n) \right] \\
&\times \sum_{L_1 L_2} (-1)^{\frac{\ell_1+\ell_2+L_1+L_2}{2}+\ell_3} I^{0\,0\,0}_{L_1 L_2 \ell_3} \\
&\times P^{\rm iso}_\zeta(k_1) j_{L_1}(k_1 y) P^{\rm iso}_\zeta(k_2) j_{L_2}(k_2 y) C j_{\ell_3}(k_3 y) \\
&\times 4 \left(\frac{4\pi}{3}\right)^3 (-1)^{m_3} \sum_{L,L',L_A=0,2} I^{01-1}_{L11} I^{01-1}_{L'11} \\
&\times I^{0\,0\,0}_{\ell_1 L_1 L} I^{0\,0\,0}_{\ell_2 L_2 L'} I^{000}_{11L_A} \left\{ \begin{matrix} L & L' & L_A \\ 1 & 1 & 1 \end{matrix} \right\} \\
&\times \sqrt{\frac{2L_A+1}{4\pi}} \sum_{M=-2}^{2} \left(\begin{matrix} L_1 & L_2 & \ell_3 \\ -m_1 - M & -m_2 + M & -m_3 \end{matrix} \right) \\
&\times \left(\begin{matrix} \ell_1 & L_1 & L \\ m_1 & -m_1 - M & M \end{matrix} \right) \left(\begin{matrix} \ell_2 & L_2 & L' \\ m_2 & -m_2 + M & -M \end{matrix} \right) \\
&\times \left(\begin{matrix} L & L' & L_A \\ M & -M & 0 \end{matrix} \right) + 2 \text{ perms.} \,. \qquad (3\cdot18)
\end{aligned}$$

By taking into account the selection rules of the Wigner symbols,[25] the multipoles and the azimuthal quantum numbers are limited as

$$\begin{aligned}
&\sum_{n=1}^{3} \ell_n = \text{even} \,, \quad \sum_{n=1}^{3} m_n = 0 \,, \\
&L_1 = |\ell_1 - 2|, \ell_1, \ell_1 + 2 \,, \quad L_2 = |\ell_2 - 2|, \ell_2, \ell_2 + 2 \,, \\
&|L_2 - \ell_3| \le L_1 \le L_2 + \ell_3 \,. \qquad (3\cdot19)
\end{aligned}$$

and the two permutations of ℓ_1, ℓ_2, ℓ_3.

3.2. *Behavior of the CMB statistically-anisotropic bispectrum*

Based on the formula (3·18), we compute the CMB bispectra for the several ℓ's and m's. Then we modify the Boltzmann Code for Anisotropies in the Microwave Background (CAMB)[35),36)] and use the Common Mathematical Library SLATEC.[37)]

In Fig. 1, the red solid lines are the CMB statistically-anisotropic bispectra of the intensity mode given by Eq. (3·18) with $C = 1$ and the green dashed lines are the statistically-isotropic one sourced from the local-type non-Gaussianity of curvature

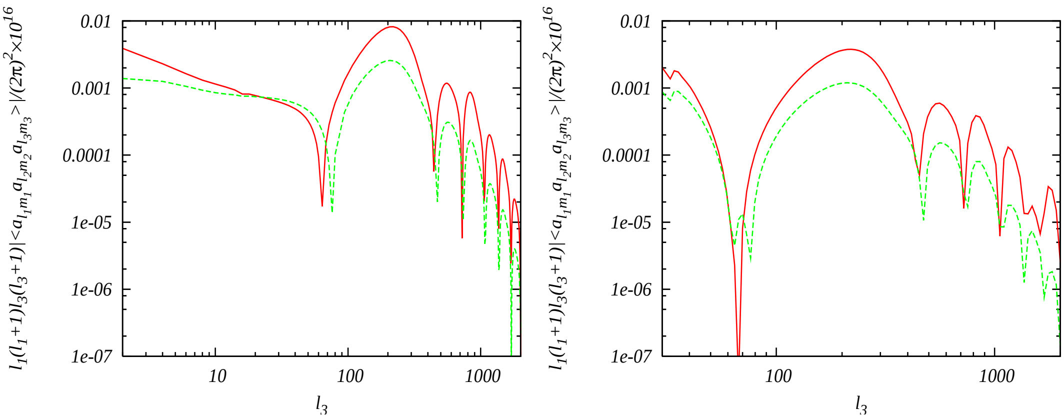

Fig. 1. Absolute values of the CMB statistically-anisotropic bispectrum of the intensity mode given by Eq. (3·18) with $C = 1$ (red solid line) and the statistically-isotropic one given by Eq. (3·20) with $f_{\rm NL} = 5$ (green dashed line) for $\ell_1 = \ell_2 = \ell_3$. The left and right figures are plotted in the configurations $(m_1, m_2, m_3) = (0, 0, 0), (10, 20, -30)$, respectively. The parameters are fixed to the mean values limited from the WMAP-7yr data as reported in Ref. 40)

perturbations given by[38)]

$$\left\langle \prod_{n=1}^{3} a_{X_n,\ell_n m_n} \right\rangle = I^{0\,0\,0}_{\ell_1\ell_2\ell_3} \left(\begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{array} \right) \times \int_0^\infty y^2 dy \left[\prod_{n=1}^{3} \frac{2}{\pi} \int_0^\infty k_n^2 dk_n \mathcal{T}_{X_n,\ell_n}(k_n) j_{\ell_n}(k_n y) \right] \times \left(P_\zeta^{\rm iso}(k_1) P_\zeta^{\rm iso}(k_2) \frac{6}{5} f_{\rm NL} + 2 \text{ perms.} \right) , \quad (3{\cdot}20)$$

with $f_{\rm NL} = 5$ for $\ell_1 = \ell_2 = \ell_3$ and two sets of m_1, m_2, m_3. From this figure, we can see that the red solid lines are in good agreement with the green dashed line in the dependence on ℓ for both configurations of m_1, m_2, m_3. This seems to be because the bispectrum of primordial curvature perturbations affected by the fluctuations of vector field given by Eq. (2·17) has not only the anisotropic part but also the isotropic part and both parts present with the same amplitude. In this sense, it is expected that the angular dependence on the vector field $\hat{\boldsymbol{A}}$ does not contribute much to a change in the shape of the CMB bispectrum. We also find that the anisotropic bispectrum for $C \sim 0.3$ is comparable in magnitude to the case with $f_{\rm NL} = 5$ for the standard local type, which corresponds to the upper bound on the local-type non-Gaussianity expected from the PLANCK experiment.[39)]

In the discussion of the CMB power spectrum, if the rotational invariance is violated in the primordial power spectrum given by Eq. (2·9), the signals in the off-diagonal configurations of ℓ also have nonzero values.[7),8),10)] Likewise, there are special configurations in the CMB bispectrum induced from the statistical anisotropy on the primordial bispectrum as Eq. (2·17). The selection rule (3·19) suggests that the statistically-anisotropic bispectrum (3·18) could be nonzero in the multipole configurations given by

$$\ell_1 = |\ell_2 - \ell_3| - 4, |\ell_2 - \ell_3| - 2, \ell_2 + \ell_3 + 2, \ell_2 + \ell_3 + 4 \ , \tag{3·21}$$

and two permutations of ℓ_1, ℓ_2, ℓ_3. In contrast, in these configurations, the isotropic bispectrum (e.g. Eq. (3·20)) vanishes due to the triangle condition of the Wigner-3j symbol $\begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$ and the nonzero components arise only from

$$|\ell_2 - \ell_3| \leq \ell_1 \leq \ell_2 + \ell_3 \ . \tag{3·22}$$

Therefore, the signals of the configurations (3·21) have the pure information of the statistical anisotropy on the CMB bispectrum.

Figure 2 shows the CMB anisotropic bispectra of the intensity mode given by Eq. (3·18) with $C = 1$ for the several configurations of ℓ's and m's as a function of ℓ_3. The red solid line and the green dashed line satisfy the special relation (3·21), namely $\ell_1 = \ell_2 + \ell_3 + 2, |\ell_2 - \ell_3| - 2$, and the blue dotted line obeys a configuration , namely $\ell_1 = \ell_2 + \ell_3$. From this figure, we confirm that the signals in the special configuration (3·21) are comparable in magnitude to those for $\ell_1 = \ell_2 + \ell_3$. Therefore, if the rotational invariance is violated on the primordial bispectrum of curvature perturbations, the signals for $\ell_1 = \ell_2 + \ell_3 + 2, |\ell_2 - \ell_3| - 2$ also can become beneficial observables. Here, note that the anisotropic bispectra in the other special configurations: $\ell_1 = \ell_2 + \ell_3 + 4, |\ell_2 - \ell_3| - 4$ are zero. It is because L_1, L_2, L, L' are fixed to a set of the values, respectively, due to the selection rules of the Wigner symbols in Eq. (3·18), and the summation of the four Wigner-3j symbols over M vanishes for all ℓ's and m's. Hence, in this anisotropic bispectrum, the additional signals arise from only two configurations $\ell_1 = \ell_2 + \ell_3 + 2, |\ell_2 - \ell_3| - 2$ and these two permutations.

§4. Summary and discussion

In this paper, we investigate the statistical anisotropy in the CMB bispectrum, by considering the modified hybrid inflation model where the waterfall field also couples with the vector field.[14)] We calculate the CMB bispectrum sourced from the non-Gaussianity of curvature perturbations affected by the vector field. In this inflation model, due to the dependence on the direction of the vector field, the correlations of the curvature perturbations violate the rotational invariance. Then, interestingly, even if the magnitude of the parameter g_β characterizing the statistical anisotropy of the CMB power spectrum is too small, the amplitude of the non-Gaussianity can become large depending on the several coupling constants of the fields.

Following the procedure of Ref. 25), we formulate the statistically-anisotropic CMB bispectrum and confirm that three azimuthal quantum numbers m_1, m_2, m_3 are not confined only to the Wigner symbol $\begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$. This is an evidence that the rotational invariance is violated in the CMB bispectrum and implies the

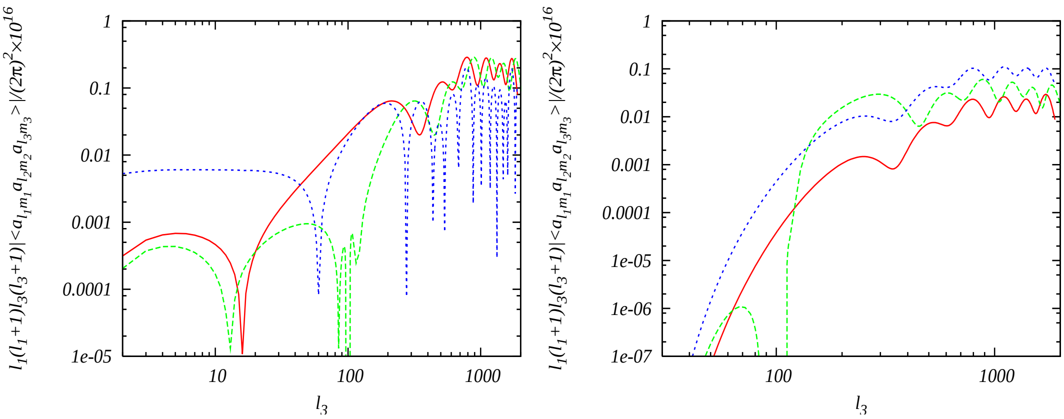

Fig. 2. Absolute values of the CMB statistically-anisotropic bispectra of the intensity mode given by Eq. (3·18) for $(m_1, m_2, m_3) = (0, 0, 0)$ (the left figure) and $(10, 20, -30)$ (the right one) as the function with respect to ℓ_3. The lines correspond to the spectra for $(\ell_1, \ell_2) = (102 + \ell_3, 100)$ (red solid line), $(|100 - \ell_3| - 2, 100)$ (green dashed line) and $(100 + \ell_3, 100)$ (blue dotted line). The parameters are identical to the values defined in Fig. 1.

existence of the signals not obeying the triangle condition of the above Wigner symbol as $|\ell_2 - \ell_3| \le \ell_1 \le \ell_2 + \ell_3$. We demonstrate that the signals of the CMB bispectrum for $\ell_1 = \ell_2 + \ell_3 + 2, |\ell_2 - \ell_3| - 2$ and these two permutations do not vanish. In fact, the statistically-isotropic bispectra are exactly zero for these configurations, hence these signals have the pure information of the statistical anisotropy. Because the amplitude of these intensity bispectra is comparable to that for $\ell_1 = \ell_2 + \ell_3$, it might be possible to detect these contributions of the statistical anisotropy in the future experiments and then it would give us novel information about the physics of the early Universe. Of course, also for the E-mode polarization, we can give the same discussions and results.

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