H-cluster stars

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The state of cold matter at supra-nuclear density depends on the non-perturbative nature of quantum chromo-dynamics (QCD) and is essential for modeling pulsars. In compact stars at a few nuclear densities and extremely low temperature, quarks could be interacting strongly with each other there. That might render quarks grouped in clusters, although the hypothetical quark-clusters in cold quark matter has not been confirmed due to the lack of clear evidence both theoretical and experimental. Motivated by recent lattice QCD simulations of the H-dibaryons (with structure *uuddss*), we are considering here a possible kind of quark-clusters, H-clusters, that could emerge inside quark stars during their cooling, as the dominant building blocks. We study the stars composed of H-clusters and derive the dependence of their maximum mass on the potential of H-H interaction, with the inclusion of the in-medium stiffening effect, showing that the maximum mass could be well above 2 $M_\odot$ under reasonable parameters. Besides a general understanding of different manifestations of compact stars, we expect further observational and experimental tests for H-cluster stars (or simply H stars) in the future.

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The study of pulsar-like compact stars opens a unique window that relates fundamental particle physics and astrophysics. At average density higher than 2 times nuclear matter density ρ_0, the quark degrees of freedom inside compact stars would not be negligible, and such stars are then called quark stars [e.g., 1, 2]. This becomes more likely if Bodmer-Witten conjecture is correct, which says that strange quark matter (composed of up, down and strange quarks) could be more stable than nuclear matter [3, 4]. The effect of non-perturbative QCD surely makes it difficult to derive the real state of cold quark matter, however the existence of quark stars cannot be ruled out neither theoretically nor observationally (see a review in [5]).

Although cold quark matter at a few nuclear densities is difficult to be created in laboratory and to be investigated by QCD calculations, some efforts have been made to understand the state of quark stars. The MIT bag model treats the quarks as relativistic and weakly interacting particles, which is the most widely used model for quark stars [2]. The color super-conductivity (CSC) state is currently focused on under perturbative QCD as well as QCD-based effective models [6]. In most of these models, quark stars are characterized by soft equations of state, because the asymptotic freedom of QCD tells us that as energy scale goes higher, the interaction between quarks will become weaker.

In cold quark matter at baryon densities of compact stars ($\rho \sim 2-10\rho_0$), however, the energy scale is far from the region where the asymptotic freedom approximation could apply, so the the ground state of realistic quark matter might not be that of Fermi gas (see a discussion given in [7]). Some evidence in heavy ion collision experiments shows that the interaction between quarks is still very strong even in the hot quark-gluon plasma [8]. It could then be reasonable to infer that quarks could be coupled strongly also in the interior of quark stars, which could make quarks to condensate in position space to form quark clusters. The observational tests from polarization, pulsar timing and asteroseismology have been discussed [9], and it is found that the idea of clustering quark matter could provide us a way to understand different manifestations of compact stars.

During the cooling of a quark star, the interaction between quarks will become stronger and stronger, then H-particles (six-quark clusters with the same structure as H dibaryons *uuddss*) would emerge due to the strong interaction between quarks. H-particles could be difficult to decay to lighter hadrons at high baryon density ($> 2\rho_0$) at which baryons could be crushed. To study quark stars composed of H-particles, i.e. H-cluster stars, we need to know the H-H interaction. In this *Letter*, we assume that the interaction between H-particles is mediated by σ and ω mesons and introduce the Yukawa potential to describe the interaction [10], and then derive the dependence of the maximum mass of H-cluster stars on the depth of potential well, taking into account the in-medium stiffening effect. More observations (e.g. pulsar-mass) could help us constrain the H-H interaction in dense matter.

Quark clusters may be analogized to hadrons, and in fact some authors did relevant researches. H dibaryon was predicted to be stable state or resonance [11], and an 18-quark cluster (quark-alpha, Q_α) being completely symmetric in spin, color and flavor space was also proposed [12]. The non-relativistic quark-cluster model was introduced to study the binding energy of H-particles [13]. The interaction between H-particles was investigated by employing one-gluon-exchange potential and an effective meson exchange potential, and a short-range repulsion was found [14]. Recently, H dibaryon has been found in lattice QCD simulations by two independent groups [15, 16], with binding energy of about

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10 to 40 MeV. Anyway, H dibaryon, although no direct evidence from experiments, provides us a specific kind of quark clusters that could be very likely to exist inside quark stars. We set the mass of H-cluster, $m_H = 2m_\Lambda - 20$ MeV $= 2210$ MeV, in our following calculations, with m_Λ the mass of Λ.

We note that H-particles could also appear in the core of a neutron star according to previous studies. The existence of H-particles inside neutron stars has been investigated in relativistic mean-field theory [10]. It was found that the Bose condensate of H-particles soften the equation of state, and stars are unstable against compression [17].

However, H-particles in H-cluster stars are more like classical particles, and the reason is as following. The Compton wavelength of an H-particle is $\lambda = h/(m_H c) \sim 0.56$ fm $(2210\ \mathrm{MeV}/m_H)$. The distance between two nearby H-particles is $d = n^{-1/3} \sim 1.1$ fm $(n/(5n_0))^{-1/3}$, with n the number density of H-clusters and n_0 the nucleon number density at saturation. The quantum effect is not very significant since $\lambda < d$, if we ignore the interaction between H-particles. Moreover, if the potential well of H-H interaction is deeper than ~ 5 MeV, then H-particles will be trapped inside the potential well and be localized, because their quantum kinetic energy is only ~ 1.9 MeV $(n/(5n_0))^{2/3}$. Consequently, the quantum effect of H-clusters there could be negligible, and the Bose condensate might not take place.

The interaction between hadrons is mediated by mesons which is characterized by repulsion at short distance and attraction at long distance. Similarly, we could write the potential between H-clusters as [10]

$$V(r) = \frac{g_{\omega H}^2}{4\pi}\frac{e^{-m_\omega r}}{r} - \frac{g_{\sigma H}^2}{4\pi}\frac{e^{-m_\sigma r}}{r}, \tag{1}$$

where $g_{\omega H}$ and $g_{\sigma H}$ are the coupling constants of H-clusters and meson fields. The numerical result of the potential between two H-dibaryons shows a minimum at $r_0 \approx 0.7$ fm with the depth $V_0 \approx -400$ MeV [14], which means that, to get the minimal point, two H-dibaryons should be very close to each other. To prevent the existence of H-dibaryons in normal nuclear matter, one has to have $V(\rho_0) \gtrsim -350$ MeV [17].

Nevertheless, the medium effect in dense matter could change those properties. In dense nuclear matter, the effective meson masses m_M^* satisfy the scaling law $m_M^* \simeq m_M(1 - \alpha_{BR} n/n_0)$, where α_{BR} is the coefficient of the scaling and m_M is the meson mass in free space. This is called Brown-Rho scaling [18], and the value of α_{BR} is found to be about 0.2 at the nuclear matter density. In the problem we are now considering, the density could reach $\sim 10\rho_0$, and we then use a modified scaling law of

$$m_M^* = m_M \exp(-\alpha_{BR} n/n_0), \tag{2}$$

which could be a good approximation of the Brown-Rho scaling at nuclear density and also shows the in-medium effect that stiffens the inter-particle potential by reducing the meson effective masses. In this case, m_σ and m_ω in Eq.(1) should be replaced by m_σ^* and m_ω^*, which makes r_0 and V_0 become larger.

Given the potential between two H-clusters, we can obtain the energy density by taking into account all of the contributions from H-clusters in the system. Note that in this problem, the interaction between H-clusters is mediated by σ and ω mesons, so the interaction at long distance is negligible. Therefore, we only consider the contributions of the nearby particles, and write the energy density as $\epsilon_I = n\ V$. Combining with Eq.(1), one has the interaction energy density ϵ_I as a function of number density n,

$$\epsilon_I = n^{4/3}\left(\frac{g_{\omega H}^2}{4\pi} e^{-m_\omega^* n^{-1/3}} - \frac{g_{\sigma H}^2}{4\pi} e^{-m_\sigma^* n^{-1/3}}\right), \tag{3}$$

and the pressure is thus

$$P = n^2 \frac{d}{dn}\left(\frac{\epsilon_I}{n}\right). \tag{4}$$

If we know the surface H-cluster number density n_s and the depth of the potential well V_0, we can determine $g_{\omega H}$ and $g_{\sigma H}$, because at the surface of stars the potential reaches its minimal value: $P(n = n_s) = 0$ and $V(n = n_s) = V_0$. Considering the uncertainty of the interaction, we take V_0 as a parameter, and fix the surface density ρ_s to be $2\rho_0$. In addition, we find that different values of m_H do not influence the equation of state significantly.

Compact stars composed of pure H-clusters are electric neutral, but in reality there could be some flavor symmetry breaking that leads to the non-equality among u, d and s, usually with less s than u and d. The positively charged quark matter is necessary because it allows the existence of electrons that is crucial for us to understand the radiative properties of pulsars. However, we note that the pressure of degenerate electrons is negligible compared to the pressure of H-clusters, so the contribution of electrons to the equation of state is neglected.

In general relativity, the hydrostatic equilibrium condition in spherically symmetry is [19]

$$\frac{1 - 2Gm(r)/c^2 r}{P + \rho c^2} r^2 \frac{dP}{dr} + \frac{Gm(r)}{c^2} + \frac{4\pi G}{c^4} r^3 P = 0, \tag{5}$$

where

$$m(r) = \int_0^r \rho \cdot 4\pi r'^2 dr', \tag{6}$$

with $\rho = \epsilon_I/c^2 + n\ m_H$. Inserting the equation of state $P(\rho)$ we can get the total mass M and radius R of an H-cluster star by numerical integration. Figure 1 shows the mass-radius and mass-central density (rest-mass energy density) curves, in the case $\rho_s = 2\rho_0$ and α_{BR}=0.2, including $V_0 = -10$ MeV (solid line) and $V_0 = -100$ MeV (dashed line). At first, M grows larger as central density increases, and eventually M reaches the maximum value, after which the increase of central density leads to

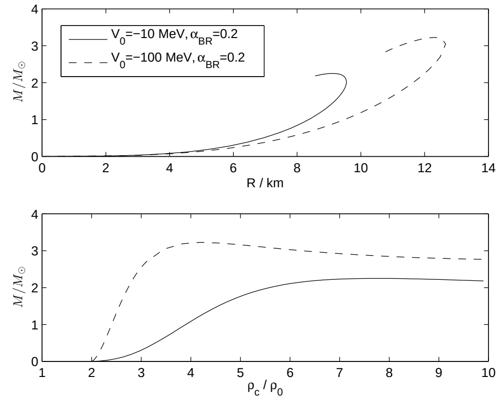

FIG. 1. The mass-radius curves and mass-central density (rest-mass energy density) curves, in the case $\rho_s = 2\rho_0$ and α_{BR}=0.2, including $V_0 = -10$ MeV (solid line) and $V_0 = -100$ MeV (dashed line).

gravitational instability. In the figure, both curves have maximum masses higher than $2M_\odot$

The observed masses of pulsars put constraints on the state of dense matter. Quark stars have been characterized by soft equations of state, because in conventional quark star models (e.g. the MIT bag model) quarks are treated as relativistic and weakly interacting particles. Recently, radio observations of a binary millisecond pulsar PSR J1614-2230 imply that the pulsar mass is 1.97±0.04 $M_\odot$ [20]. Although this high mass could rule out conventional quark star models with soft equations of state, some other models of stars with quark matter could be consistent with the observation of the high mass pulsar, such as color-superconducting quark matter model [21] and hybrid star models[6, 22]. Quark stars with quark clusters could also have maximum mass $M_{\rm max} > 2M_\odot$ because of stiff equation of state [23–25].

In this *Letter*, we study quark stars composed of H-clusters and apply the potential model which is widely used in nuclear physics to describe the H-H interaction. The depth V_0 and position r_0 of potential well should be meaningful for study the properties of cold quark matter with H-clusters. The coefficient of Brown-Rho scaling α_{BR} is also unknown in quark matter, whose value could be different from that used in nuclear matter. We constrain the parameters V_0 and α_{BR} in the context of H-cluster stars by the maximum mass of pulsars $M_{\rm max}$. The interaction between H-dibaryons was studies previously and the related parameters were derived by fitting data in experiments of nucleon-nucleon interaction and hypernucleus events (e.g. see [14] and references therein); however, whether the two-particle interaction data are adequate in determining the properties of quark matter is uncertain. Our model for H-H interaction could provide us another way to study the properties of H-clusters in quark matter, although giving wide ranges of parameters due to the uncertainty of $M_{\rm max}$. Figure 2 shows the dependence of $M_{\rm max}$ on V_0 and α_{BR}, in the case $\rho_s = 2\rho_0$. To make comparison, we also plot the result when $\alpha_{BR} = 0$. The discrepancy between different values of non-zero α_{BR} is not very significant, and under a wide range of parameter-space $M_{\rm max}$ can be well above $2M_\odot$.

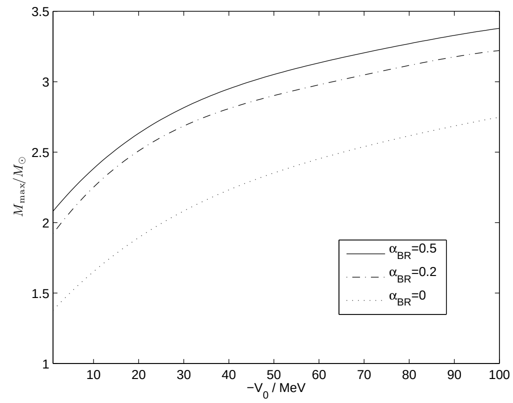

FIG. 2. The dependence of $M_{\rm max}$ on V_0 and α_{BR}, in the case $\rho_s = 2\rho_0$, including $\alpha_{BR} = 0.5$ (solid line), $\alpha_{BR} = 0.2$ (dashed line) and $\alpha_{BR} = 0$ (dotted line).

Could H-clusters decay inside quark stars? The density range for a gravitationally stable H-cluster star in our model is from 2 to $\sim 8\rho_0$. Certainly, H-clusters themselves could be so crowded that they press the nearby ones, but if the size of each H-cluster is not larger than that of a nucleon, they would not be crushed. On the other hand, H-clusters could be energetically favored inside pulsars, because the gravitational energy gained in going over to H-clusters can compensate the loss of rest-mass energy if the mass of the system is high enough [26]. In the reaction $2n + 2\pi \leftrightarrow 2\Lambda \leftrightarrow H$, we find that the energy defects in creating one H-cluster from nuclear matter is about 60 MeV, so for a star with mass M the total energy defects is $\sim 2.4 \times 10^{51}{\rm erg}(M/M_\odot)$. Assume that the H-cluster star has a constant density $2\rho_0$, then the gravitational energy of the homogeneous sphere of mass M is $\sim -1.7 \times 10^{53}{\rm erg}(M/M_\odot)^{5/3}$ in Newtonian gravity. A star composed of H-clusters is more stable than that composed of nuclear matter when $2.4 \times 10^{51}{\rm erg}(M/M_\odot) < 1.7 \times 10^{53}{\rm erg}(M/M_\odot)^{5/3}$, that is $M > 0.05M_\odot$. In reality, the general relativity effect should be taken into account, in which the effective gravity is stronger and then the conclusion would become firmer. Therefore when the mass of a star is much larger than this critical mass, the formation of H-clusters with higher densities is energetically favored. It is also possible that there could be normal matter surrounding a self-bound H-cluster star, but initially the surroundings would not remain because of energetic exploding [27–29].

Composed of non-relativistic H-clusters with interaction in the form of Eq.(1), quark stars could have stiff equation of state and high maximum mass. Under some certain range of parameters, the equation of state could be so stiff that the adiabatic sound speed $c_s = \sqrt{dP/d\rho}$ is larger than the speed of light, c. The probability that the speed of sound exceeding the speed of light in ultra-dense matter was studied extensively [30], and the issue regarding the causality and speed of sound was discussed from several theoretical points of view [31]. Microscopic theories consistent with special relativity prevent any real particle or signal moving faster than light, but ultrabaric matter with that adiabatic speed $c_s > c$ is not necessarily superluminal [32]. In Newtonian hydrodynamics, the value of c_s is related to the thermal velocity and reflects the thermodynamic properties of the medium, so taking c_s to be the signal propagation speed is meaningful. In the model that we use here, however, we do not consider the finite temperature effect, then the value of c_s coming from Eq.(3) and Eq.(4) has nothing to do with the thermodynamic properties of the system and does not reflect the dynamics of the medium. As a consequence, the adiabatic sound speed c_s in our model is not a dynamically meaningful speed, but reflects the local stiffness. The interaction is mediated by mesons, so the real speed of interaction is obviously finite, and the signal propagation speed remains subluminal.

H-particle has been the subject of many theoretical and experimental studies, and pulsar-like compact stars provide high density and relatively low temperature conditions in which quark matter with H-clusters could emerge. We can constrain on H-H interaction by observations; for example, if a pulsar with mass higher than $3M_\odot$ is found, then we would have $-V_0 > 60$ MeV. Certainly, besides equation of state, the highest mass of pulsars would also be meaningful for researches of γ-ray bursts (GRBs) and gravitational waves since GRB X-ray Flares may originate from massive pulsars produced by compact star mergers [33]. In addition, the peculiar nature of self bound surface and global rigidity of H-cluster stars would have profound implication for the studies of pulsar magnetosphere activity and compact star cataclysmic bursts [7], which could be tested by future observations. In conclusion, although the state of cold quark matter at a few nuclear densities is still an unsolved problem in low energy QCD, various pulsar phenomena would be helpful to study the nature of elemental strong interaction.

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