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Nullity of Measurement-induced Nonlocality

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Measurement-induced nonlocality (MiN) is a new measure of nonlocality introduced by Luo and Fu [Phys. Rev. Lett. **106**, 120401(2011)]. In this letter, we study MiN further and obtain necessary and sufficient conditions for a state to have nullity of measurement-induced nonlocality and for a state to be classical-quantum, in terms of commutativity, for both finite- and infinite-dimensional systems. These results reveal that MiN and quantum discord are raised from noncommutativity rather than entanglement. MiN is the most essential nonlocality among MiN, quantum discord and entanglement. The set of states with zero MiN is a proper subset of the set of zero discordant states, and both of them are zero-measure sets. Thus, there exist not only quantum nonlocality without entanglement but also quantum nonlocality without quantum discord.

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Quantum nonlocality, whereby particles of spatially separated quantum systems can instantaneously influence one another irrespective of the distance between them, is one of the most elusive features in quantum theory [1–3]. Different manifestations of it, such as quantum entanglement [4, 5], quantum discord (QD) [6, 7] and measurement-induced nonlocality (MiN) [8], have been studied. It is the key to our understanding of quantum physics, and, in particular, it is essential for the powerful applications of quantum information and quantum computation. Entanglement lies at the heart of quantum information theory [4, 5]. QD can be used in quantum computation [9, 10]. It is indicated in [8] that MiN may also be applied in quantum cryptography, general quantum dense coding [11, 12], remote state control [13, 14], etc.. The quantifying of nonlocality, for instance, entanglement measure (such as entanglement of formation [15, 16], concurrence [15, 16], Schmidt number entanglement measure [17], etc) and computation of quantum discord [9, 18–22], has been discussed intensively.

Measurement-induced nonlocality was firstly proposed by Luo and Fu [8], which can be viewed as a kind of nonclassical correlation from a geometric perspective based on the local von Neumann measurements from which one of the reduced states is left invariant. Let ρ be a bipartite state acting on the associated state space $H_A\otimes H_B$ with $\dim H_A\otimes H_B<+\infty$, $\rho_{A/B}=\mathrm{Tr}_{B/A}(\rho)$ be the reduced states of ρ. The MiN of ρ, denoted by $N(\rho)$, is defined in [8] by

$$N(\rho)=\max_{\Pi^A}\|\rho-\Pi^A(\rho)\|_2^2, \tag{1}$$

where $\|\cdot\|_2$ is the Hilbert-Schmidt norm (that is $\|A\|_2=[\mathrm{Tr}(A^\dagger A)]^{\frac{1}{2}}$), and the max is taken over all local von Neumann measurement $\Pi^A=\{\Pi_k^A\}$ with $\sum_k\Pi_k^A\rho_A\Pi_k^A=\rho_A$, $\Pi^A(\rho)=\sum_k(\Pi_k^A\otimes I_B)\rho(\Pi_k^A\otimes I_B)$. MiN is different from, and in some sense dual to, the *geometric measure of quantum discord*(GMQD) [8]

$$D_G(\rho):=\min_{\Pi^A}\|\rho-\Pi^A(\rho)\|_2^2$$

where Π^A runs over *all* local von Neumann measurements (GMQD is originally introduced in [9] as $D_G(\rho):=\min_\chi\|\rho-\chi\|_2^2$ with χ runs over all zero QD states and proved in [22] that the two definitions coincide). We recall that the quantum discord, which can be viewed as a measure of the minimal loss of correlation in the sense of quantum mutual information, is defined in [6] by

$$D(\rho)=\min_{\Pi^A}\{I(\rho)-I(\rho|\Pi^A)\}, \tag{2}$$

where the min is taken over all local von Neumann measurements Π^A. $I(\rho)=S(\rho_A)+S(\rho_B)-S(\rho)$ is interpreted as the quantum mutual information, where $S(\rho)=-\mathrm{Tr}(\rho\log\rho)$ is the von Neumann entropy, $I(\rho|\Pi^A)\}:=S(\rho_B)-S(\rho|\Pi^A)$, $S(\rho|\Pi^A):=\sum_k p_kS(\rho_k)$, and $\rho_k=\frac{1}{p_k}(\Pi_k^A\otimes I_B)\rho(\Pi_k^A\otimes I_B)$ with $p_k=\mathrm{Tr}[(\Pi_k^A\otimes I_B)\rho(\Pi_k^A\otimes I_B)]$, $k=1,2,\ldots,\dim H_A$. Throughout this paper, all logarithms are taken to base 2. QD is nonnegative [6, 21]. It is known that a state has zero QD if and only if it is a *classical-quantum*(CQ) state. Recall that a state ρ is said to be a CQ state if it has the form of

$$\rho=\sum_i p_i|i\rangle\langle i|\otimes\rho_i^B, \tag{3}$$

for some orthonormal basis $\{|i\rangle\}$ of H_A, where ρ_i^Bs are states of the subsystem B, $p_i\geq 0$, $\sum_i p_i=1$. Some conditions for nullity of quantum discord may be found in [9, 19, 21].

Mathematically, quantumness is always associated with noncommutativity while classical mechanics displays commutativity in some sense [23–25]. With this idea in mind, we describe quantum nonlocality mathematically. In this letter, we consider the two-mode system labeled by A+B which is described by a complex Hilbert space $H=H_A\otimes H_B$ with $\dim H_A\otimes H_B\leq+\infty$. We denote by $\mathcal{S}(H_A\otimes H_B)$ the set of all states acting on $H_A\otimes H_B$. The aim of this letter is twofold: (i) Analyzing the condition for quantum states having zero MiN in terms of commutativity mathematically; (ii) Comparing nullity of MiN with that of QD mathematically, which helps us understand better these different kinds of quantum correlations.

Firstly, with the same spirit as that of the finite-dimensional case, we can generalize MiN, QD and CQ states to infinite-dimensional bipartite systems straightforward.

Measurement-induced nonlocality- Let $\dim H_A\otimes H_B = +\infty$, $\rho\in\mathcal{S}(H_A\otimes H_B)$. Let $\Pi^A=\{\Pi_k^A=|k\rangle\langle k|\}$ be a set of mutually orthogonal rank-one projections that sum up to the identity of H_A. Similar to the finite-dimensional case, we call such $\Pi^A=\{\Pi_k^A\}$ a local von Neumann measurement. Note that $\sum_k(\Pi_k^A\otimes I_B)^\dagger(\Pi_k^A\otimes I_B) = \sum_k\Pi_k^A\otimes I_B = I_{AB}$, here the series converges under the strong operator topology [26]. We define the MiN of ρ by

$$N(\rho) := \sup_{\Pi^A}\|\rho-\Pi^A(\rho)\|_2^2, \tag{4}$$

where the supremum is taken over all local von Neumann measurement $\Pi^A=\{\Pi_k^A\}$ that satisfying $\sum_k\Pi_k^A\rho_A\Pi_k^A=\rho_A$, $\|\cdot\|_2$ denotes the Hilbert-Schmidt norm: $\|A\|_2=[\mathrm{Tr}(A^\dagger A)]^{\frac{1}{2}}$. The following properties are straightforward. (i) $N(\rho)=0$ for any product state $\rho=\rho_A\otimes\rho_B$. (ii) $N(\rho)$ is locally unitary invariant, namely, $N[(U\otimes V)\rho(U^\dagger\otimes V^\dagger)]=N(\rho)$ for any unitary operators U and V acting on H_A and H_B, respectively. (iii) $N(\rho)>0$ whenever ρ is entangled since $\Pi^A(\rho)$ is always a classical-quantum state and thus is separable. (iv) $0\le N(\rho)<4$. The MiN of a pure state is easily calculated. Let $|\psi\rangle\in H_A\otimes H_B$ and $|\psi\rangle=\sum_k\lambda_k|k\rangle|k'\rangle$ be its Schmidt decomposition. For the finite-dimensional case, Luo showed in [8] that $N(|\psi\rangle)=1-\sum_k\lambda_k^4$. It is also true for infinite-dimensional case. Similarly, one can define MiN with respect to the second subsystem B, and the corresponding properties are valid. It is easily seen that these two MiNs are asymmetric, namely, the MiN with respect to subsystem A is not equal to the one with respect to subsystem B generally. In this paper we consider the former one.

The quantum discord for infinite-dimensional systems was firstly discussed in [21]. For readers' convenience, we list it below again.

Quantum discord- Let $\dim H_A\otimes H_B=+\infty$, $\rho\in\mathcal{S}(H_A\otimes H_B)$. Let

$$I(\rho)=S(\rho_A)+S(\rho_B)-S(\rho)$$

denote the quantum mutual information of ρ, where $S(\rho)=-\mathrm{Tr}(\rho\log\rho)$ denotes the von Neumann entropy of the state ρ (remark here that $S(\rho)$ maybe $+\infty$). Let $\Pi^A=\{\Pi_k^A=|k\rangle\langle k|\}$ be a local von Neumann measurement. We perform Π^A on ρ, the outcome $\Pi^A(\rho)=\sum_k p_k\rho_k$, where $\rho_k=\frac{1}{p_k}(\Pi_k^A\otimes I_B)\rho(\Pi_k^A\otimes I_B)$ with probability $p_k=\mathrm{Tr}[(\Pi_k^A\otimes I_B)\rho(\Pi_k^A\otimes I_B)]$. Define $I(\rho|\Pi^A):=S(\rho_B)-S(\rho|\Pi^A)$ and $S(\rho|\Pi^A):=\sum_k p_kS(\rho_k)$. The difference

$$D(\rho):=I(\rho)-\sup_{\Pi^A}I(\rho|\Pi^A) \tag{5}$$

is defined to be the quantum discord of ρ, where the sup is taken over all local von Neumann measurement. It is proved in [21] that $D(\rho)\ge 0$ for any state $\rho\in\mathcal{S}(H_A\otimes H_B)$ (remark that it holds since the von Neumann entropy is strongly subadditive for both finite- and infinite-dimensional cases, see [21] for detail). One can check that QD can also be calculated by

$$D(\rho)=I(\rho)-\sup_{\Pi^A}I[\Pi^A(\rho)]. \tag{6}$$

Namely, QD is defined as the infimum of the difference of mutual information of the pre-state ρ and that of the post-state $\Pi^A(\rho)$ with Π^A runs over all local von Neumann measurements.

For finite-dimensional systems, the CQ states attracted much attention since they can be used for quantum broadcasting [27]. We extend it to the infinite-dimensional case via the same scenario.

Classical-quantum state- Similar to Eq.(3), for $\rho\in\mathcal{S}(H_A\otimes H_B)$, $\dim H_A\otimes H_B=+\infty$, if ρ has the following form

$$\rho=\sum_k p_k|k\rangle\langle k|\otimes\rho_k^B, \tag{7}$$

where $\{|k\rangle\}$ is an orthonormal set of H_A, ρ_k^Bs are states of the subsystem B, $p_k\ge 0$ and $\sum_k p_k=1$, then we call ρ a classical-quantum (briefly, CQ) state. It is clear that every CQ state has zero QD.

Let

$$\begin{aligned}\mathcal{S}_N^0&=\{\rho\in\mathcal{S}(H_A\otimes H_B):N(\rho)=0\},\\ \mathcal{S}_C&=\{\rho\in\mathcal{S}(H_A\otimes H_B):\rho\text{ is CQ}\},\\ \mathcal{S}_D^0&=\{\rho\in\mathcal{S}(H_A\otimes H_B):D(\rho)=0\}\end{aligned}$$

and $\mathcal{S}_{sep}$ be the set of all separable states acting on $H_A\otimes H_B$. Then

$$\mathcal{S}_N^0\subseteq\mathcal{S}_C\subseteq\mathcal{S}_D^0\subseteq\mathcal{S}_{sep}. \tag{8}$$

It is known that, for the finite-dimensional case, $\mathcal{S}_D^0$ is a zero-measure set [28] (that is, each point of this set can be approximated by a sequence of states that not belong to this set with respect to the trace norm), and, for the infinite-dimensional case, $\mathcal{S}_{sep}$ is a zero-measure set [29]. Thus $\mathcal{S}_N^0$ is a zero-measure set in both finite- and infinite-dimensional cases. (Consequently $\mathcal{S}_D^0$ is also a zero-measure set in infinite-dimensional cases, which answers a question raised in [21].)

Let $\dim H_A\otimes H_B\le+\infty$, and $\{|k\rangle\}$, $\{|i'\rangle\}$ be any orthonormal bases of H_A, H_B, respectively. Denote $F_{ij}=|i'\rangle\langle j'|$. Then, for any $\rho\in\mathcal{S}(H_A\otimes H_B)$, we can write ρ as

$$\rho=\sum_{i,j}A_{ij}\otimes F_{ij} \tag{9}$$

where A_{ij}s are trace-class operators acting on H_A and the series converges in the trace norm[30]. It is proved in [31] that, for any density matrix $\rho\in\mathcal{S}(H_A\otimes H_B)$ with $\dim H_A\otimes H_B<+\infty$, if $\rho=\sum_{ij}A_{ij}\otimes F_{ij}$ with A_{ij}s are mutually commuting normal matrices, then ρ

is separable. We will prove below that such state ρ is not only a separable state but also a CQ state. In fact, we show that ρ is a CQ state if and only if it has the above form. Moreover, this result is also valid for the infinite-dimensional cases.

Theorem 1. *Let* $\dim H_A \otimes H_B \leq +\infty$, $\rho \in \mathcal{S}(H_A \otimes H_B)$. *Write* $\rho = \sum_{ij} A_{ij} \otimes F_{ij}$ *as in Eq.(9) with respect to some given bases of* H_A *and* H_B. *Then* ρ *is a CQ state if and only if* A_{ij}*s are mutually commuting normal operators acting on* H_A.

Proof. The 'if' part. Assume that A_{ij}s are mutually commuting normal operators; then A_{ij}s are simultaneously diagonalizable. Thus there exist diagonal operators D_{ij}s and a unitary operator U acting on H_A such that $(U^\dagger \otimes I_B)\rho(U \otimes I_B) = \sum_{i,j} D_{ij} \otimes F_{ij}$. With no loss of generality, we may assume $\rho = \sum_{i,j} D_{ij} \otimes F_{ij}$. It turns out that ρ can then be rewritten as $\rho = \sum_i \tilde{E_{ii}} \otimes B_{ii}$, where $\tilde{E_{ii}}$s are rank-one orthogonal projections. Now it is obvious that $B_{ii} \geq 0$ since $\rho \geq 0$, $i = 1, 2, \ldots$. Hence, ρ is a classical-quantum state.

The 'only if' part. If ρ is a CQ state, then $\rho = \sum_k p_k |k\rangle\langle k| \otimes \rho_k^B$, $p_k \geq 0$, $\sum_k p_k = 1$ for some orthonormal set $\{|k\rangle\}$ of H_A. Write $\rho = \sum_{i,j} A_{ij} \otimes F_{ij}$ as in Eq.(9). If Π^A is the von Neumann measurement induced from $\{|k\rangle\langle k|\}$, then it follows from $\Pi^A(\rho) = \sum_k (|k\rangle\langle k| \otimes I_B)(\sum_k p_k |k\rangle\langle k| \otimes \rho_k^B)(|k\rangle\langle k| \otimes I_B) = \rho$ that $\sum_k (|k\rangle\langle k| \otimes I_B)(\sum_{i,j} A_{ij} \otimes F_{ij})(|k\rangle\langle k| \otimes I_B) = \sum_{i,j} A_{ij} \otimes F_{ij}$. This leads to $\sum_k |k\rangle\langle k|A_{ij}|k\rangle\langle k| = \sum_k \langle k|A_{ij}|k\rangle |k\rangle\langle k| = A_{ij}$ for any i, j, that is, every A_{ij} is a diagonal operator with respect to the same orthonormal base $\{|k\rangle\}$. Therefore, A_{ij}s are mutually commuting normal operators acting on H_A. □

Theorem 1 implies that CQ stems from noncommutativity not from entanglement. We can also find this kind of noncommutativity from another perspective: For finite-dimensional case, it is proved in [28] that if $\rho \in \mathcal{S}_C (= \mathcal{S}_D^0)$ then $[\rho, \rho_A \otimes I_B] = 0$. It is easy to check that this is also valid for infinite-dimensional systems as well:

Proposition 1. *Let* $\dim H_A \otimes H_B \leq +\infty$, $\rho \in \mathcal{S}(H_A \otimes H_B)$. *Then*

$$\rho \in \mathcal{S}_C \Rightarrow [\rho, \rho_A \otimes I_B] = 0. \tag{10}$$

That is, if $\rho = \sum_{ij} A_{ij} \otimes F_{ij}$ as in Eq.(9) with respect to some given bases of H_A and H_B and ρ is a CQ state, then $\rho_A = \sum_i A_{ii}$ commutes with A_{ij} for any i and j. So noncommutativity signals quantumness of the state. The converse is not true since for any state with maximal marginal state we have Eq.(10) holds in the finite-dimensional case [28]. One can check that it is not true for infinite-dimensional case, either.

Let us now begin to discuss the nullity of MiN. The following is the main result of this letter.

Theorem 2. *Let* $\dim H_A \otimes H_B \leq +\infty$, $\{|k\rangle\}$ *and* $\{|i'\rangle\}$ *be orthonormal bases of* H_A *and* H_B, *respectively, and* $\rho \in \mathcal{S}(H_A \otimes H_B)$. *Write* $\rho = \sum_{i,j} A_{ij} \otimes F_{ij} \in \mathcal{S}(H_A \otimes H_B)$ *as in Eq.(9) with respect to the given bases. Then* $N(\rho) = 0$ *if and only if* A_{ij}*s are mutually commuting normal operators and each eigenspace of* ρ_A *contained in some eigenspace of* A_{ij} *for all* i *and* j.

Proof. By the definition of $N(\rho)$, it is clear that the condition $N(\rho) = 0$ is equivalent to the condition that $\Pi^A(\rho) = \rho$ holds for any local von Neumann measurement that makes ρ_A invariant.

The 'if' part. If each eigenspace of ρ_A is a one-dimensional space, then $\rho_A = \sum_i p_i |i\rangle\langle i|$ for some orthonormal base $\{|i\rangle\}$ and $\{p_i\}$ with $p_i > 0$, $p_i \neq p_j$ if $i \neq j$. Obviously, for any local von Neumann measurement $\Pi^A = \{\Pi_k^A\}$, $\sum_k \Pi_k^A \rho_A \Pi_k^A = \rho_A$ implies that, for each k, $|k\rangle = |i\rangle$ for some i. Thus Π^A is introduced in fact by $\{|i\rangle\}$. Now it is clear that $\Pi^A(\rho) = \rho$ as every A_{ij} commutes with ρ_A. Denote by $E(\lambda^A) = \mathrm{Ker}(\lambda^A - \rho_A)$ the eigenspace of ρ_A with respect to the eigenvalue λ^A (here, $\mathrm{Ker}(X)$ denotes the kernel of the operator X). If $\dim \mathrm{Ker}(\lambda^A - \rho_A) \geq 2$ for some nonzero eigenvalue λ^A of ρ_A, then the restricted operator of ρ_A on $E(\lambda^A)$, denoted by $\rho_A|_{E(\lambda^A)}$, satisfying $\rho_A|_{E(\lambda^A)} = \lambda^A I_{E(\lambda^A)}$, where $I_{E(\lambda^A)}$ denotes the identity operator on $E(\lambda^A)$. As A_{ij}s are mutually commuting normal operators and each eigenspace of ρ_A contained in some eigenspace of A_{ij} for all i and j, $C_{ij} = A_{ij}|_{E(\lambda^A)} = \lambda^{(ij)} I_{E(\lambda^A)}$ for some eigenvalue $\lambda^{(ij)}$ of A_{ij} for any i and j. This leads to $\sum_k \Pi_k^A A_{ij} \Pi_k^A = A_{ij}$ for any local von Neumann measurement $\Pi^A = \{\Pi_k^A\}$ that doesn't disturb ρ_A locally, so we have $\Pi^A(\rho) = \rho$.

The 'only if' part. If $\Pi^A(\rho) = \rho$ for any local von Neumann measurement Π^A that leave ρ_A invariant, then Π^A satisfying $\sum_k \Pi_k^A A_{ij} \Pi_k^A = A_{ij}$ for any i, j. This forces that A_{ij}s are mutually commuting normal operators. We show that each eigenspace of ρ_A contained in some eigenspace of A_{ij} for all i and j. Or else, we may assume with no loss of generality that $\dim \mathrm{Ker}(\lambda^{(i_0 j_0)} - A_{i_0 j_0}) = 1$ and $\dim \mathrm{Ker}(\lambda^A - \rho_A) = 2$ for some nonzero eigenvalue $\lambda^{(i_0 j_0)}$ of $A_{i_0 j_0}$ and nonzero eigenvalue λ^A of ρ_A. It turns out that there must exist a orthonormal basis of $\mathrm{Ker}(\lambda^A - \rho_A)$, denoted by $\{|e_1\rangle, |e_2\rangle\}$, such that the Π_α^A induced from $\{|e_1\rangle, |e_2\rangle\}$ makes $A_{i_0 j_0}$ variant, i.e., $\sum_\alpha \Pi_\alpha^A A_{i_0 j_0} \Pi_\alpha^A \neq A_{i_0 j_0}$. A contradiction. □

Theorem 2 indicates that the phenomenon of MiN is a manifestation of quantum correlations due to noncommutativity rather than due to entanglement as well. And we claim that the commutativity for a state to have zero MiN is 'stronger' than that of zero discordant state. We illustrate it with the following example.

Example. We consider a $3 \otimes 2$ system. Let

$$\rho = \left(\begin{array}{ccc|ccc} a & \cdot & \cdot & e & \cdot & \cdot \\ \cdot & a & \cdot & \cdot & f & \cdot \\ \cdot & \cdot & b & \cdot & \cdot & g \\ \hline \bar{e} & \cdot & \cdot & c & \cdot & \cdot \\ \cdot & \bar{f} & \cdot & \cdot & c & \cdot \\ \cdot & \cdot & \bar{g} & \cdot & \cdot & d \end{array}\right).$$

(Here, dots denotes the vanished matrix elements.) It is clear that ρ is a CQ state for any positive numbers a, b,

c, d and complex numbers e, f, g that make ρ be a state. However, taking $\Pi^A = \{|\psi_i\rangle\langle\psi_i|\}_{i=1}^3$ with

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}, |\psi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}, |\psi_3\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix},$$

it is easy to see that $\sum_k \Pi_k^A \rho_A \Pi_k^A = \rho_A$ and $\Pi^A(\rho) \neq \rho$ whenever $e \neq f$. If $a+c = b+d$, one can easily conclude that $N(\rho) = 0$ if and only if $a = b$, $c = d$ and $e = f = g$. Hence, there are many CQ states with nonzero MiN.

The above example shows that, $\mathcal{S}_N^0$ is a proper subset of $\mathcal{S}_D^0$. In addition, for $0 \leq \epsilon \leq 1$, ρ_1, $\rho_2 \in \mathcal{S}_N^0$ do not imply $\epsilon\rho_1 + (1-\epsilon)\rho_2 \in \mathcal{S}_N^0$ in general, so $\mathcal{S}_N^0$ is not a convex set. Similarly, $\mathcal{S}_D^0$ (or $\mathcal{S}_C$) is not convex, either.

Furthermore, equivalent to Theorem 2, one can check that $N(\rho) = 0$ if and only if $\rho = \sum_k p_k |k\rangle\langle k| \otimes \rho_k^B$ (as in Eq.(7)) with the property that $\rho_k^B = \rho_l^B$ whenever $p_k = p_l$. Comparing with Eq.(7), we get a more transparent picture of these two different quantum correlations.

Reviewing the proof of Theorem 2, the following is clear:

Proposition 2. *Let* $\dim H_A \otimes H_B \leq +\infty$, $\rho \in \mathcal{S}(H_A \otimes H_B)$. *Suppose that each eigenspace of* ρ_A *is of one-dimension and* $\rho_A = \sum_k p_k |k\rangle\langle k|$ *is the spectral decomposition. Then the local von Neumann measurement* Π^A *that makes* ρ_A *invariant is uniquely (up to permutation) induced from* $\{|k\rangle\langle k|\}$, *and vice versa .*

In Ref.[8], for finite-dimensional case, the authors claim that $N(\rho)$ =0 for any classical-quantum state $\rho = \sum_k p_k |k\rangle\langle k| \otimes \rho_k^b$ whose marginal state $\rho^a = \sum_k p_k |k\rangle\langle k|$ is nondegenerate (here, a matrix A is said to be nondegenerate provided that each eigenspace of A is of one-dimension). It is also valid for infinite-dimensional case:

Corollary 1. *Let* $\dim H_A \otimes H_B \leq +\infty$ *and* $\rho \in \mathcal{S}_C$. *Then* $N(\rho) = 0$ *provided that each eigenspace of* ρ_A *is of one-dimension.*

Summarizing, the zero MiN states and the CQ states are characterized in terms of commutativity for both finite- and infinite-dimensional systems. We argue that MiN is the most essential nonlocality among MiN, QD and entanglement. They all originated from the *supposition* of the states (since for pure state ρ, it is separable if and only if $N(\rho) = D(\rho) = 0$). Nonlocality is ubiquitous: Almost all quantum states have quantum nonlocality. In other words, as a resource, we get more states valid in tasks of quantum processing based on nonlocality. Our results suggest many further studies and applications. An important issue, for example, is to discuss whether the relation $\mathcal{S}_C = \mathcal{S}_D^0$ holds for infinite-dimensional case as well (we conjecture that it is true). In addition, it is worthy to compare $N(\rho)$ with $D(\rho)$ and some entanglement measure (such as concurrence or entanglement of formation) for any arbitrarily given state ρ. Another interesting task is to investigate an analytical formula of $N(\rho)$ for an arbitrary state ρ for both finite- and infinite-dimensional cases.

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