Electron loss of fast heavy projectiles in collision with neutral targets

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The multiple electron loss of heavy projectiles in fast ion-atom collisions has been studied in the framework of the sudden perturbation approximation. Especially, a model is developed to calculate the cross sections for the loss of any number of electrons from the projectile ion, including the ionization of a single electron and up to the complete stripping of the projectile. For a given collision system, that is specified by the (type and charge state of the) projectile and target as well as the collision energy, in fact, the experimental cross sections for just three final states of the projectile are required by this model in order to predict the loss of any number, N, of electrons for the same collision system, or for any similar system that differs only in the energy or the initial charge state of the projectile ion. The model is simple and can be utilized for both, the projectile and target ionization, and without that large computer resources are requested. Detailed computation have been carried out for the multiple electron loss of Xe^{18+} and $U^{6+,\,10+,\,28+}$ projectiles in collision with neutral Ar and Ne gas targets.

I. INTRODUCTION

In recent years, the loss of electrons from heavy projectiles in fast collisions with neutral target atoms has attracted a great deal of interest because of its importance for the design of new heavy-ion accelerators and storage rings [1–3]. In such rings, any ionization or capture of electrons by the projectiles typically leads to a loss of the ions from the beam, thus limiting the intensities that are to be obtained eventually. For the International Facility of Antiproton and Ion Research (FAIR) at the GSI in Darmstadt, for example, reliable predictions are required especially for the multiple electron loss of fast uranium ions in collision with neutral rest gas atoms in order to estimate the beam intensities that can be achieved and manipulated at the SIS and the subsequent experimental storage rings [4, 5]. Indeed, many atomic and nuclear experiments that are planned for this facility will depend on the availability of having intensive beams with projectile energies of several ten MeV/u to a few GeV/u, and on the excellent control of all the beam parameters [6, 7].

For these reasons, several experiments have been performed in the past in order to measure and analyze the multiple loss of electrons for different projectiles and various rest gas and foil targets [8–11]. In particular, a series of measurements were carried out for highly-charged uranium ions in collision with neon, argon and krypton targets in order to make available a systematic set of total ionization cross sections [8, 9]. Most of these measurements were found in a reasonable-to-good agreement with Classical Trajectory Monte Carlo (CTMC) computations. Moreover, multiple and total (absolute) cross section have been measured also for 1.4 MeV/u uranium ions in collision with Ne, Ar and N_2 targets [10]. In these experiments, it was shown especially that, for 1.4 MeV/u $U^{4,6,10+}$ ions, the cross sections for a simultaneous loss of electrons first decrease rather slowly in magnitude if only up to about 10 electrons are stripped from the projectile, while they fall off much more rapidly for all higher final charge states of the projectiles. In addition, Watson *et al.* [11] have measured recently the electron loss cross sections for 6 MeV/u Xe^{18+} projectiles in collisions with (noble) He, Ne, Ar, Kr, and Xe gas targets.

From the viewpoint of atomic theory, of course, the main difficulty in predicting the (simultaneous) loss of several electrons in collisions of high-Z projectiles with light rest gas atoms or target material arises from the break-down of perturbation theory if the charge (state) of the final ions is increased successively. For U^{10+} ions, for example, a perturbation approach breaks down already for all collision energies smaller than about E=2.5 MeV/u. For the ion beams at the FAIR facility in Darmstadt, therefore, the Born approximation cannot be applied since its basic condition, $Z/v \ll 1$, is no longer fulfilled for high-Z projectiles even for rather moderate velocities. For this reason, it is necessary to explore further models that can be utilized beyond the Born approximation, and to estimate the total and differential

cross sections on more reliable grounds.

Until now, several computational models have been developed to calculate the multiple electron loss of fast projectiles in collision with different target materials: Apart from the classical trajectory Monte Carlo method [12] and the computer code Loss, implemented by Shevelko and coworkers [13], the (so-called) sudden perturbation approximation has been worked out recently [14–16] that enables one in an efficient way to estimate the inelastic transition amplitudes and cross sections in collisions of fast projectiles with different target atoms. In this previous work [16], however, only the active electrons were taken into account in the theoretical treatment, i.e. those which are ionized in course of the collision, ignoring the remaining charge density at the projectile. In the present work, we now release this restriction and account for all of the electrons in order to re-analyze the energy-dependence of different electron loss cross sections which are known from experiment. Moreover, we have extended the sudden approximation to take into account the multiple electron loss from both, either the projectile or the target ions, if some measured cross sections are known already for just three final states of the ions from prior experiments. In fact, the model described below is simple and can be utilized in order to predict the loss of any number, N, of electrons for either the same collision system (as specified by the type and incoming charge states of the projectile and target atoms as well as their collision energy), or for any similar system that only differs in the energy, the initial charge state of the projectiles, or in the choice of the target atoms.

The paper is organized as follows: In the next section, we first recall the basic ideas of the *sudden* (perturbation) approximation and how this method can be utilized in order to calculate the cross sections for a multiple electron loss from fast projectiles. During the last years, this method was developed especially for studying the multiple ionization processes of either the target and/or the projectiles and for the case, that the (orbital) motion of the bound electrons can be neglected when compared with the relative velocity of the collisions partners [16]. Apart from the transition amplitude for the projectile electron loss, this section also formulates a recipe how the cross section for a multiple loss of electrons can be predicted, if analogue cross sections are known already for the same collision partners but for a different charge state or energy of the projectiles ions. In Section III, this method is then utilized to calculate the multiple electron loss cross sections for fast uranium and xenon projectiles. Results are shown for different collision energies, several initial charges states of the projectile ions and for Ar and Ne targets, respectively. Finally, a few conclusions are drawn in section IV.

II. THEORETICAL BACKGROUND

Originally, the sudden perturbation approximation was developed to efficiently describe the multiple electron loss from target atoms in fast ion-atom collisions [16, 17]. This approximation is based on the assumption that a (not necessarely weak) time-dependent perturbation acts on a collision system for a time which is *much shorter* than the period of all the electrons to be ionized in course of the collision. Making use of this assumption, the transition amplitudes for the projectile ionization/excitation can be evaluated without that the time-dependent Schrödinger or Dirac equation would need to be solved explicitly for the ion-atom collision. In a first application, the sudden perturbation approximation was employed to the target excitation and ionization [17], including the simultaneous stripping of several electrons, but has been utilized more recently also in order to calculate the electron loss of heavy projectiles in fast collisions with neutral targets [14–16, 18]. In the present work, we aim to extent this formalism to the case that several or even all electrons are ionized simultaneously from the projectile and that all the electrons are taken into account in terms of their mean-field.

To describe quantitatively a multiple ionization of N electrons from fast projectiles, let us consider a collision system in which a projectile with charge Z_p and initially N_p ($\geq N$) electrons collides with a neutral target with charge Z_a and N_a electrons. In the following, moreover, let us assign to the electrons of the projectile and target the coordinates $\mathbf{r}_p$ ($p = 1, 2, ..., N_p$) and $\mathbf{r}_a$ ($a = 1, 2, ..., N_a$), and which are defined in the projectile and target frame, respectively. If the projectile nucleus moves with constant (relative) $\mathbf{v}$ and impact parameter (closest approach) $\mathbf{b}$ along a semi-classical trajectory, the interaction potential between the target and projectile can then be written as

$$V = -\sum_{a=1}^{N_a} \frac{Z_p}{|\mathbf{R}(t) + \mathbf{r}_a|} - \sum_{p=1}^{N_p} \frac{Z_a}{|\mathbf{R}(t) + \mathbf{r}_p|} + \sum_{p,a=1}^{N_p,\, N_a} \frac{1}{|\mathbf{R}(t) + \mathbf{r}_p - \mathbf{r}_a|}\,, \qquad (1)$$

if $\mathbf{R}(t) = \mathbf{b} + \mathbf{v}t$ denotes the (time-dependent) distance between the two nuclei, and where we make use of atomic units (if not stated otherwise). Obviously, this interaction potential consists of three terms where the first one describes the interaction of the target electrons with the nucleus of the projectile, the second —*vice versa*— the interaction between the projectile electrons and the target nucleus, and the third term finally the pairwise repulsion between the electrons with one belonging to the projectile and the other to the target, respectively.

In order to make use of the sudden approximation, we shall assume moreover that the (interaction) collision time t_c between the projectile and the target is much

shorter than the period t_s of the most inner (and fastest) electron to be ionized from the projectile, i.e.

$$t_c \ll t_s \,. \tag{2}$$

Since the collision time is roughly given by $t_c \approx a/v$, with $a \approx 1$ (in atomic units) being the characteristic size of the collision partners and $v = |\mathbf{v}|$ their relative velocity, and since the period of all orbital is $t_s \lesssim 1$ for all (positive) ions, the condition (2) becomes $t_c \approx 1/v \ll 1$ or, equivalently,

$$v \gg 1.$$

This means that, as mentioned before, the sudden approximation can be safely applied only if the relative projectile-target velocity is (much) larger than the velocity of the electrons to be ionized during the collision.

For a large enough distance of the two collision partners, of course, wave functions can be assigned independently to both, the projectile and target. Below, let us suppose to have a complete set of wave functions $\psi_k = \psi_k(\{\mathbf{r}_p\}) \equiv \psi_k(\mathbf{r}_1, \ldots, \mathbf{r}_{N_p})$ for the projectile electrons, and a certain number of wave functions (state vectors) $\varphi_n = \varphi_n(\{\mathbf{r}_a\}) \equiv \varphi_n(\mathbf{r}_1, \ldots, \mathbf{r}_{N_a})$ for the target, including with ψ_0, respectively, ϕ_0 the corresponding (and undisturbed) ground states in both cases. With this notation, the probability for any excitation or ionization of the target from state φ_0 to φ_n and the projectile from ψ_0 to ψ_k is given in perturbation theory by

$$w_{0\to k}^{0\to n} = |\langle \varphi_n \psi_k | \exp\left(-i \int_{-\infty}^{+\infty} V dt\right) |\psi_0 \varphi_0\rangle|^2. \tag{3}$$

Since, in the following, we are not further interested in the final state of the target, we then obtain the probability for a transition of the projectile from the ground state ψ_0 to some excited and/or ionized state ψ_k by a summation over all final states

$$W_{0\to k} \;=\; \sum_n w_{0\to k}^{0\to n} \,.$$

As shown in further detail in Ref. [16], this excitation-ionization probability of the projectile can be expressed as function of the impact parameter $\mathbf{b}$ as

$$W_{0\to k} = \langle \varphi_0 | \left| \langle \psi_k | \exp\left(-i \int_{-\infty}^{+\infty} U_a dt\right) |\psi_0\rangle \right|^2 |\varphi_0\rangle \,, \tag{4}$$

if we assume that the sudden perturbation approximation is valid and if the interaction potential

$$U_a = -\sum_{p=1}^{N_p} \frac{Z_a}{|\mathbf{R}(t) + \mathbf{r}_p|} + \sum_{p,a=1}^{N_p,N_a} \frac{1}{|\mathbf{R}(t) + \mathbf{r}_p - \mathbf{r}_a|} \,, \tag{5}$$

with $\mathbf{R}(t) = \mathbf{b} + \mathbf{v}t$, now includes only the interaction between the projectile electrons and the target nucleus as well as the interelectronic repulsion.

Despite of its approximate validity, the direct use of formula (4) is hardly feasible, especially if there are many electrons involved in the target and projectile. Therefore, in order to further simplify this expression, we shall assume in the following that the positions of the projectile electrons *do not* change with regard to the target nucleus during the time of the collision. In this case, it becomes possible to average over the interaction potential between the target nucleus and the projectile electrons and over the initial state of the target electrons. Here, moreover, we shall suppose also that the state of the target electrons are well described by means of the one-electron orbitals as obtained from the Dirac-Hartree-Fock model, forming a mean-field target density. Under these assumption a much simpler formula were derived earlier [16]

$$W_{0\to k} \;=\; \left| \langle \psi_k | \exp\left(-i \int_{-\infty}^{+\infty} \overline{U}_a dt\right) |\psi_0\rangle \right|^2 , \tag{6}$$

in which the averaged potential, $\overline{U}_a$, is given by

$$\begin{aligned} \overline{U}_a \;&=\; \langle \varphi_0(\{\mathbf{r}_a\}) | U_a | \varphi_0(\{\mathbf{r}_a\}) \rangle \\ &=\; -\sum_{p=1}^{N_p} \frac{Z_a}{|\mathbf{R}(t) - \mathbf{r}_p|} \sum_{i=1}^{3} A_i e^{-\alpha_i |\mathbf{R}(t) - \mathbf{r}_p|} \,, \end{aligned} \tag{7}$$

and where the second line was obtained by applying the parameterized Dirac-Hartree-Fock-Slater ground state wavefunctions [19]. Let us note here that Eq. (6) formally coincides also with Glauber approximation in which the energy differences between the projectile states are neglected and the target was supposed to be frozen in its initial state (see Ref. [20] for further details). In Eq. (7), moreover, the averaged potential $\overline{U}_a$ does no longer depend on the coordinates $\{\mathbf{r}_a\}$ of the target electrons but only on some tabulated constants A_i and α_i as listed for different atoms in Ref. [19]. In fact, taking the average in Eq. (7) implies that the target has a ground-state charge density $-\rho_a(r)$ of the form [16]

$$\rho_a(\mathbf{r}) \;=\; \frac{Z_a}{4\pi|\mathbf{r}|} \sum_{i=1}^{3} A_i {\alpha_i}^2 e^{-\alpha_i |\mathbf{r}|} \,, \tag{8}$$

as parameterized by means of the Dirac-Hartree-Fock-Slater model.

Using Eq. (8), it can be shown [17] moreover that the integral in Eq. (6) can be written also as a sum of 'eikonal phases' of the projectile electrons

$$\int_{-\infty}^{+\infty} \overline{U}_a dt = \sum_{p=1}^{N_p} \chi(\mathbf{b}, \mathbf{r}_p) \,, \tag{9}$$

with the functions $\chi(\mathbf{b}, \mathbf{r}_p)$ given by

$$\chi(\mathbf{b}, \mathbf{r}_p) \;=\; -\frac{2Z_a}{v} \sum_{i=1}^{3} A_i \, K_0(\alpha_i |\mathbf{b} - \mathbf{s}_p|) \,. \tag{10}$$

In this formula, $\mathbf{s}_p$ denotes the projection of $\mathbf{r}_p$ onto the plane that is perpendicular to the velocity (vector) of the projectile and K_0 is the lowest-order McDonald function. The (total) cross section for the ionization of the projectile, and averaged over all states of the target electrons, is then obtained by integrating the probability in Eq. (6) over all impact parameters

$$\sigma \;=\; \int d^2\,\mathbf{b}\; W(b) \;\equiv\; \int d^2\,\mathbf{b}\; \sum\!\!\!\!\!\!\int_{[k]} W_{0\to k}\,, \qquad (11)$$

and where the restricted summation (integration) over $[k]$ runs over those final state of the projectile electrons, where a given number of electrons, N, have left the projectile. In Eq. (11), this means that we are not interested in the momenta of the outgoing electron but only in the dependence of the cross sections on the charge Z_p and the (relative) velocity v for just the simultaneous loss of N electrons from the projectile ions.

Using Eqs. (6)-(11), the cross section for a single-electron loss has been calculated especially for hydrogen-like Pb^{81+} and Au^{78+} projectiles in collision with neutral target atoms at collision energies of 160 GeV/u and 10.8 GeV/u, respectively [15]. Based on these equations, moreover, a method was developed for calculating the 'energy loss' of fast, heavy projectiles in collision with neutral targets [21].

In principle, Eq. (11) could be applied together with the probabilities in Eq. (6) in order to compute the cross sections for the loss of electrons from the projectile and for both, a single or multiple ionization of electrons. In practice, however, this is still hardly feasible for many-electron projectiles since the summation over $[k]$ in Eq. (11) cannot be carried out explicitly in this case. Nevertheless, both equations (6) and (11) can be utilized in order to obtain an expression that relates the cross section for the loss of $N < N_p$ electrons, σ^{N+}, to the cross section σ^{N_p+} for the simultaneous ionization of all electrons. Together with the assumptions from above, it was shown especially in Ref. [16] that the probability $W^{N+}(\mathbf{b})$ for the loss of N electrons at a given impact parameter $\mathbf{b}$ can be written in terms of the single-electron form factors $p_i(\mathbf{b})$ as

$$W^{N+}(\mathbf{b}) \;=\; \frac{N_p}{(N_p-N)!N!} \prod_{i=1}^{N_p-N} p_i(\mathbf{b}) \prod_{j=N_p-N+1}^{N_p} (1-p_j(\mathbf{b}))\,, \qquad (12)$$

and where these form factors are given by

$$p_i(\mathbf{b}) \;=\; \int d^3k_i \left| \int d^3r_i \Psi^*_{\mathbf{k}_i}(\mathbf{r}_i) \exp\{-i\chi_i(\mathbf{b},\mathbf{r}_i)\}\phi_i(\mathbf{r}_i) \right|^2 . \qquad (13)$$

Following Ref. [16], we next replace the single-electron form factors by those as obtained from the average over all electrons with orbital angular momentum l and magnetic projection m of the given shell with principal quantum number n,

$$p(b) \;=\; \frac{1}{n_0}\sum_{n=1}^{n_0}\frac{1}{n^2}\sum_{l,m}\int d^3k \left| \int d^3r\, \Psi^*_{\mathbf{k}} \exp\{-i\,\chi(\mathbf{b},\mathbf{r})\}\phi_{nlm}(\mathbf{r}) \right|^2 , \qquad (14)$$

and which has the advantage to depend only on the modulus b of the impact parameter. By taking this average, the probability $W^{N+}(\mathbf{b})$ for the loss of N electrons can then be expressed as

$$W^{N+}(b) \;=\; \frac{N_p}{(N_p-N)!N!}\sum_{m=0}^{N}(-1)^m \frac{N!}{(N-m)!m!}\,\{p(b)\}^{N_p-N+m}\,, \qquad (15)$$

while the integration over the impact parameter b gives finally rise to

$$\sigma^{N+} \;=\; \frac{N_p!\,\sigma^{N_p+}}{(N_p-N)!\,N!}\sum_{m=0}^{N_p-N}(-1)^m \left(\frac{Z_p+N_p}{Z_p+N+m}\right)^2 \frac{(N_p-N)!}{(N_p-N-m)!\,m!}\left(\frac{N_p}{N+m}\right)^{\kappa}\{p\,(b_0,\,E)\}^{\,N-N_p+m}. \qquad (16)$$

In this cross section expression for the loss of N electrons, b_0 hereby refers to the value of the impact parameter for which the inelastic form factor takes its maximum, and κ is a quantity that characterizes the behavior of the function $p(b)$ near to this maximum, while $\Psi_{\mathbf{k}}$ is the final-state wave function of the projectile electron.

In general, neither the total cross section σ^{N_p+} for the loss of all electrons, nor the characteristic exponent κ, nor the inelastic form factor $p(b)$ are known with sufficient accuracy from (ab-initio) theory in order to make direct use

of Eq. (16) and to calculate from it the cross sections for the loss of N electrons. However, since for any given collision energy E, all the cross sections σ^{N+} $(N = 1, ..., N_p)$ only depend on these three quantities, Eq. (16) can be utilizes together with three experimentally known cross sections in order to determine (numerically) the values of σ^{N_p+}, κ and $p(b_0, E)$ within the framework as outlined above. From these values, then, all the other cross sections can be easily determined by applying Eq. (16). While, for such a set of non-linear equations (16), formally of course quite different solutions may exist for σ^{N_p+}, κ and $p(b_0, E)$, all these parameters should be real and positive for physical reasons. To our experience and up to the present, there has been found only *one* solution which fulfills this requirement.

More often than not, three cross sections σ^{N_1+}, σ^{N_2+} and σ^{N_3+} are either know or can be measured for a given collision system. For the collision of U^{28+} projectiles with neutral argon (rest gas) atoms, for example, the cross section for the loss of 1, 2, and up to 15 electrons have been measured in the Ref. [12], and were compared with the results of LOSS code calculations and the classical trajectory Monte Carlo method. By applying the method above, moreover, further cross sections for the multiple loss of electrons have been calculated also for U^{28+} and U^{10+} projectiles [16]. Here, we shall not follow these prior computational lines further but extend the method in order to apply it for different collision energies and/or targets.

To do so, let us note first that the cross section for the loss of N_p electrons is for $N_p \gg 1$ proportional to the N_p-th power of the one-electron inelastic form factor, $\sigma^{N_p+} \sim p(b, E)^{N_p}$ [16]. Therefore, the ratio

$$\frac{\sigma^{N_p+}}{[p(b_0)]^{N_p}} = a \qquad (17)$$

is found to be independent not only from the energy and the initial charge state of the projectile but also with regard to the type of the target atoms. To make further use of this observation, let us introduce the term 'collision system' in order to denote a particular reaction scenario with given (type and charge state of the) projectile and target atoms as well as with given collision energy. If, for such a system, the cross sections are known for the loss of *three* different numbers of electrons, Eq. (16) can be utilized (as said before) to determine the cross sections also for any other number of electrons. In addition, we can use the cross section of some collision system A to determine the cross section of any other collision system B, provided that *two* cross sections are known already for this system. The third cross section value for system B is then simply obtained from the relation (17) or, equivalently, from

$$\frac{\sigma_A^{N_p+}}{[p_A]^{N_p}} = \frac{\sigma_B^{N_p+}}{[p_B]^{N_p}} . \qquad (18)$$

In Section III, we shall apply this recipe to derive the cross section for the loss of N electrons for systems for

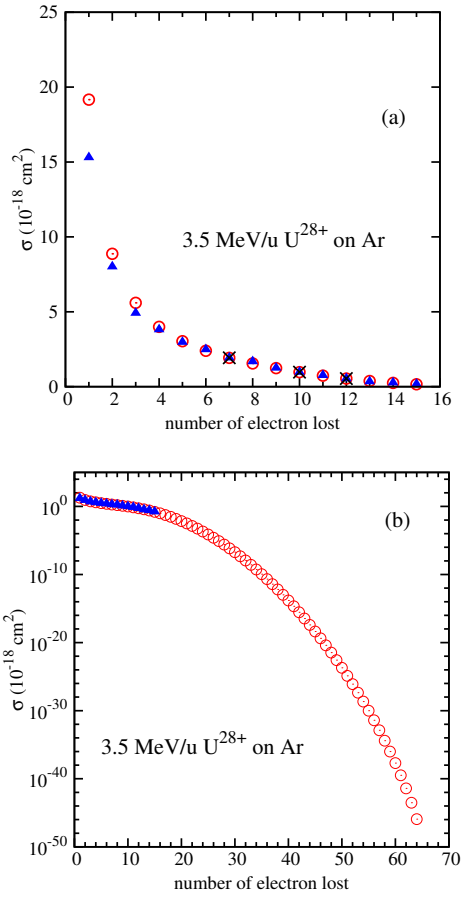

FIG. 1: (Color online): Cross sections (in 10^{-18} cm^2) for a multiple loss of projectile electrons in collisions of U^{28+} ion with neutral Ar atoms at collision energy E = 3.5 MeV/u, and as function of the number N of lost electrons; (a) for $N = 1, ..., 15$ and (b) for $N = 1, ..., 64$. Computations from this work (open circles) are compared with the experimental data (filled triangles) from Ref. [12]. The crosses indicate the experimental data that were utilized for the computations.

which only two cross sections are (supposed to be) known from experiment. The validity of this approach can be then tested easily by comparing the cross section with the data as obtained from the prior knowledge of 'three' cross sections for the system. Again, the determination of the three parameters σ^{N_p+}, κ and $p(b_0, E)$ then follows lines similar as discussed above and by making use of the relation (17) as the additional (third) equation.

Finally, if two collision systems A and B differ only by the (collision) energy $E_A \neq E_B$, we have the additional condition $\kappa_A = \kappa_B$, and the prescription above simplifies considerably. In this case, only *one* experimental cross section need to be known for the energy E_B, while the other two cross sections (parameters in Eq. (16)) can be utilized from the collision of the projectile and target at the energy E_A. In the next section, results will be shown and discussed especially for the multiple electron loss of xenon and uranium ions in collision with different target

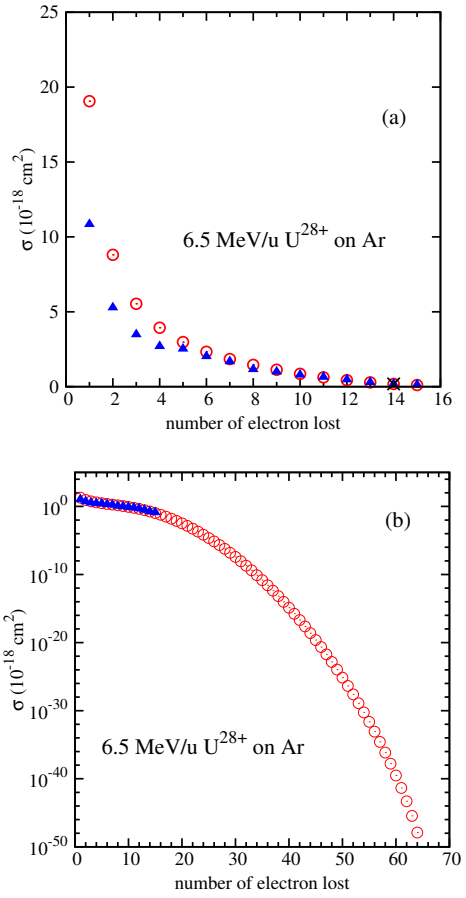

FIG. 2: (Color online): The same as Fig. 1 but for the collision energy E = 6.5 MeV/u. For the theoretical cross sections, only the single experimental value for the loss of $N = 14$ electrons has been utilized at this energy; see text for discussion.

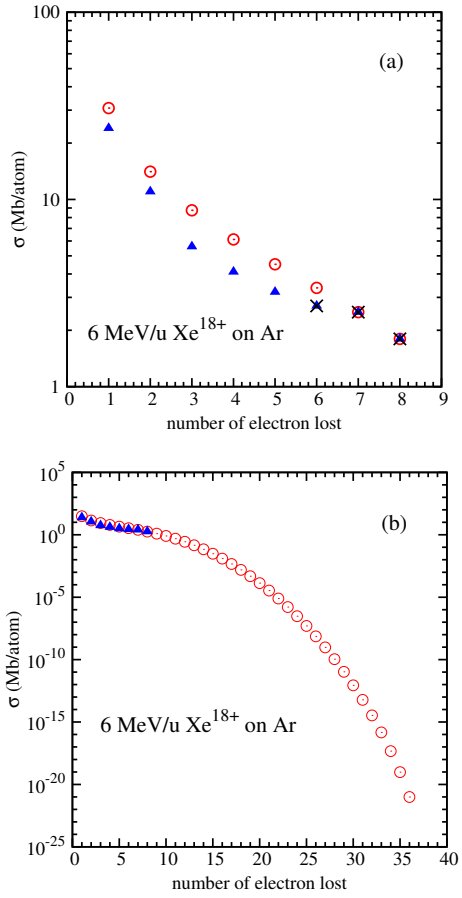

FIG. 3: (Color online): The same as in the Figs. 1 but for the collision systems $Xe^{18+} \rightarrow$ Ar and the energy E = 6 MeV/u; (a) for $N = 1, ..., 8$ and (b) for $N = 1, ..., 36$.

materials. These ions are important for the design of the FAIR facility in Darmstadt, and most experimental data are available for them.

III. RESULTS AND DISCUSSION

Cross sections for the electron loss of fast, heavy projectiles have been measured for a number of collision systems. For example, Olson *et al.* [12] display measured cross sections for the electron loss of up to 15 electrons for collisions of U^{28+} ions with neutral Ar atoms. These cross sections are shown in Fig. 1(a) for the projectile energy E = 3.5 MeV/u and in Fig. 2(a) for 6.5 MeV/u, respectively, and are compared with our theoretical prediction following the prescription above. To generate these theoretical values, the experimental cross sections for the (simultaneous) loss of 7, 10, and 12 electrons have been utilized [cf. crosses in Figs. 1(a)]. Theoretical data are shown for the loss of $N = 1, ..., 15$ electrons in Fig. 1(a) and for $N = 1, ..., 64$ in Fig. 1(b). Very similar results within a few percent were obtained if just three other experimental cross sections are applied. In general, however, the cross sections for a loss of $N < 5$ electrons should not be used in order to obtain the cross sections for large N because they often appear more sensitive to many-electron effects that are not incorporated into the model. Fig. 2 displays analogue cross sections for $U^{28+} \rightarrow$Ar collisions but for the collision energy E = 6.5 MeV/u. For the theoretical cross sections in Fig. 2, only the (single) cross section for the loss of $N = 14$ electrons have been utilized from the experiments at 6.5 MeV/u, while the other information were obtained by using the ratio (17) and κ from the data for E =3.5 MeV/u in Fig. 1. Typically, good agreement between theory and experiment is found, and only the cross sections for the loss of just a very few electrons ($N = 1, ..., 4$) are slightly overestimated by our model. As expected, the electron-loss cross sections for the collision energy 6.5 MeV/u are larger than for 3.5 MeV/u, independent of how many electrons are lost from the projectile.

Our model from above works well also if we consider two collision systems that differ in the target atom. This is seen from Figs. 3-4 which display the projectile

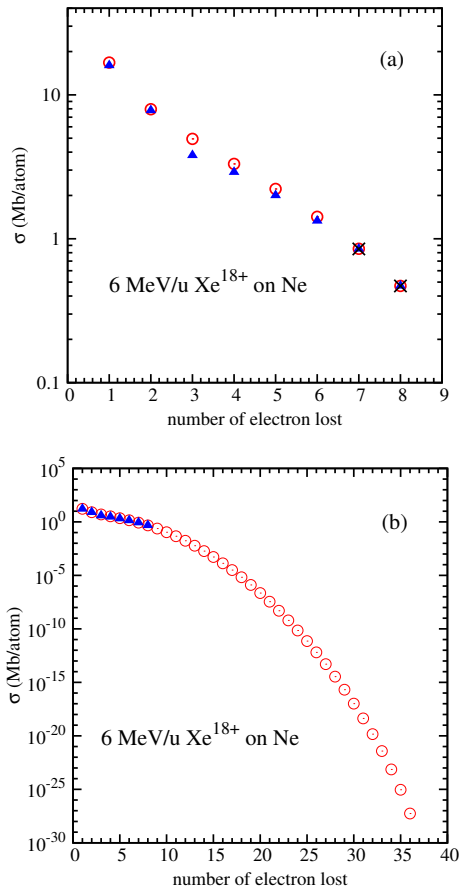

FIG. 4: (Color online): The same as in the Fig. 3 but for the collision systems $Xe^{18+} \rightarrow Ne$ and the energy E = 6 MeV/u; (a) for $N = 1, ..., 8$ and (b) for $N = 1, ..., 36$.

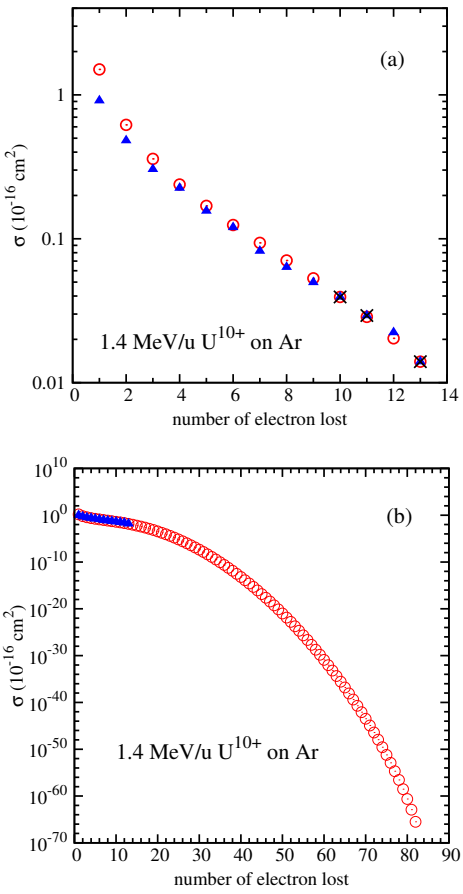

FIG. 5: (Color online): The same as in the Fig. 3 but for the collision systems $U^{10+} \rightarrow Ar$ and the collision energy E = 1.4 MeV/u; (a) for $N = 1, ..., 13$ and (b) for $N = 1, ..., 82$. Theoretical cross sections are compared with the measurements by DuBois *et al.* [10] as far as available.

electron-loss cross sections for the two systems $Xe^{18+} \rightarrow Ar$ (cf. Figs. 3(a) and 3(b)) and $Xe^{18+} \rightarrow Ne$ (cf. Figs. 4(a) and 4(b)) at the collision energy E = 6 MeV/u. In Fig. 3 (a,b), again, the theoretical cross sections are based on three measured data by Watson *et al.* [11] for the loss of 6, 7, and 8 electrons from the Xe^{18+} projectile, while the ratio (17) and only two cross sections (for the loss of 7 and 8 electrons) are used in Figs. 4(a,b). The theoretical cross sections for $Xe^{18+} \rightarrow Ar$ collisions are slightly overestimated for the loss of just a few electrons but are in excellent agreement with experiment for the $Xe^{18+} \rightarrow Ne$ collisions in Figs. 4(a,b). In Figs. 5 and 6, theoretical results are shown for the collision of $U^{10+} \rightarrow Ar$ and $U^{6+} \rightarrow Ar$ at the energy E = 1.4 MeV/u, i.e. for two different initial charges states of the projectile. Experimental cross sections from Ref. [10] were utilized in order to derive the theoretical data. Very good agreement between theory and the experiments by DuBois *et al.* [10] is found in both cases.

So far, we have always considered the multiple loss of electrons from the projectile ions in collision with different target atoms and for various collision energies. As mentioned before, the same model can be utilized also to predict the ionization of the target. For this, only the nuclear charge $Z_p \leftrightarrow Z_a$ and the (initial) number of bound electrons $N_p \leftrightarrow N_a$ need to be interchanged in Eq. (17), while the rest of the computational procedure remains rather unchanged. In Fig. 7, our theoretical prescription is applied to predict the cross sections for the multiple loss of target electrons from argon in $U^{28+} \rightarrow Ar$ collisions at E = 15 MeV/u. Cross sections are displayed for single ionization and up to the complete (18-fold) stripping of all electrons from Ar. The theoretical data are compared with the experiments by Olson *et al.* [12] and are found in excellent agreement. This confirms in a practical manner that our model can be applied for the multiple loss of electrons from both, the projectile and target.

IV. CONCLUSIONS

The multiple electron loss of heavy projectiles in fast ion-atom collisions has been investigated. Based on the

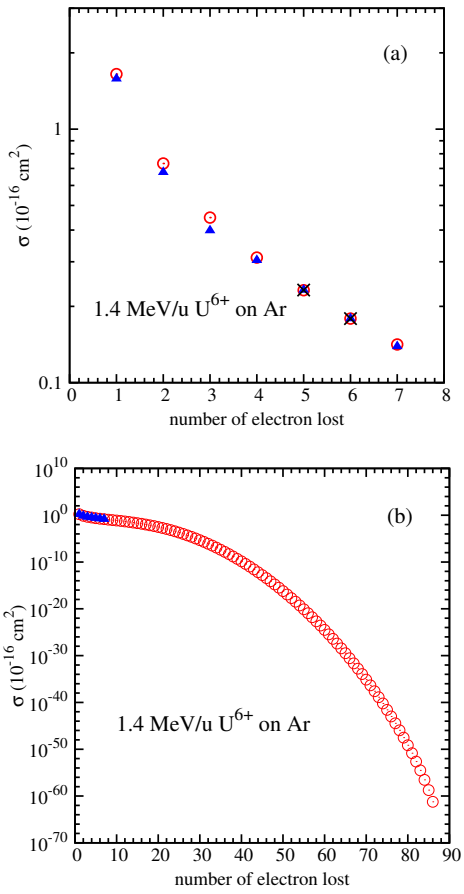

FIG. 6: (Color online): The same as in the Fig. 5 but for the collision systems $U^{6+} \rightarrow$ Ar and the collision energy E = 1.4 MeV/u; (a) for $N = 1, ..., 7$ and (b) for $N = 1, ..., 86$. Theoretical cross sections are compared with the measurements by DuBois *et al.* [10] as far as available.

sudden perturbation approximation, a model is developed to estimate the cross sections for a multiple loss of electrons from both, the projectile and target atoms, and up to their complete ionization. In this model, only three (measured) cross sections are needed from experiment in order to predict the loss of any number N of electrons for a given collision system. Moreover, the model can be applied to different projectiles and targets if the cross sections for the loss of at least two different numbers of electrons are known, while the third cross section value can be taken from any other system. Only a single measured cross section is needed, moreover, if (two) other values for the same projectile-target collision system are known at some different collision energy. By making use of this model, calculations have been carried out especially for the multiple electron-loss cross sections of $U^{28+} \rightarrow$ Ar, $U^{10+} \rightarrow$ Ar, $U^{6+} \rightarrow$ Ar as well as for $Xe^{18+} \rightarrow$ Ar and $Xe^{18+} \rightarrow$ Ne, and for different energies. The model is simple and can be utilized without that large computer resources are required.

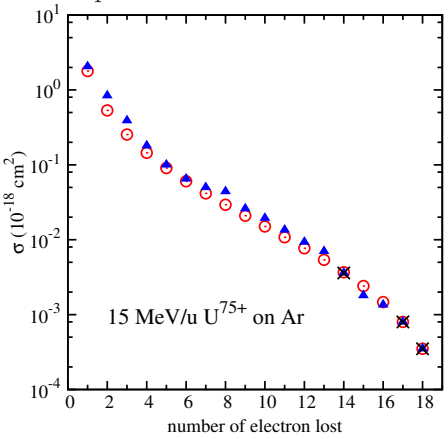

FIG. 7: (Color online): Cross sections (in 10^{-18} cm^2) for a multiple loss of *target* electrons in $U^{28+} \rightarrow$ Ar collisions at E = 15 MeV/u, and as function of the number N of lost electrons. The notation is the same as in Fig. 1. Computations from this work are compared again with the experimental data from Ref. [12].

ACKNOWLEDGMENTS

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