Gauge invariance, correlated fermions, and Meissner effect in 2+1 dimensions

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Abstract

We present a 2+1 dimensional quantum gauge theory model with correlated fermions that is exactly solvable by bosonization. This model gives an effective description of partially gapped fermions on a square lattice that have density-density interactions and are coupled to photons. We show that the photons in this model are massive due to gauge-invariant normal-ordering, similarly as in the Schwinger model. Moreover, the exact excitation spectrum of the model has two gapped and one gapless mode. We also compute the magnetic field induced by an external current and show that there is a Meissner effect. We find that the transverse photons have significant effects on the low-energy properties of the model even if the fermion-photon coupling is small.

The possible violation of Landau's Fermi liquid theory in models of strongly interacting fermions has been an actively researched problem for many years. Interest in this topic quickly grew with the discovery of the cuprate high-temperature superconductors in 1986 [1], and the realization that these materials display many properties not described by Fermi liquid theory [2]. Early on, it was suggested that models of Hubbard-type capture the strongly correlated physics of cuprates [3, 4, 5, 6]. While it has proven very difficult to do reliable computations for two dimensional (2D) such models, an excellent theoretical understanding has been obtained for the corresponding one-dimensional (1D) cases. This understanding is largely based on models that are exactly solvable. A famous example is the *Luttinger model* [7, 8], which provides an effective description of interacting fermions on a 1D lattice, and which has become a prototype of non-Fermi-liquid behavior [9, 10].

It has been known for quite some time that fermions coupled to dynamical photons can have non-Fermi-liquid behaviour [11, 12]. One argument against this mechanism being relevant for real materials is the smallness of the fine-structure constant ($\alpha \approx \frac{1}{137}$), which governs the strength of interactions between matter and transverse photons. However, various scenarios have been proposed and explored in which effective photon-like gauge fields arise in the low-energy limit of models for strongly correlated fermions [13, 14, 15];

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see also [10]. In these instances, the effective coupling constant need not be small. The computations to explore this mechanism in 2D are usually based on approximations that are difficult to justify and, again, things are much better understood in 1D due to the existence of exactly solvable prototype models. For example, the (1+1)D quantum gauge theory obtained by minimally coupling the Luttinger model to dynamical photons is exactly solvable [16]. This model is a generalization of the *Schwinger model* [17] and, similarly as in the latter model [18], the photons have a non-zero mass that arises due to gauge-invariant normal ordering.

Despite a concerted effort, much remains to be understood when it comes to strongly correlated fermions in 2D. We believe that there is a need for models that are amenable to exact computations. One pioneer in this direction is Mattis who, already in 1987, presented a 2D interacting fermion model that can be mapped exactly to non-interacting bosons [19]. Despite its potential, Mattis' model did not receive much attention (notable exceptions are [20, 21, 22]). In recent work, we derived an effective model describing fermions on a square lattice with local interactions and close to half filling [23, 24, 25]. Since this model is similar to Mattis' [19], we refer to it as the *Mattis model*. In [26] we obtained a complete solution of the Mattis model, including all Green's functions. We also proved that the fermion two-point functions have algebraic decay with exponents that depend on the interaction strength, which is a hallmark of Luttinger liquids [9]. The Mattis model is not relativistically invariant but, as already noted in [19], this is natural for effective models of square-lattice fermions.

In this Letter, we present a (2+1)D quantum gauge theory obtained by coupling the Mattis model to a dynamical $U(1)$ gauge field. We refer to the latter as *photon field*, even though other physical interpretations are possible. Our main result is that this model is exactly solvable by bosonization. We also show that this model has some remarkable properties, including a non-zero photon mass due to gauge-invariant normal ordering, and a Meissner effect. We find that there is a renormalization of the *bare charge*, which determines the strength of fermion-photon interactions. Furthermore, the gauge theory contains a gapless mode that is strongly affected by the presence of the dynamical photons even if the fermion-photon coupling is weak. The idea of mapping fermions to free bosons has been used before to study 2D interacting fermions coupled to gauge fields [27, 28], but no exactly solvable fermion model was clearly identified before.

(*Notation*: $\mu, \nu = 0, +, -$ are space-time indices, $s, s' = +, -$ space indices, $r, r' = \pm$ chirality indices; the space-time metric signature is $(-, +, +)$; $x = (ct, \mathbf{x})$ are space-time coordinates with 2D positions $\mathbf{x} = (x^+, x^-)$; c is the velocity of light; $\partial_s = \partial/\partial x^s$ are spatial derivatives; $\psi^{(\dagger)}_{r,s}$ are standard fermion field operators; A_μ is the gauge potential, E_s are the electric field components, and $B = \partial_+ A_- - \partial_- A_+$ is the magnetic field. Common argument $\mathbf{x}$ of field operators are suppressed whenever possible.)

The relation of the Mattis model to square-lattice fermions is described using Figure 1, which shows the Brillouin zone corresponding to a square lattice (dashed large square) divided into regions of non-equal sizes. Close to half filling, mean field theory predicts that the system is partially gapped, and there is an underlying Fermi surface in the so-called *nodal* regions (the four tilted rectangles) [26]. We model this underlying surface by straight

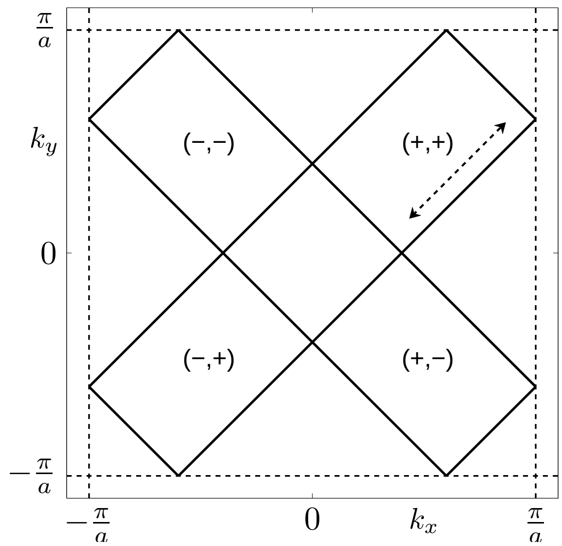

Figure 1: The Brillouin zone corresponding to a 2D square lattice with lattice constant a. The quantum gauge theory provides a low-energy description of fermions with momenta in one of the four tilted rectangles labeled by (r, s), $r, s = \pm$. Fermion degrees of freedom outside the rectangles are assumed to be gapped.

arcs, which either corresponds to a truncated Fermi surface or the portion of a closed Fermi pocket having dominant momentum occupation. The Mattis model describes the fermion degrees of freedom in the vicinity of these arcs. It is written in terms of four fermion field operators $\psi_{r,s}$, $r, s = \pm$, in one-to-one correspondence with the nodal regions. The quantum field theory limit making this model amenable to bosonization amounts to removing the momentum cutoff orthogonal to the arcs (indicated by the arrow in the nodal $(+, +)$-region, for example), which is possible after normal ordering [24]. The arc-picture underlying our derivation of the Mattis model is supported by renormalization group studies of weakly coupled 2D Hubbard-like systems [29]. It is also a signature feature of the pseudogap phase as observed in angle-resolved photoemission experiments on hole-doped cuprates [30].

The Hamiltonian of the Mattis model is

$$H_M = \sum_{r,s=\pm} \int \mathrm{d}^2 x \left(r v_F :\psi^\dagger_{r,s}(-\mathrm{i}\partial_s)\psi_{r,s}: + \sum_{r',s'=\pm} g_{r,s,r',s'} :\psi^\dagger_{r,s}\psi_{r,s}::\psi^\dagger_{r',s'}\psi_{r',s'}: \right) \tag{1}$$

with $\{\psi_{r,s}(\mathbf{x}), \psi^\dagger_{r',s'}(\mathbf{y})\} = \delta_{r,r'}\delta_{s,s'}\delta^2(\mathbf{x}-\mathbf{y})$, etc., and colons denoting standard fermion normal ordering with respect to a Dirac vacuum [26]. These definitions are formal since an important UV regularization has been suppressed: while the x_s-component of $\mathbf{x}$ in $\psi^{(\dagger)}_{r,s}(\mathbf{x})$ is continuous, the x_{-s}-component is discretized to integer multiples of a UV cutoff $\tilde{a}$, and thus the integrals and Dirac deltas have to be interpreted as partial Riemann sums and Kronecker deltas [26]. The coupling constants scale with the UV cutoff as $g_{r,s,r',s'} = \tilde{a}\pi v_F\big(\gamma_1\delta_{s,s'}\delta_{r,-r'} + \gamma_2\delta_{s,-s'}/2\big)$, with the Fermi velocity $v_F > 0$ and dimension-less constants $\gamma_{1,2}$ such that $|\gamma_1| < 1$ and $|\gamma_2| < |1+\gamma_1|$. The above scaling of the coupling constants is not only obtained by deriving the model from lattice fermions [24], but it also ensures that H_M has a non-trivial limit as $\tilde{a} \to 0$. The restrictions on $\gamma_{1,2}$ are to ensure stability of the Dirac vacuum [26].

The Hamiltonian of our quantum gauge theory model is obtained by coupling the Mattis Hamiltonian in (1) to a dynamical electromagnetic field,

$$
\begin{aligned}
H = \sum_{r,s=\pm} \int \mathrm{d}^2x \, \Big(r v_F \overset{\circ}{\circ} \psi^\dagger_{r,s}(-\mathrm{i}\partial_s + e_0 A_s)\psi_{r,s} \overset{\circ}{\circ} + \sum_{r',s'=\pm} g_{r,s,r',s'} \overset{\circ}{\circ} \psi^\dagger_{r,s}\psi_{r,s} \overset{\circ}{\circ}\overset{\circ}{\circ} \psi^\dagger_{r',s'}\psi_{r',s'} \overset{\circ}{\circ} \Big) \\
+ \frac{1}{2} \int \mathrm{d}^2x \overset{\times}{\times} \Big(E_+^2 + E_-^2 + c^2 B^2 \Big) \overset{\times}{\times}
\end{aligned}
\tag{2}
$$

with $[A_s(\mathbf{x}), E_{s'}(\mathbf{y})] = \mathrm{i}\delta_{s,s'}\delta^2(\mathbf{x}-\mathbf{y})$, etc., e_0 the bare charge, $\overset{\times}{\times} \cdots \overset{\times}{\times}$ boson normal ordering, and $\overset{\circ}{\circ} \cdots \overset{\circ}{\circ}$ a gauge-invariant generalization of fermion normal ordering (see below). The Gauss law operators generating gauge transformations $A_s \to A_s + \partial_s \chi$, etc., are

$$
G[\chi] = \int \mathrm{d}^2x \, \chi \sum_{s=\pm} \Big(-\partial_s E_s + e_0 \sum_{r=\pm} \overset{\circ}{\circ} \psi^\dagger_{r,s}\psi_{r,s} \overset{\circ}{\circ} \Big), \tag{3}
$$

and the physical states are those annihilated by $G[\chi]$, for arbitrary real-valued functions $\chi(\mathbf{x})$. Note that, except for the normal-ordering procedures, the Hamiltonian in (2) is obtained from the Mattis Hamiltonian by standard minimal coupling: introduce the action corresponding to the Mattis Hamiltonian in (1); couple to Abelian gauge fields using the substitution $-\mathrm{i}\partial_\mu \to -\mathrm{i}\partial_\mu + e_0 A_\mu$; add the Maxwell term $c^2 F_{\mu\nu}F^{\mu\nu}/4$; perform the usual Dirac procedure for systems with constraints [31] to obtain the formal Hamiltonian (i.e. without normal ordering) and the formal Gauss law constraint. We note in passing that the gauge field operators are well-defined without UV regularization (details will be spelled out elsewhere).

As already indicated, fermion normal ordering plays a key role for our model. Indeed, formulating a sensible quantization of the classical theory is non-trivial, even in the absence of gauge fields: In order for the Hamiltonian to be bounded from below (have a ground state), we need to normal-order all fermion bilinears with respect to the Dirac vacuum in which all negative energy states are occupied. An important consequence of normal-ordering is that the fermion densities $J_{r,s} \equiv :\psi^\dagger_{r,s}\psi_{r,s}:$ obey the anomalous commutator relations [32]

$$
[J_{r,s}(\mathbf{x}), J_{r',s'}(\mathbf{y})] = r\delta_{r,r'}\delta_{s,s'}(2\pi\mathrm{i}\tilde{a})^{-1}\partial_s \delta^2(\mathbf{x}-\mathbf{y}), \tag{4}
$$

and the non-interacting part of (1) can be expressed in terms of these densities using the operator identity

$$
\int \mathrm{d}^2x \; :\psi^\dagger_{r,s} r \left(-\mathrm{i}\partial_s\right) \psi_{r,s}: = \pi\tilde{a} \int \mathrm{d}^2x \overset{\times}{\times} J^2_{r,s} \overset{\times}{\times} \tag{5}
$$

(see Propositon 2.1 in [26] for proofs of these statements). The commutation relations in (4) imply that

$$
\partial_s \Phi_s = \sqrt{\pi\tilde{a}}\big(J_{+,s} + J_{-,s}\big), \qquad \Pi_s = \sqrt{\pi\tilde{a}}\big(-J_{+,s} + J_{-,s}\big) \tag{6}
$$

define boson operators obeying the usual canonical commutator relations $[\Phi_s(\mathbf{x}), \Pi_{s'}(\mathbf{y})] = \mathrm{i}\delta_{s,s'}\delta^2(\mathbf{x}-\mathbf{y})$, etc. It follows, using (5), that the Mattis Hamiltonian in (1) can be written in terms of free bosons, and this is the key step towards the exact solution of the Mattis model [26].

The gauge fields are quantized as usual by a partial gauge fixing $A_0 = 0$, postulating the canonical commutation relations (see below (2)), and imposing the Gauss law operator constraint on the Hilbert space [33]. However, the corresponding quantum Hamiltonian is *not* obtained as the straightforward quantization of the minimally coupled classical Hamiltonian: due to the anomalous commutators in (4), the Gauss law operators $G[\chi]$ would in this case no longer commute with the Hamiltonian, and the theory would thus not be gauge invariant. The remedy of this problem is to introduce a manifestly gauge-invariant normal-ordering prescription. We use the point-splitting method pioneered by Schwinger [32]: start with the gauge-invariant expression

$$\psi^\dagger_{r,s}(\mathbf{x} - \epsilon\mathbf{e}_s/2)\,\mathrm{e}^{\mathrm{i}e_0\int_{-\epsilon/2}^{\epsilon/2}A_s(\mathbf{x}+\xi\mathbf{e}_s)d\xi}\psi_{r,s}(\mathbf{x} + \epsilon\mathbf{e}_s/2), \tag{7}$$

which includes a line integral of the gauge field. We define our gauge-invariant fermion normal ordering of bilinears, $\circ\!\!\circ\, \psi^\dagger_{r,s}\psi_{r,s}\, \circ\!\!\circ$, as the limit $\epsilon \to 0$ of (7) after first subtracting off its singular part $r/(2\pi\mathrm{i}\tilde{a}\epsilon)$ (see Equation(4.9) in [26]). The result is

$$\circ\!\!\circ\, \psi^\dagger_{r,s}\psi_{r,s}\, \circ\!\!\circ = J_{r,s} + re_0A_s/(2\pi\tilde{a}), \tag{8}$$

and similarly

$$\int d^2x\ \circ\!\!\circ\, \psi^\dagger_{r,s}r\,(-\mathrm{i}\partial_s + e_0A_s)\,\psi_{r,s}\, \circ\!\!\circ = \int d^2x\left(:\psi^\dagger_{r,s}r(-\mathrm{i}\partial_s)\psi_{r,s}: + re_0A_sJ_{r,s} + \frac{e_0^2}{4\pi\tilde{a}}A_s^2\right) \tag{9}$$

(computational details will be provided elsewhere). An important feature of (9) is the "bare" photon mass term in the second line; as noted, this is a direct consequence of gauge-invariant normal-ordering. One can verify that (8) and (9) are gauge-invariant expressions.

The Hamiltonian and the Gauss law operators of the gauged model are now bosonized using the above results:

$$\begin{aligned} H = \frac{1}{2}\int \mathrm{d}^2x\ {}^\times_\times\Big(v_F\sum_{s=\pm}\Big[&(1-\gamma_1)(\Pi_s - e_RA_s)^2 + (1+\gamma_1)(\partial_s\Phi_s)^2 \\ &+\gamma_2(\partial_s\Phi_s)(\partial_{-s}\Phi_{-s})\Big] + E_+^2 + E_-^2 + c^2B^2\Big)\,{}^\times_\times \end{aligned} \tag{10}$$

and $G[\chi] = \int \mathrm{d}^2x\,\chi\sum_{s=\pm}\partial_s(-E_s + e_R\Phi_s)$, with the *renormalized* charge $e_R = e_0/(\sqrt{\pi\tilde{a}})$. Note that the scaling of the model parameters and boson fields are such that our gauge theory model remains well-defined in the UV limit $\tilde{a} \to 0^+$. The charge renormalization $e_0 \to e_R$ shows that photons can have stronger influence on physical properties than what superficial arguments might suggest. Under a gauge transformation, $A_s \to A_s + \partial_s\chi$, $E_s \to E_s$, $\Phi_s \to \Phi_s$ and $\Pi_s \to \Pi_s + e_R\partial_s\chi$, such that $\Pi_s - e_RA_s$, and thus H in (10), are gauge invariant.

The Hamiltonian in (10) is quadratic in boson operators and can therefore be diagonalized by a Bogoliubov transformation. To this end, we perform a Fourier transformation, $E_s(\mathbf{x}) \to \hat{E}_s(\mathbf{p})$, and define longitudinal- and tranverse fields by $|\mathbf{p}|\hat{E}_L(\mathbf{p}) = \mathrm{i}p_+\hat{E}_+(\mathbf{p}) + \mathrm{i}p_-\hat{E}_-(\mathbf{p})$ and $|\mathbf{p}|\hat{E}_T(\mathbf{p}) = \mathrm{i}p_+\hat{E}_-(\mathbf{p}) - \mathrm{i}p_-\hat{E}_+(\mathbf{p})$ (similarly for $\hat{A}_s$, $\hat{\Pi}_s$, and $\hat{\Phi}_s$). The Gauss law constraint then implies that $|\mathbf{p}|(-\hat{E}_L + e_R\hat{\Phi}_L)$ is zero on the physical space. Fixing the Coulomb gauge, $\hat{A}_L = 0$, and solving the Gauss law, $\hat{E}_L = e_R\hat{\Phi}_L$, we obtain the gauge-fixed Hamiltonian $H_{g.f.}$ describing transverse photons ($\hat{A}_T$) coupled to longitudinal- and transverse *plasmons* ($\hat{\Phi}_L$ and $\hat{\Phi}_T$, respectively). By straightforward methods, this Hamiltonian can be written in the diagonal form $H_{g.f.} = E_0 + \sum_{j=1,2,3}\sum_{\mathbf{p}} \omega_j(\mathbf{p}) b_j^\dagger(\mathbf{p}) b_j(\mathbf{p})$ with standard boson operators, i.e. $[b_j(\mathbf{p}), b_{j'}^\dagger(\mathbf{p}')] = \delta_{j,j'}\delta_{\mathbf{p},\mathbf{p}'}$, etc. (see e.g. Appendix C in [26] for further details), and the groundstate energy E_0. The exact dispersion relations $\omega_j(\mathbf{p})$ are computed from the eigenvalues of a certain 3×3 matrix. We obtain the following characteristic polynomial of this matrix whose zeros $\lambda = \lambda_j$ are equal to $\omega_j(\mathbf{p})^2$

$$\lambda(\lambda - \Theta^2 - c^2|\mathbf{p}|^2)(\lambda - \Theta^2 - \tilde{v}_F^2|\mathbf{p}|^2) + |\mathbf{p}|^4 S^2 v_-^2 (v_+^2(\lambda - c^2|\mathbf{p}|^2) - c^2\Theta^2) \tag{11}$$

with $v_\pm^2 = v_F^2(1-\gamma_1)(1-\gamma_1 \pm \gamma_2)/2$, $\tilde{v}_F^2 = v_F^2(1-\gamma_1^2)$, $\Theta = \sqrt{v_F(1-\gamma_1)}|e_R|$, and $S = |\sin(2\varphi)| = |2p_+p_-|/|\mathbf{p}|^2$ ($|\mathbf{p}|^2 \equiv p_+^2 + p_-^2$). We thus obtain two gapped modes with the same gap proportional to the renormalized charge: $\omega_1(\mathbf{0}) = \omega_2(\mathbf{0}) = \Theta$, whereas the third mode is gapless, $\omega_3(\mathbf{0}) = 0$. Moreover, for $c \gg v_F$ and $v_F^2|\mathbf{p}|^2 S \ll 1$, $\omega_1(\mathbf{p}) \approx \sqrt{\Theta^2 + c^2|\mathbf{p}|^2}$, $\omega_2(\mathbf{p}) \approx \sqrt{\Theta^2 + \tilde{v}_F^2|\mathbf{p}|^2}$, and

$$\omega_3(\mathbf{p}) \approx |\mathbf{p}|^2 S \sqrt{\frac{c^2 v_-^2(\Theta^2 + v_+^2|\mathbf{p}|^2)}{(\Theta^2 + c^2|\mathbf{p}|^2)(\Theta^2 + \tilde{v}_F^2|\mathbf{p}|^2)}}. \tag{12}$$

This shows that ω_1 gives the energy spectrum of the dressed transverse photon, whereas ω_2 and ω_3 give those of the dressed transverse- and longitudinal plasmons, respectively. Remarkably, the behavior of the last mode is *qualitatively* different for $e_R = 0$ and $e_R \neq 0$: in the former case, $\omega_3(\mathbf{p}) \approx |\mathbf{p}| S\sqrt{v_+v_-}/\tilde{v}_F$, which is equal to the lowest-energy mode of the Mattis model [26], whereas in the latter case, $\omega_3(\mathbf{p}) \approx |\mathbf{p}|^2 S c v_-/\Theta$ for $p < \Theta/c$. Thus the photons can affect the low-temperature thermodynamical properties of the system, no matter how small $|e_R| \neq 0$. For example, we found that the temperature (T) dependence of the specific heat at low T is linear for $e_R = 0$: $C_v \propto T$, but there are logarithmic corrections for $e_R \neq 0$: $C_v \propto T\ln(T_0/T)$ with $T_0 = (\pi/\tilde{a})^2 c\sqrt{v_-}/(1.423\Theta)$.

We also studied the magnetic field response to an external current J^μ with $J^0 = 0$, i.e., we computed the linear response function $\hat{K}(\omega,\mathbf{p})$ in the relation $\langle \hat{B}(\omega,\mathbf{p})\rangle = \hat{K}(\omega,\mathbf{p})|\mathbf{p}|\hat{J}_T(\omega,\mathbf{p}) + O(\hat{J}_T^2)$ ($|\mathbf{p}|\hat{J}_T$ is the Fourier transform of $\partial_+J_- - \partial_-J_+$). The exact result is

$$\hat{K}(\omega,\mathbf{p}) = \frac{\omega_+^2(\omega_+^2 - \Theta^2 - \tilde{v}_F^2|\mathbf{p}|^2) + v_-^2 S^2|\mathbf{p}|^2(\Theta^2 + v_+^2|\mathbf{p}^2|)}{(-\omega_+^2 + \omega_1(\mathbf{p})^2)(-\omega_+^2 + \omega_2(\mathbf{p})^2)(-\omega_+^2 + \omega_3(\mathbf{p})^2)} \tag{13}$$

with $\omega_+^2 \equiv (\omega + \mathrm{i}0^+)^2$ and $\omega_j(\mathbf{p})$ the dispersion relations given above. Expanding $\hat{K}(\omega,\mathbf{p})$ in partial fractions and inserting the long-distance approximations of $\omega_j(\mathbf{p})$ given above, we obtain for $c \gg v_F$,

$$\langle \hat{B}(\omega,\mathbf{p})\rangle \approx \frac{1}{-\omega_+^2 + c^2|\mathbf{p}|^2 + \Theta^2}|\mathbf{p}|\hat{J}_T(\omega,\mathbf{p}), \tag{14}$$

ignoring $O(\hat{J}_T^2)$-terms. This proves that there is a Meissner effect with a London penetration depth $\lambda_L = c/\Theta$. The question if our model has other features of a superconductor, e.g. a fermion condensate, is under investigation.

The exact solution of the model presented in this paper provides a reliable starting point to explore the physics of 2D correlated fermion systems. If the gauge field in this model is to be interpreted as a physical electromagnetic field, one should extend it to three dimensions. Furthermore, if the model is to describe correlated fermions in real materials, one should introduce spin degrees of freedom, and investigate if the gapped fermions affect the ones in the nodal regions. Exactly solvable extensions of our model addressing these remarks will be presented elsewhere.

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